Validating future gravity missions via optical clock networks:
Scientific Requirements

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Abstr.: D1737 | EGU2020-1998
Session G4.2: Modern Concepts for Gravimetric Earth Observation
Thursday, 7 May 2020, 16:15-18:00
1 - Motivation

- Since 2002 GRACE observes the Earth’s gravity field variations with unprecedented accuracy
- Mission start of its successor GRACE Follow-On: 2018
- Further missions are suggested (e.g. Pail et al., 2015)

- However, it is difficult to validate GRACE results

Tapley et al. (2019)
WHY is validation of GRACE important?

- required to identify/understand possible problems in sensor system and data analysis
- required to better understand/quantify/calibrate GRACE error estimates
- helps to understand resolution limits of GRACE

- Especially important over Europe, where the signal is comparably small

Tapley et al. (2019)
The CLOck NETwork Services (CLONETS) project aims at developing an optical atomic clock network over Europe connected by fibre links.

- How would it benefit time-variable gravity field determination?

- Which accuracies does it require to detect mass load variations?
GRACE has been validated with **GNSS** measurements

**Main problems:**

- GNSS Comparison to GRACE requires an elastic loading model of the Earth
- Short wavelength signals like *local groundwater discharge and recharge* affects the GNSS but *does not follow elastic loading theory*, i.e. can not be detected with GRACE
- Technique specific errors (e.g. Ray, 2006)
If and how **Superconducting Gravimeters** (SG) can be used to validate GRACE, is disputed (see e.g. the discussion between Van Camp et al., 2014 and Crossley et al., 2014)

**Main problem:**
- Local hydrology and wet air mass affect the gravimeters, but not GRACE; they are hard to model
Validating GRACE with SG’s

Another problem: heterogenous distribution of SG’s
Our objectives

- Here, we suggest that **optical atomic clocks will soon be a third ground measurement tool for GRACE validation**, which is much less affected by local phenomena than GNSS and SG’s.

- In order to test this hypothesis, in this presentation we will **simulate atmospheric and hydrologic effects on gravity potential**.

An optical atomic clock (Takamoto et al., 2015)
Our objectives

- Lisdat et al. (2016) observed gravitational redshift between Paris and Braunschweig due to **height** (and thus potential) **difference**

- We would like to take this one step further and investigate this relativistic effect due to **time-variable potential changes**
### Our objectives

<table>
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<th>Optical atomic clock network</th>
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<td>1 month (daily/weekly solutions disregarded)</td>
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<td>&gt; 330 x 330 km ( \triangleq l_{\text{max}} = 120 ) This is rather an optimistic view considering that filters are used afterwards</td>
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<td>mm geoid height, averaged over 330 x 330 km</td>
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2 - Methods

\[
\frac{\delta f}{f} = \frac{\delta U}{c^2} = \frac{\delta N - \delta h}{c^2} \frac{GM}{R^2}
\]

- \(U\) ... gravity potential
- \(N\) ... geoid height
- \(h\) ... vertical land motion
- \(\frac{\delta f}{f}\) ... relative frequency difference

That means: 1\(\text{cm}\) change in geoid height equals \(\frac{\delta f}{f}\) of \(\sim 1\text{e-18}\)

- But: Vertical land motion acts similarly on potential
Poli et al. (2014) show the rapid development of optical atomic clock uncertainty.
So, how good are the clocks now?

Differentiate between a clock’s **accuracy** and **stability**

**Accuracy**: Common in geodesy; limited by systematic effects/drifts

**Stability**: same as precision/repeatability; sometimes referred to as instability (because this term is proportional to the number)
Case of Bothwell et al. (2019): **Accuracy of 2.0e-18**; this value is reached by the stability \((4.8e^{-17}/\sqrt{\tau}, \tau \text{ are seconds})\) after <10 minutes of averaging.

Depending on how fast the accuracy value is reached by averaging, we could obtain **several values of potential difference per hour**.

- Here, we will simulate **errors at 1e-18** (little bit better than state-of-the-art), **1e-19**, and **1e-20**.
- We will simulate only **one data point per day** here.
3 - Results

- Optical fibre links are already established between some European National Metrology institutes (NMI) and other (possible) optical clock locations.
- We show simulations for some of the existing and planned clocks and links.
We focus on hydrology and atmosphere

Community Land Model (CLM) Total water storage variability in 2007, expanded to spherical harmonics ($l_{\text{max}}=720$), RMS computed from daily values.

Atmospheric mass variability computed after Forootan et al. (2013), from ERA-5 data, expanded to spherical harmonics ($l_{\text{max}}=180$); we consider the elastic loading of the Earth’s crust via the LLN approach; RMS computed from daily values; dry+wet air.
Let’s take a look at some time series at certain clock locations.

Atmospheric part is much more variable at shorter time scales and has higher overall magnitude.

So what does this mean for fractional frequency measurements?
Hydrologic: EWH change + resulting elevation, geoid, and $\frac{\delta f}{f}$

$$\frac{\delta f}{f} = \frac{\delta U}{c^2} = \frac{\delta N - \delta h}{c^2} \frac{GM}{R^2}$$

Clear annual signal; geoid change and elevation go in opposite directions, but their effect on potential goes in the same direction.

To infer geoid change from $\frac{\delta f}{f}$, we have to correct for land elevation change.
Effects like local groundwater changes do not follow elastic loading theory:

- Vertical land motion that is not associated with much geoid change

GNSS as correction for the resulting potential change is inevitable

Effects on vertical land motion apart from elastic loading.
Source: Anna Klos, NEROGRAV project presentation (2019).
Atmosphere for comparison

- 3-4 times higher effect on $\frac{\delta f}{f}$
- Let’s take one time series and make some assumptions for errors
PTB Braunschweig

- Braunschweig (Physikalisch-Technische Bundesanstalt) – time series and amplitude spectrum
- Modelled white noise with $\sigma = 1e^{-18}$ (19, 20)
- Signal larger than noise
- But: We are only observing a single clock ...

How does it look for a clock comparison?

![Graph showing time series and amplitude spectrum with frequencies below 1/month and GRACE-observable.](image)
White **noise is now larger** \( (\sigma = \sqrt{2}e - 18 \ (19, \ 20)) \)

- **Signal is smaller** because large scale/low degree signals vanish
- Only a few frequencies of the amplitude spectrum visible for clock uncertainty of \( 1e-18 \frac{\delta f}{f} \)
An optical clock network at NMI locations with clock uncertainty of 1e-18 is not able to detect time-variable gravity over Europe

Let’s go down one magnitude
- The $\sqrt{2}e^{-19}$ noise in orange can hardly be seen on the time series
- It’s also way below almost all frequencies in the amplitude spectrum

Clocks with uncertainty of $1e^{-19}$ could very well detect atmospheric changes over Europe
For **hydrology** the noise of $1e^{-19}$ becomes critical as the variations in $\frac{\delta f}{f}$ are at $\pm 2e^{-19}$.

Clocks with an uncertainty of $1e^{-19}$ would be just about able to detect **hydrologic changes over Europe**; which is small compared to other continents.
Compare that to GRACE: simulate GRACE measurements + errors at clock locations

We assumed to be able to correct for land elevation change below the mm-level
4 - Summary

- Which accuracies do we need for proper validation?
  - largest time-variable signals detectable at 1e-18 (~1cm geoid height) accuracy of clocks
  - To actually use an optical atomic clock network to validate GRACE or next generation gravity missions (NGGM) we need to go below that level: At 1e-19 (~1mm) we reach a point where only short time hydrological variations pose a problem.
    Here, the correction factor GNSS reaches its limits as well
## Resolution and Uncertainty

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<td>At least daily, maybe even &lt; 1 hour</td>
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<td><strong>Spatial resolution</strong></td>
<td>&gt; 330 x 330 km ≅ ( l_{\text{max}} = 120 )</td>
<td>&lt; 3000 km (depending on extent of the network) ( l_{\text{min}} = 13 )</td>
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<td>This is rather an optimistic view considering that filters are used afterwards</td>
<td>&gt; 100 km (depending on clock distribution) ( l_{\text{max}} = 400 )</td>
</tr>
<tr>
<td><strong>Uncertainty</strong></td>
<td>mm geoid height, averaged over 330 x 330 km</td>
<td>Right now: Few cm geoid height (but rapidly improving, possibly at 1mm in the next decade), point-wise</td>
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## Validation tools

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<th>SG</th>
<th>GNSS</th>
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<tr>
<td>Measures gravity variations</td>
<td>Measures (vertical) land motion</td>
<td>Measures geopotential differences</td>
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<tr>
<td>Often highly affected by direct gravitational effect of local hydrologic changes</td>
<td>Good distribution of stations with free available data</td>
<td>Rapidly improving technology</td>
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<td>Affected by local land motion that is not aligned with elastic loading</td>
<td>Clocks sitting on the surface measure the vertical land motion induced potential change</td>
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<td>Combination allows for direct estimation of geoid height variations</td>
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<tr>
<td>Combination might be interesting e.g. for a better SG signal separation</td>
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Thank you for taking the time to go through our slides!

We are looking forward to the online discussions before, after, and during the actual **session at May 7th**

You can also write me an Email: schroeder@geod.uni-bonn.de
References

- Bothwell et al. (2019): JILA SrI optical lattice clock with uncertainty of $2.0 \times 10^{-18}$. https://doi.org/10.1088/1681-7575/ab4089
- Forootan et al. (2013): Comparisons of atmospheric data and reduction methods for the analysis of satellite gravimetry observations. https://doi.org/10.1002/jgrb.50160
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