ON THE NECESSITY OF ENERGY BALANCED NON-HYDROSTATIC PRESSURE MODELS FOR FREE SURFACE FLOWS OVER COMPLEX TOPOGRAPHY

I. Echeverribar[1] / P. Brufau[1] / P. García-Navarro[1]
[1] University of Zaragoza, email: echeverribar@unizar.es

Universidad Zaragoza 1542

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MOTIVATION

SWE widely used for numerical geophysical applications. Limitations arise when dealing with dispersive effects

Dispersive effects can be found when dealing with complex topographies and a non-hydrostatic profile appears.

A SW model is compared with a non-hydostatic pressure (NHP) model to analyse their behaviour in different test cases

NON-HYDROSTATIC PRESSURE MODEL EQUATIONS

The depth-averaged system is based on conservation laws as in [1]:

- Mass and *x* momentum equations:

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial (hu)}{\partial x} = 0 \\[2ex] \dfrac{\partial (hu)}{\partial t} + \dfrac{\partial}{\partial x}\left(hu^2 + \dfrac{1}{2}gh^2\right) + gh\dfrac{\partial z_b}{\partial x} = \underbrace{-\dfrac{1}{2}\left(h\dfrac{\partial p}{\partial x} + p\dfrac{\partial (h+2z_b)}{\partial x}\right)}_{\text{Non-hydrostatic terms}} \end{cases}$$

- Developement of the continuity equation [1] that must be fullfiled and is used to solve *p*:

$$h\frac{\partial (hu)}{\partial x} + 2hw + (hu)\frac{\partial}{\partial x}(h + 2z_b) = 0$$

- And the *z* momentum equation negleting convective terms [1]

$$\frac{\partial w}{\partial t} = \frac{p}{h}$$

NUMERICAL SCHEME

The system is solved in two steps:

- First, the hydrostatic part is solved with a FV first-order scheme [2] as a pure SW system, updating each cell with a ARoe solver [2]:

$$(h)_i^* = (h)_i^{n+1} = (h)_i^n - \Delta t/\Delta x\left[\delta \mathbf{f}^+_{i-1/2} + \delta \mathbf{f}^-_{i+1/2}\right]$$

$$(hu)_i^* = (hu)_i^n - \Delta t/\Delta x\left[\delta \mathbf{f}^+_{i-1/2} + \delta \mathbf{f}^-_{i+1/2}\right]$$

- Secondlly, the Pressure Correction Method is solved implicitly so the *p* can be computed at cell edges from continuity eq.
- Once p is obtained, the velocity field in x and z is updated up to *n+1*

$$(hu)_i^{n+1} = (hu)_i^* - \Delta t/2\left[h_i\frac{p_{i+1/2} - p_{i-1/2}}{\Delta x} + p_i\frac{(h+2z)_{i+1} - (h+2z)_{i-1}}{2\Delta x}\right]$$

$$w_{i+1/2}^{n+1} = w_{i+1/2}^n + \Delta t\frac{p_{i+1/2}^{n+1}}{h_{i+1/2}^{n+1}}$$

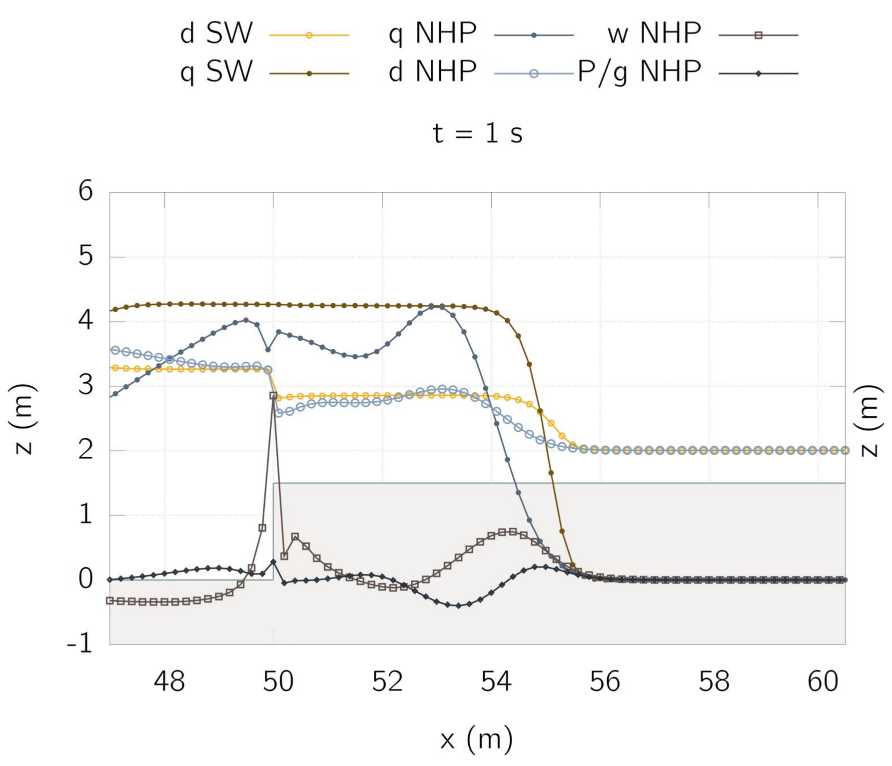
- Note the index of the variables. Since a staggered grid is used, *p* and *w* are edge values, whereas *h* and *hu* are cell centered.

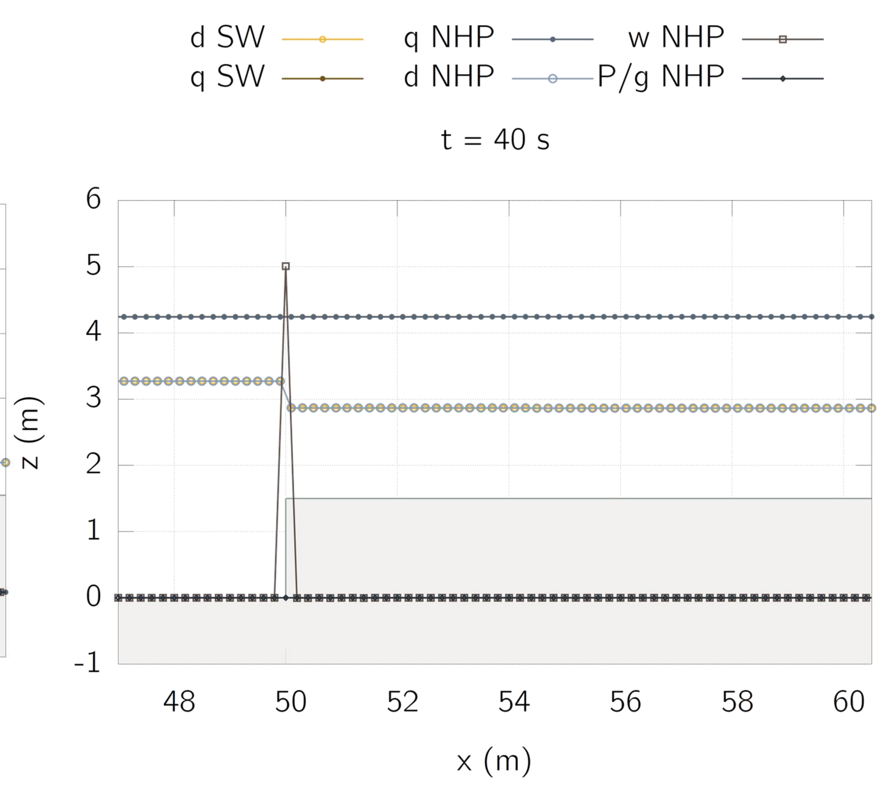
RESULTS

IDEALIZED DAMBREAK (AUGMENTED RIEMANN PROBLEM, RP)

- Aug. RP converges to same steady state w/ different unsteady.

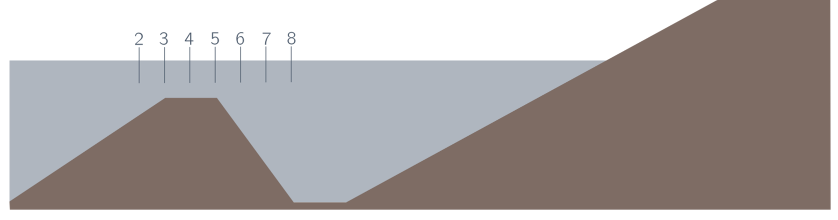
Ini. Cond	h (m)	z (m)	u (m/s)
Left	4,0	0	0,1
Right	0,505	1,5	0

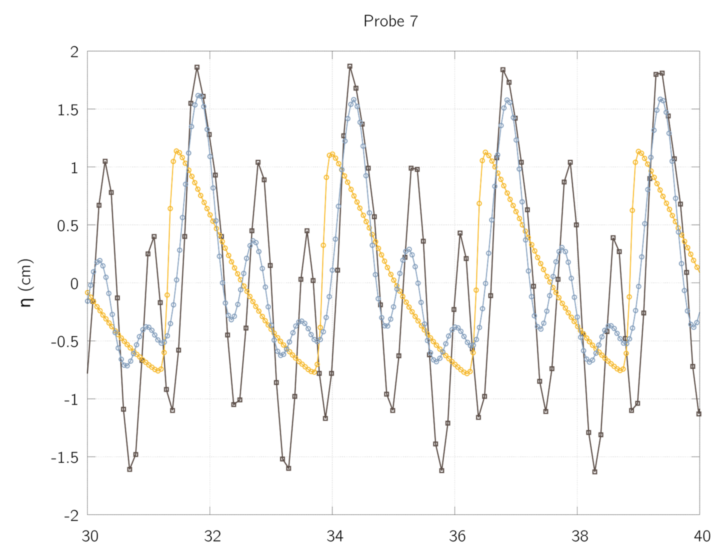


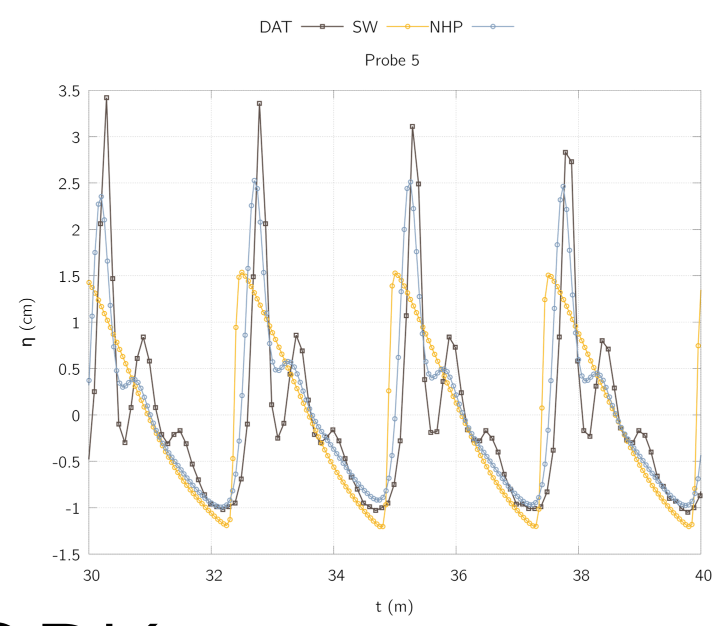


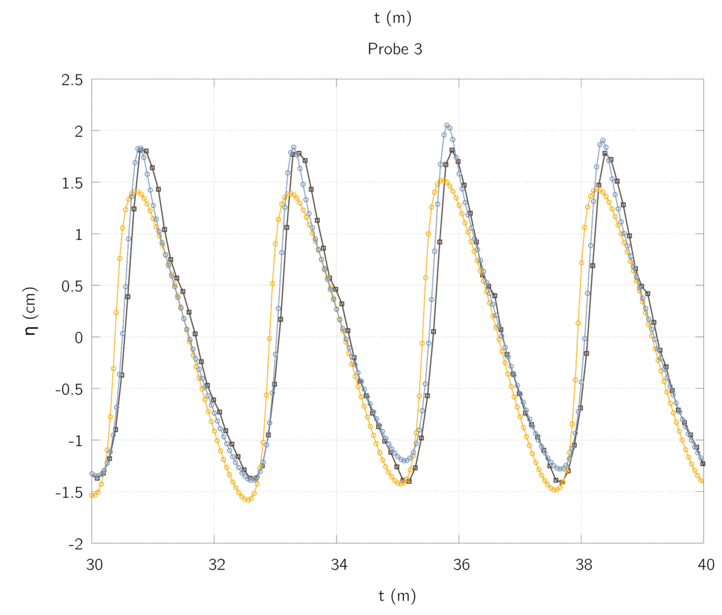
WAVE PROPAGATION OVER IRREGULAR TOPOGRAPHY

- Surface evolution between data, SW and NHP models is compared at probes in experimental case [3].



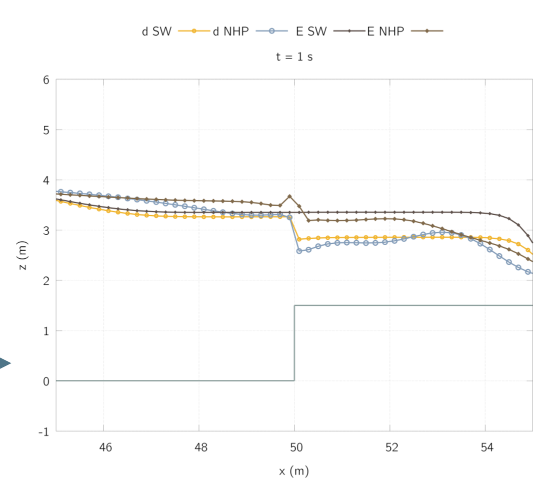


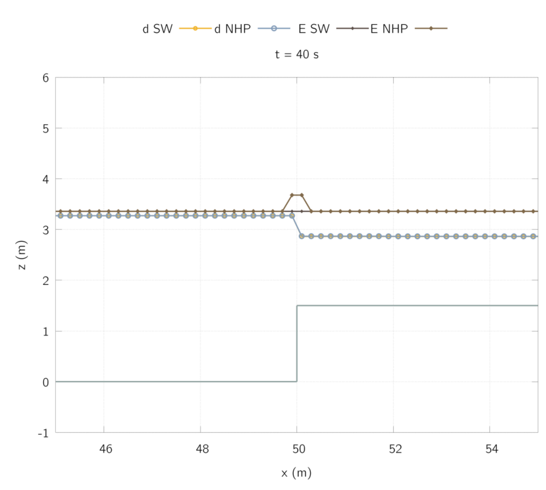




CONCLUDING REMARKS AND FUTURE WORK

- The NHP model shows a good agreement with experimental data in [3]. However, some discrepancies are observed in high frecuency waves due to an energy transfer [3] that is not reproduced.
- Augmented RP are properly solved with NHP models. However, they do not preserve energy balance in steady states if the approach in [2] is used.
- An adaptation of numerical energy balance approach must be found for NHP models





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[2] J. Murillo and P. García-Navarro, , “Energy balance numerical schemes for shallow water equations with discontinuous topography”, Journal of Computational Physics, vol. 236, pp.119–142, 2013.
[3] S. Beji and J. A. Battjes , “Experimental investigation of wave propagation over a bar”, Coastal Engineering, vol. 19, pp.151–162, 1993