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New modified and extended stability functions for the stable boundary layer based on SHEBA data

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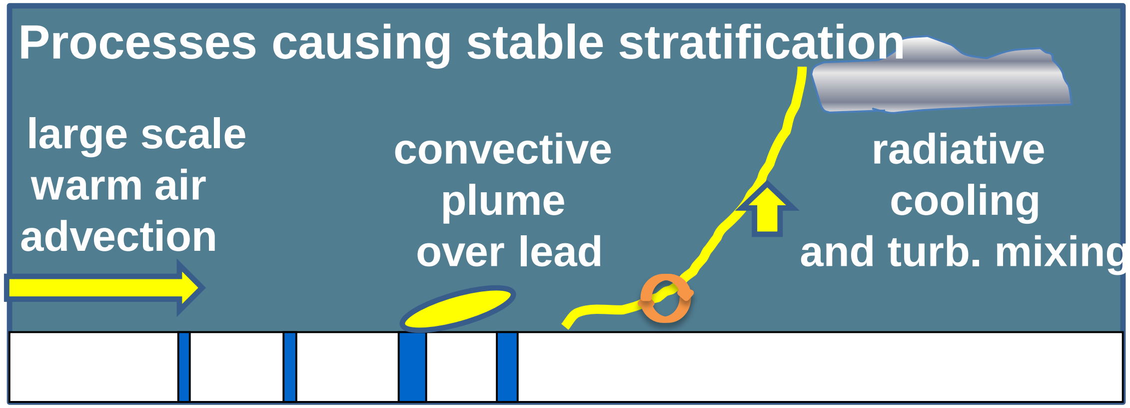
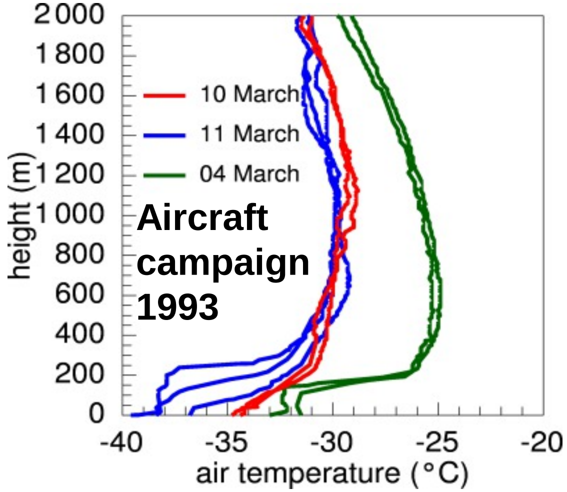
Summary

New stability functions are proposed for the stable boundary layer, which are based on the SHEBA measurements but avoid the complexity of the SHEBA functions proposed by Grachev et al. (2007). The new functions are superior to the former ones with respect to the representation of the measured relationship between the Obukhov length and the bulk Richardson number. Moreover, the resulting transfer coefficients agree slightly better with the SHEBA observations in the very stable range. Nevertheless, the functions fulfill the same criteria of applicability as the earlier functions and contain furthermore as an extension a dependence on the neutral Prandtl number. Applying the new functions, an efficient non-iterative parametrization of the near-surface turbulent fluxes of momentum and heat is developed where transfer coefficients result as a function of the bulk Richardson number (Ri_b) and roughness parameters. The new transfer coefficients, which are recommended for weather and climate models, agree well with the SHEBA data in a large range of stability ($0< Ri_b<0.5$) and with those based on the Dyer-Businger functions in the range $Ri_b <0.08$.

Stable stratification is a common feature of the atmospheric boundary layer over polar sea ice



measured temperature



Examples of measured temperature profiles (AWI aircraft campaign)

Calculation of fluxes based on Monin Obukhov similarity theory (MOST)

$$M = -C_d U^2$$ **momentum flux**

$$H = -\rho c_p C_h U [\Theta(z) - \Theta_s]$$ **heat flux**

$$C_d = C_{dn} f_m \qquad C_h = C_{hn} f_h$$ **transfer coefficients**

Normalized stability dependent transfer coefficients

$$f_m = \left[1 - \frac{\psi_m(\zeta) - \psi_m(\zeta/\varepsilon)}{\ln \varepsilon}\right]^{-2}$$

$$f_h = \left[1 - \frac{\psi_m(\zeta) - \psi_m(\zeta/\varepsilon)}{\ln \varepsilon}\right]^{-1} \left[1 - \frac{\psi_h(\zeta) - \psi_h(\zeta/\varepsilon_t)}{\ln \varepsilon_t}\right]^{-1}$$

$\varepsilon = z / z_0$, $\varepsilon_t = z / z_t$,

z_o = momentum roughness length

z_t = scalar roughness length

Drawbacks:

- Since ζ depends on M and H, iteration is necessary with high costs
- ψ-function for sea ice conditions (SHEBA) very complex
- ψ and Φ-functions of different authors show large variability

SHEBA stability functions of Grachev et al. (2007)

$$\varphi_m = 1 + \frac{a_m \zeta (1+\zeta)^{1/3}}{1 + b_m \zeta}, \quad \varphi_h = 1 + \frac{a_h \zeta + b_h \zeta^2}{1 + c_h \zeta + \zeta^2}$$

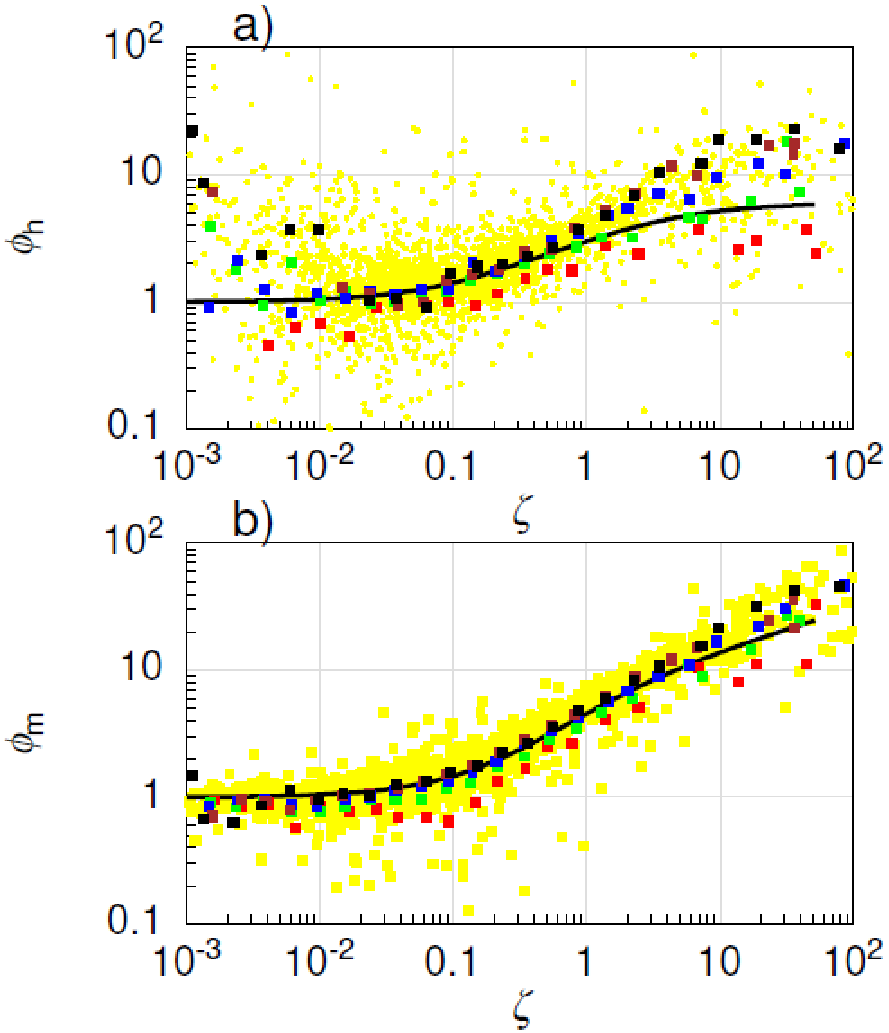
and related stability correction functions

$$\psi_m(\zeta) = -\frac{3a_m}{b_m}(x-1) + \frac{a_m B_m}{2b_m}\left[2\ln\frac{x+B_m}{1+B_m} - \ln\frac{x^2 - xB_m + B_m^2}{1 - B_m + B_m^2} + 2\sqrt{3}\left(\arctan\frac{2x - B_m}{\sqrt{3}B_m} - \arctan\frac{2 - B_m}{\sqrt{3}B_m}\right)\right]$$

$$\psi_h(\zeta) = -\frac{b_h}{2}\ln\left(1 + c_h\zeta + \zeta^2\right) - \left(\frac{a_h}{B_h} - \frac{b_h c_h}{2B_h}\right) \times \left(\ln\frac{2\zeta + c_h - B_h}{2\zeta + c_h + B_h} - \ln\frac{c_h - B_h}{c_h + B_h}\right)$$

Most accurate, but complex formulation causing high numerical costs

Stability functions versus SHEBA data

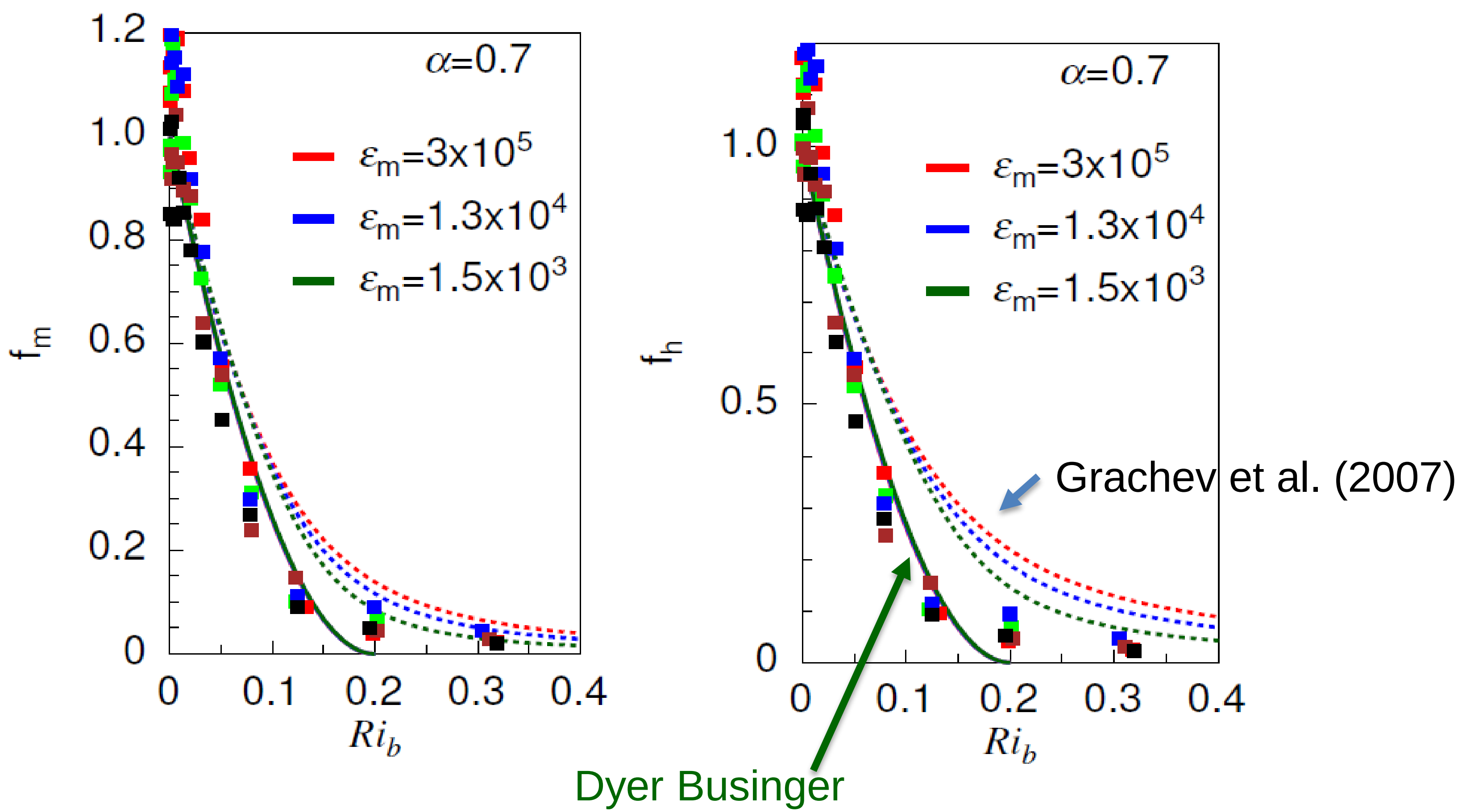


Colours of squares refer to
bin averaged results
at different measurement heights

Black solid lines refer to
the Grachev et al. (2007)
stability functions

Despite the very good agreement of the Grachev-functions with SHEBA observations, they cause also slight drawbacks as shown on the next two pages.

Normalized transfer coefficients versus SHEBA data

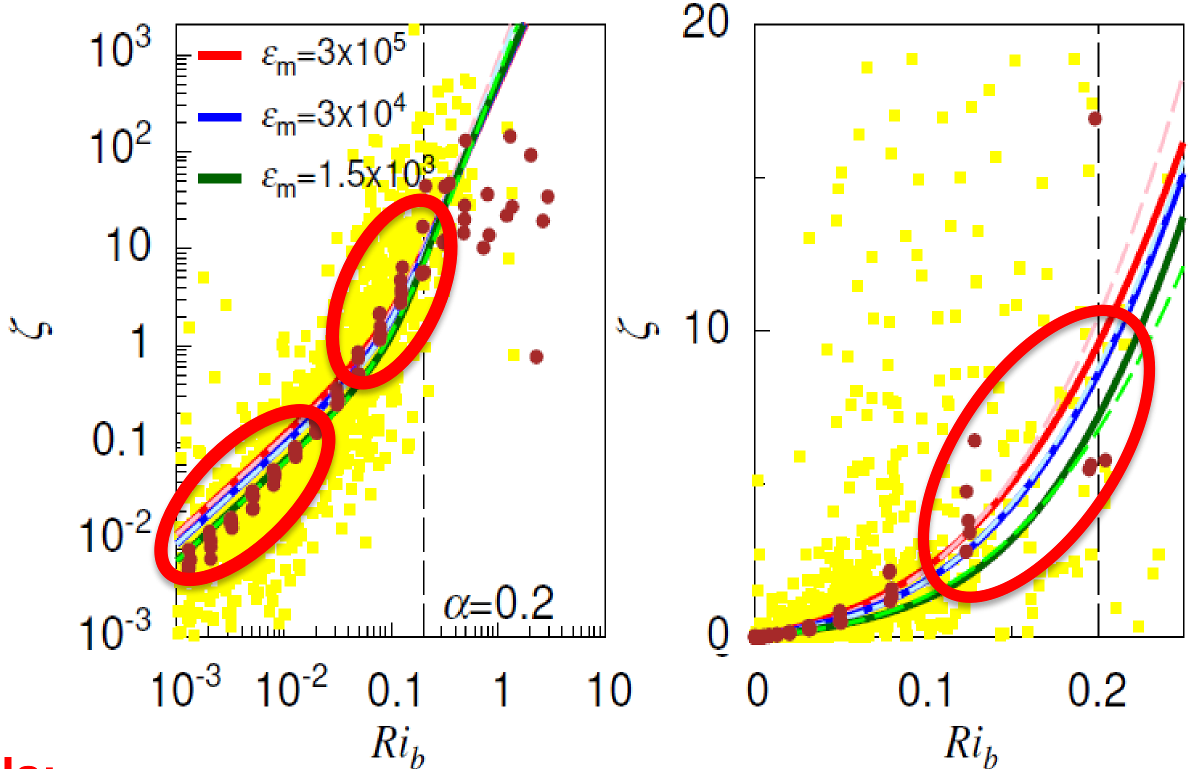


Dyer Businger (1974): no turbulence for Rib > 0.2

Grachev functions: overestimation for Ri_b> 0.05

Governing MOST equation

$$\hat{Ri}_b = \frac{(1-1/\varepsilon_m)^2}{1-1/\varepsilon_t}\,\zeta\,\frac{\ln\varepsilon_t - Pr_0^{-1}[\psi_h(\zeta)+\psi_h(\zeta/\varepsilon_t)]}{[\ln\varepsilon_m - \psi_m(\zeta)+\psi_m(\zeta/\varepsilon_m)]^2}, \qquad \hat{Ri}_b = \frac{Ri_b}{Pr_0}$$



Drawbacks:
$Ri_b \rightarrow 0$
Overestimation

$0.05 < Rib < 0.2$
Underestimation

Goals:
Reduce drawbacks of Grachev functions by defining modified functions, minimizing differences to three observed features
i) stability functions, (ii) $\xi(Ri_b)$ and (iii) transfer coefficients.

New modified and extended stability functions

$$\phi_m(\zeta) = 1 + \frac{a_m\,\zeta}{(1+b_m\zeta)^{2/3}}$$

$$\phi_h(\zeta) = Pr_0\left(1 + \frac{a_h\,\zeta}{1+b_h\zeta}\right)$$

$$\psi_m(\zeta) = -3\,\frac{a_m}{b_m}\left[(1+b_m\,\zeta)^{1/3} - 1\right]$$

$$\psi_h(\zeta) = -Pr_0\,\frac{a_h}{b_h}\,\ln(1+b_h\zeta).$$

Fulfills same constraints as Grachev functions,

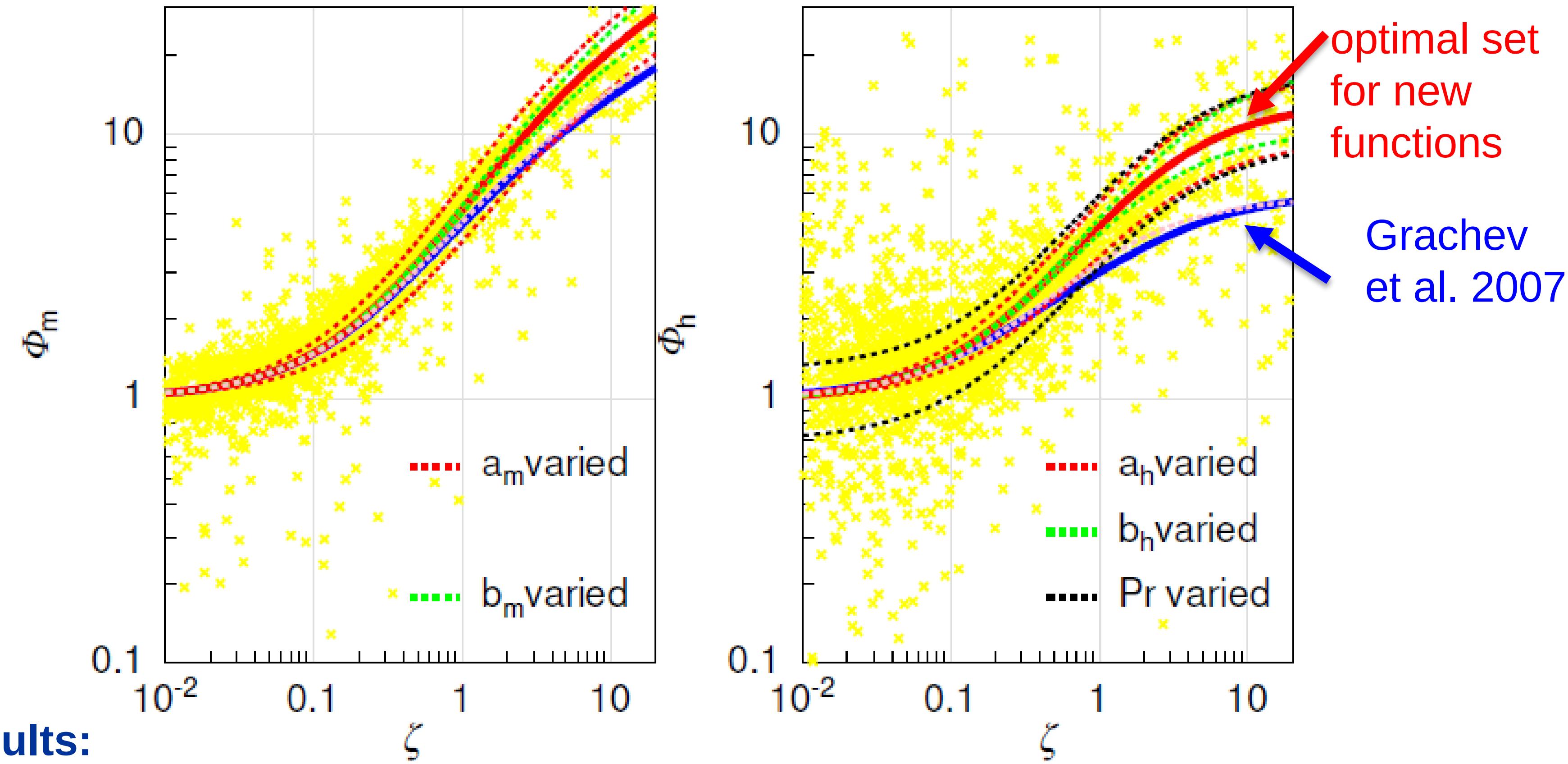
e.g., same limits for $\zeta \rightarrow 0$ (linear function)
similar limit for $\zeta \rightarrow \infty$

Challenge: Define values for set of constants P_{r0}, a_m, b_m, a_h, b_h

Strategy: minimize differences to measured
- **flux gradient relationship (phi-functions)**
- **stability parameter – Ri_b relationship**
- **transfer coefficients**

For $Ri_b \rightarrow 0$, solution should approximate Dyer Businger → $a_m = 5.0$
P_{r0} should not be larger than 1

Sensitivity study (ϕ-functions, variation of constants by 30 %)

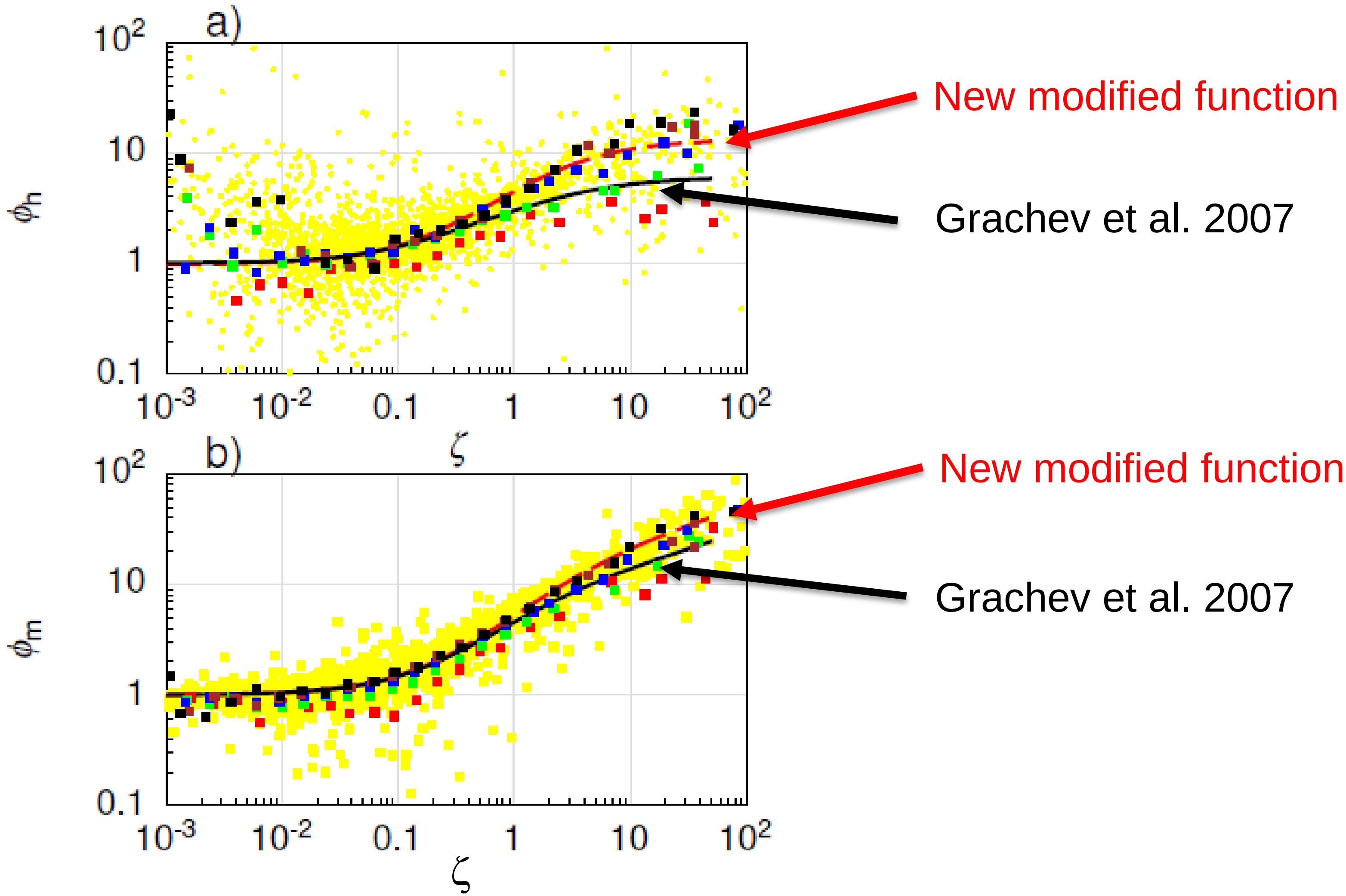


Results:

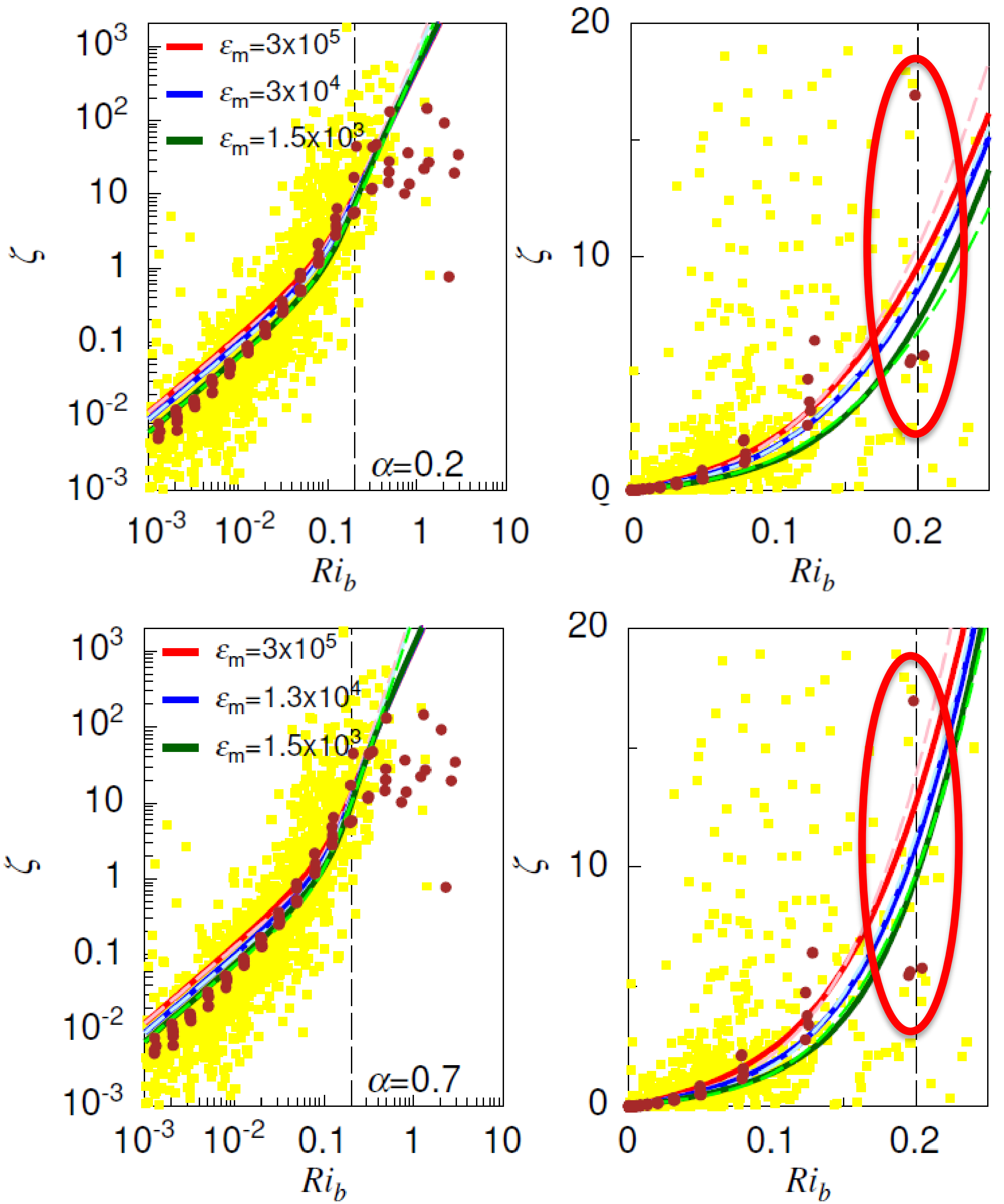
- $Ri_b < 0.1$: P_{r0} has by far the largest impact
- a_m can be varied in a wide range without a large effect on ϕ_m, ϕ_h for $Ri_b < 0.05$

- There is a combination of constants, for which Grachev et al. (2007) fuctions are reproduced with only very little differences (pink line), but optimal constants for requirements (i)-(iii) (see previous slide 8) are different.

Stability functions versus SHEBA data



P_{r0} = 0.98, a_m = 5.0, a_h = 5.0, b_m = 0.3, b_h = 0.4



Grachev et al. 2007

New modified

After application of a semi-analytical method by Gryanik and Lüpkes (2018) to derive a non-iterative scheme based on MOST, we obtain:

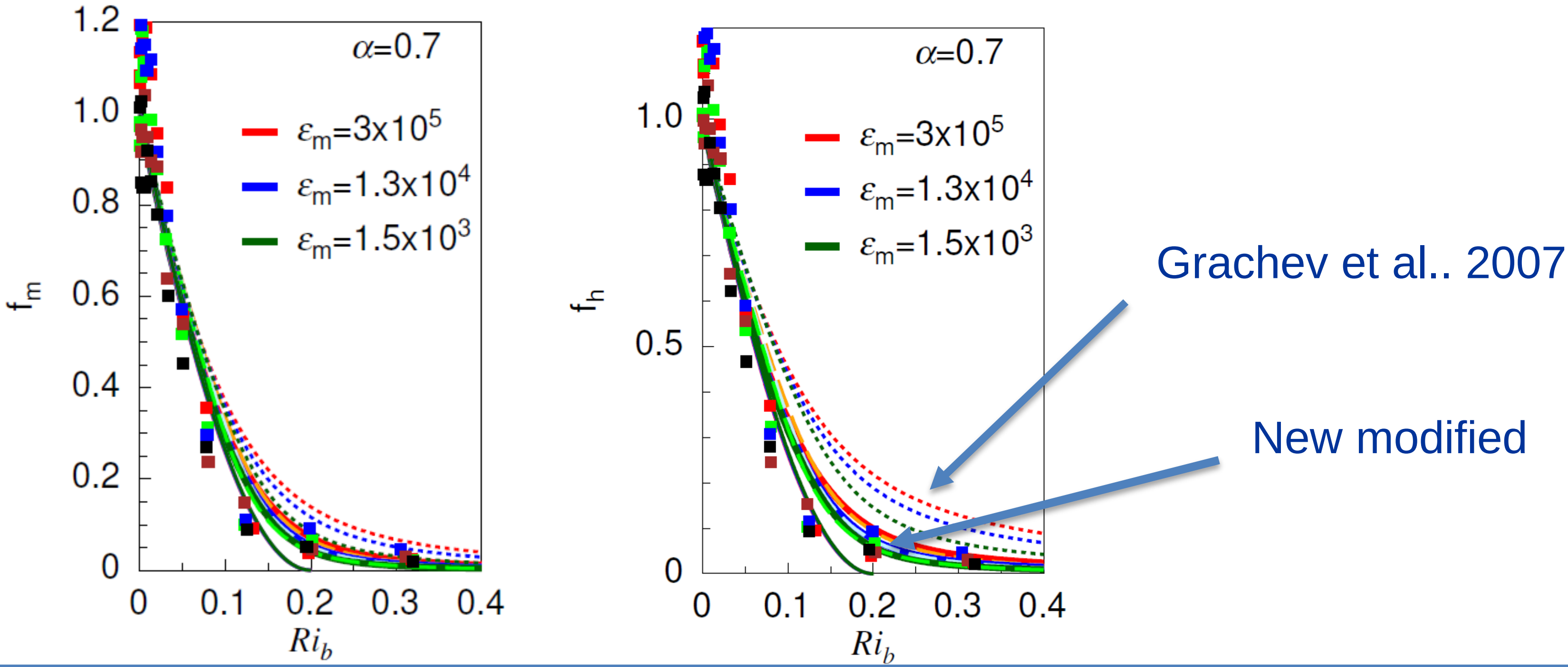
New normalized transfer coefficients based on SHEBA data using the new, modified stability functions

$$C_d = \frac{\kappa^2}{\left[\ln \varepsilon_m + 50.0\left[\left(1 + 0.3\left(\frac{\ln^2 \varepsilon_m}{\ln \varepsilon_t}\hat{R}i_b + A\hat{R}i_b^{3.625}\right)\right)^{1/3} - 1\right]\right]^2}$$

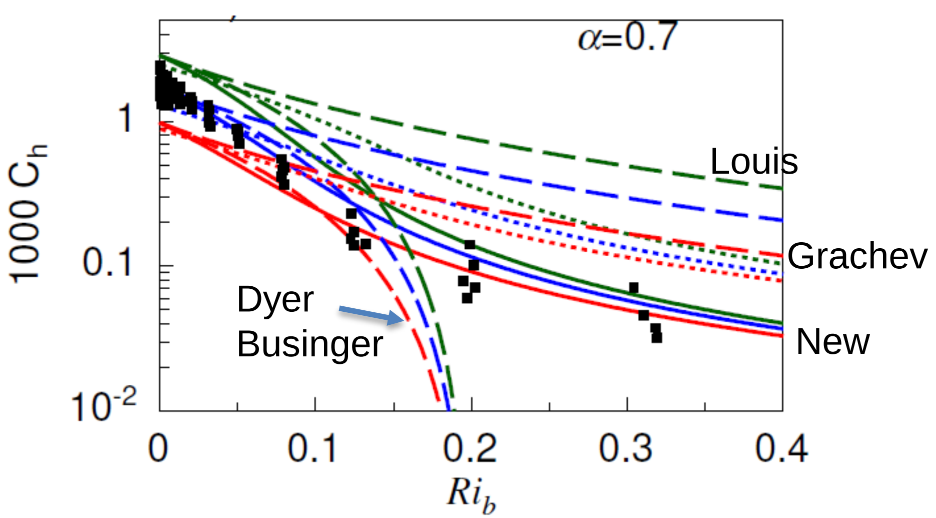
$$C_h = \frac{\kappa C_d^{1/2}}{\ln \varepsilon_t + 12.5 \ln\left[1 + 0.40\left(\frac{\ln^2 \varepsilon_m}{\ln \varepsilon_t}\hat{R}i_b + A\hat{R}i_b^{3.625}\right)\right]}$$

$$A = \frac{(\ln \varepsilon_m + 23.50)^{5.25}}{181.3\,(\ln \varepsilon_t + 16.67)^{2.625}}\left[\frac{(\ln \varepsilon_m + 23.50)^2}{\ln \varepsilon_t + 16.67} - \frac{\ln^2 \varepsilon_m}{\ln \varepsilon_t}\right]$$

Normalized Transfer Coeffcients



Heat Transfer Coefficient



Conclusions

- New stability functions: same accuracy as Grachev et al. (2007) ‚SHEBA-functions', but less complex
- New transfer coefficients agree slightly better with measurements in the very stable range
- Prandtl number is included in the new functions
- Parametrization of transfer coefficients based on Gryanik and Lüpkes (2018) with new functions less complex than with ‚SHEBA-functions'
- New functions should be compared with the data obtained during the current drift of FS Polarstern through the Arctic (MOSAiC)

The content of this contribution is part of a new paper:
Gryanik, Lüpkes, Grachev, and Sidorenko (2020) New modified and extended stability functions for the stable boundary layer based on SHEBA and parametrizations of bulk transfer coefficients for climate models, J. Atmos. Sci., under review