A new parameterization of gravity waves for atmospheric circulation models based on the radiative transfer equation

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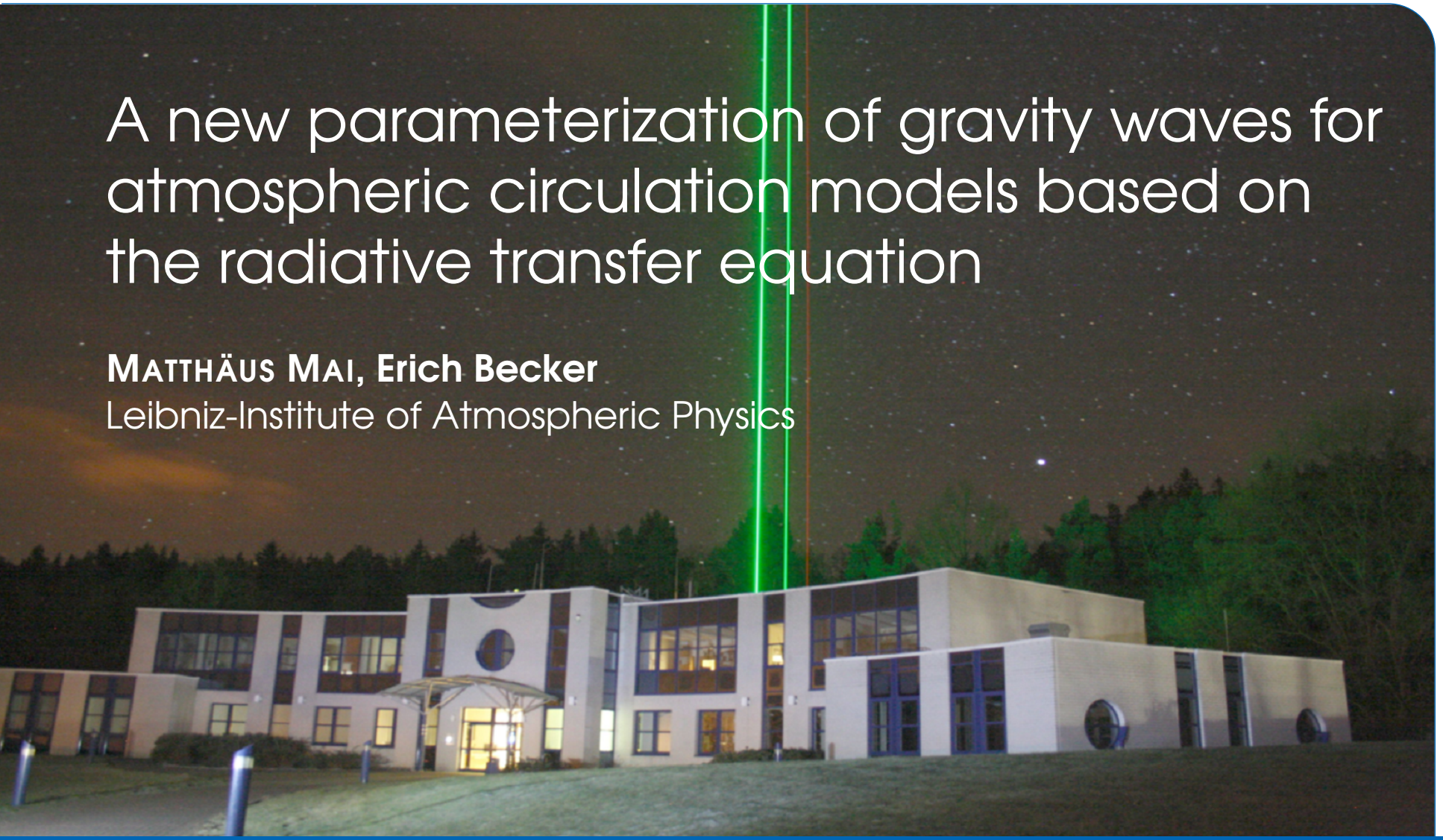


Solving the radiative transfer equation for Gravity Waves (GW) using the Gaussian variation principle

Radiative transfer equation [Olbers, Eden 2013 *Journal of Physical Oceanography*]:

$$\partial_t \mathcal{E} + \partial_z (\dot{z}\mathcal{E}) + \partial_m (\dot{m}\mathcal{E}) + \partial_{\omega_I} (\dot{\omega}_I \mathcal{E}) = \frac{\dot{\omega}_I}{\omega_I}\mathcal{E} - m^2 \mathcal{D}\mathcal{E} + \mathcal{S} \tag{1}$$

Decomposition of spectral energy density: $\mathcal{E}(z, t, \omega_I, m) = \mathcal{E}_0(z, t) A(m, m^*) B(\omega_I)$

Assumptions/approximations:

Single column

$k = const$
$\dot{k} = 0$

Desaubies spectrum

$A(m, m^*) = \mathcal{N}_A \frac{m/m^*}{1+(m/m^*)^4}$
$B(\omega_I) = \mathcal{N}_B \omega_I^{-2}$

Mid-frequency, Boussinesq for GW

$\omega_I^2 = \frac{k^2 N^2}{m^2}$

Definition of error squared:

$$\chi^2(z, t) := \int\int A(m, m^*) B(\omega) \left(\frac{\textit{radiative transfer equation}}{A(m, m^*) B(\omega)} \right)^2 dm d\omega$$

Variation of χ^2 with respect to $\partial_t \mathcal{E}_0(z, t)$ and $\partial_t m^*(z, t)$ yields the best approximate solution to the radiative transfer equation:

$$\partial_t \mathcal{E}_0 = +C_{\mathcal{E}0} \partial_z \mathcal{E}_0 + C_{\mathcal{E}1} \partial_z m^* \mathcal{E}_0 - C_{\mathcal{E}2} \mathcal{E}_0 \frac{\partial_z U}{N} + C_{\mathcal{E}3} \mathcal{E}_0 \frac{\partial_z N}{N} - C_{\mathcal{E}4} \mathcal{D}\mathcal{E}_0 + C_{\mathcal{E}5} \mathcal{S} \tag{2}$$

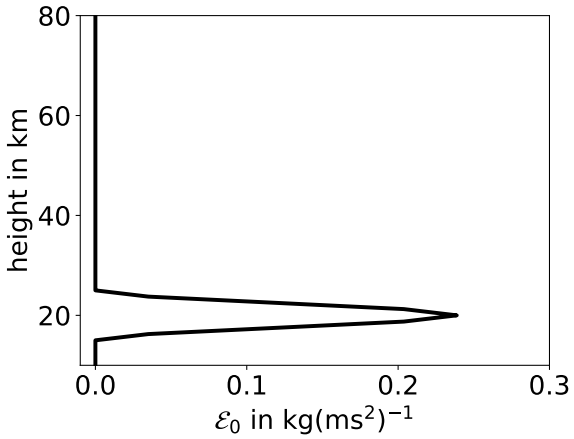
$$\partial_t m^* = +C_{m0} \frac{\partial_z \mathcal{E}_0}{\mathcal{E}_0} + C_{m1} \partial_z m^* - C_{m2} \frac{\partial_z U}{N} + C_{m3} \frac{\partial_z N}{N} - C_{m4} \mathcal{D} + C_{m5} \frac{\mathcal{S}}{\mathcal{E}_0} \tag{3}$$

$C_i = \mathcal{C} \iint f_i(m, m^*, \omega_I) dm d\omega_I$

The Setting for the offline simulation

Wave field parameters:

Initial energy density:



Initial characteristic vertical wave number:

$$m^*(z,0) = -\frac{2\pi}{3000}\mathrm{m}^{-1}$$

Spectral parameters:

$$\omega_I \in [0.001826, 0.001916]\mathrm{s}^{-1}$$

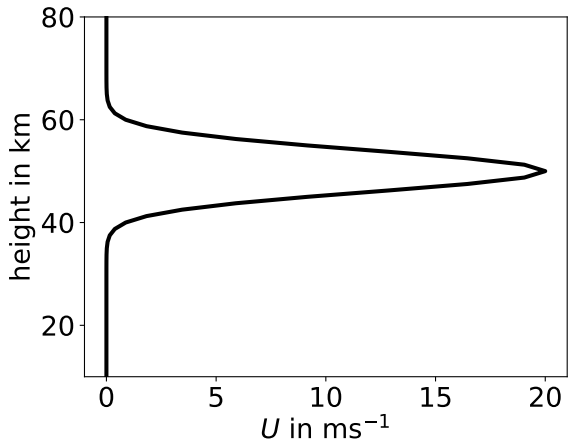
$$m \in \left[-\frac{2\pi}{10}, -\frac{2\pi}{80000}\right]\mathrm{m}^{-1}$$

Source parameter:

$$\mathcal{S}(z,t) = 0$$

Background parameters:

Eastward mesospheric wind jet:



Brunt-Väisälä frequency:

$$N(z,t) = 0.0187\mathrm{s}^{-1}$$

Density:

$$\rho(z) = 1.2\exp(-z/7000m)\mathrm{kgm}^{-3}$$

Dissipation by saturation:

Static instability if:

$$\frac{1}{\rho N^2}\mathcal{E}_0 \iint m^2 A(m)B(\omega_I)dmd\omega_I > \alpha^2$$

$$\mathcal{C}_D\mathcal{E}_0 > \alpha^2$$

When the lhs is bigger than α^2 the saturation is reached and dissipation sets in:

$$\mathcal{D} = D_0(\mathcal{C}_D\mathcal{E}_0 - \alpha^2)$$

case 1:

$$D_0 = 0$$

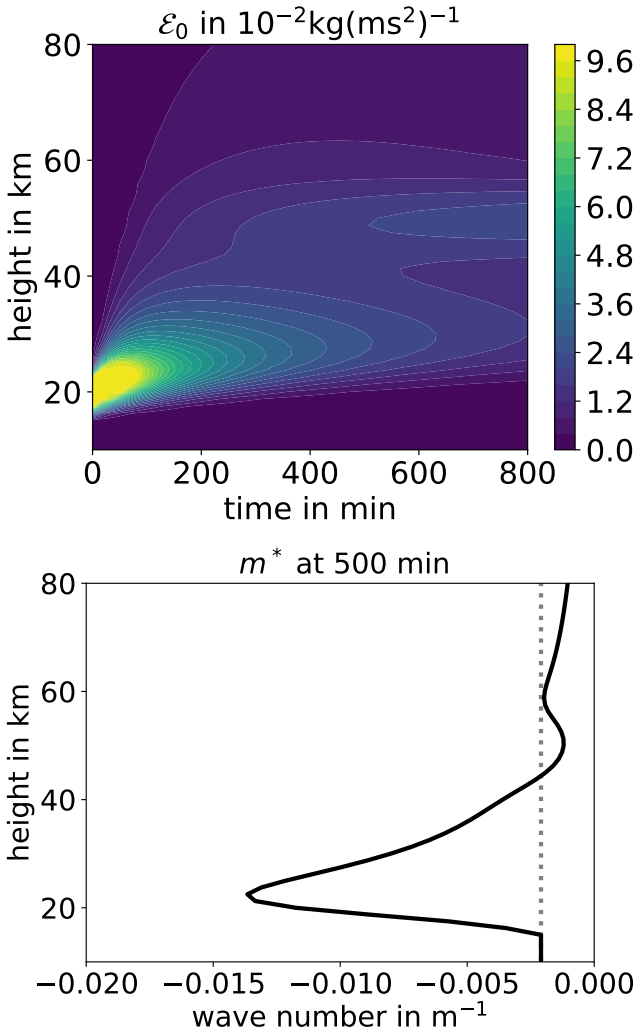
case 2:

$$D_0 = 100\mathrm{m}^2\mathrm{s}^{-1}$$

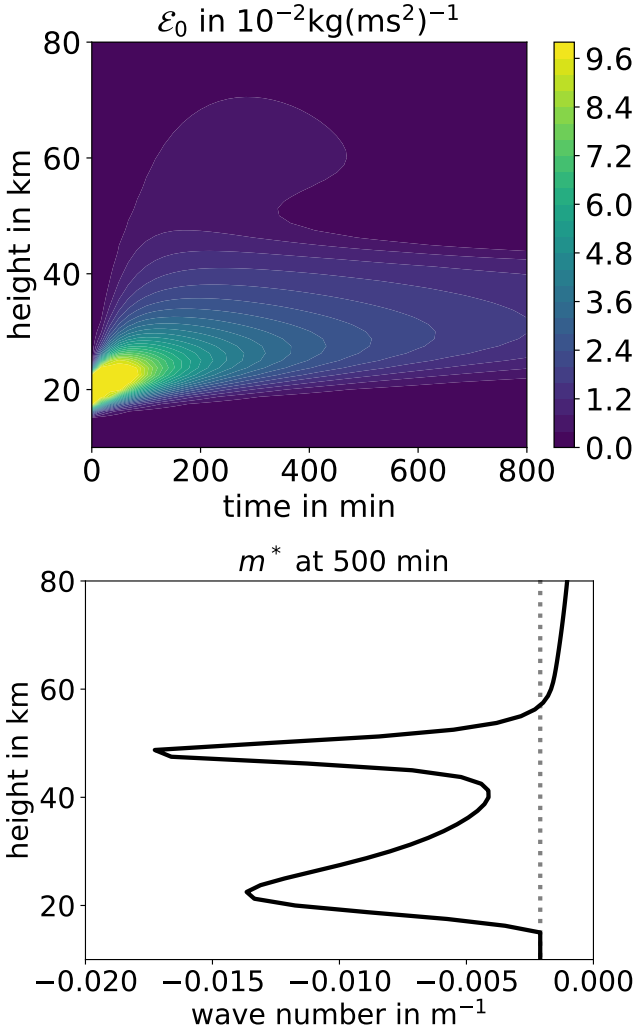
$$\alpha^2 = 0.5$$

The scheme describes wave refraction and even critical layer (case 1)

Horizontal wave propagation against the windjet, $k < 0$

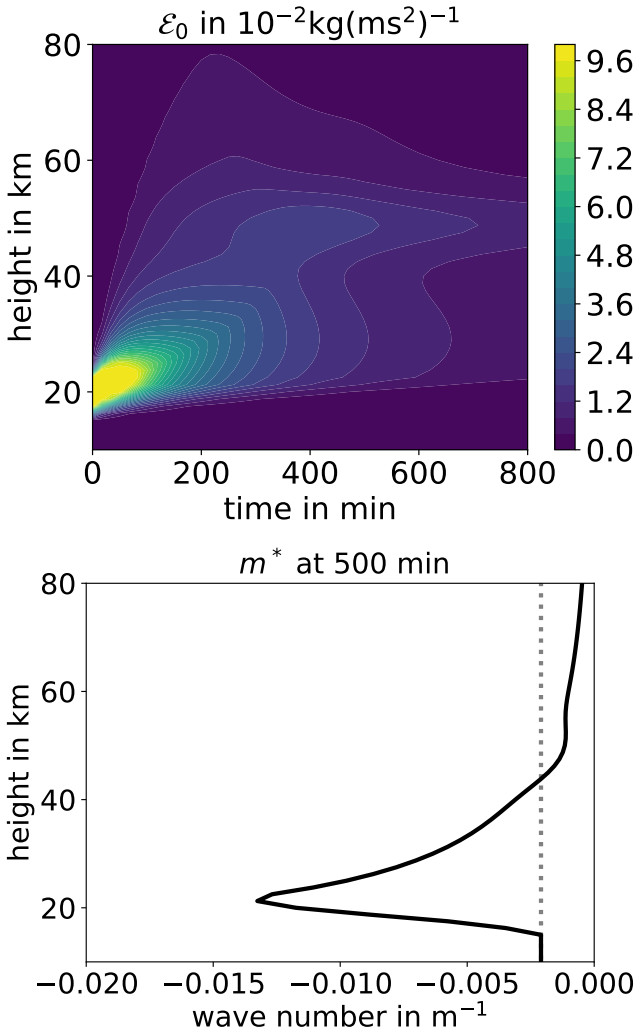


Horizontal wave propagation in the direction of the windjet, $k > 0$ (critical layer)



The scheme describes wave refraction and even critical layer (case 2)

Horizontal wave propagation against the windjet, $k < 0$



Horizontal wave propagation in the direction of the windjet, $k > 0$ (critical layer)