A new parameterization of gravity waves for atmospheric circulation models based on the radiative transfer equation

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Solving the radiative transfer equation for Gravity Waves (GW) using the Gaussian variation principle

Radiative transfer equation [Olbers, Eden 2013 Journal of Physical Oceanography]:

\[ \partial_t \mathcal{E} + \partial_z (\dot{z} \mathcal{E}) + \partial_m (\dot{m} \mathcal{E}) + \partial_{\omega_I} (\dot{\omega_I} \mathcal{E}) = \frac{\dot{\omega_I}}{\omega_I} \mathcal{E} - m^2 D \mathcal{E} + S \]  

(1)

Decomposition of spectral energy density: \( \mathcal{E}(z, t, \omega_I, m) = \mathcal{E}_0(z, t)A(m, m^*)B(\omega_I) \)

Assumptions/approximations:

Single column text

\[ k = \text{const} \]
\[ \dot{k} = 0 \]

Desaubies spectrum

\[ A(m, m^*) = N_A \frac{m/m^*}{1+(m/m^*)^4} \]

\[ B(\omega_I) = N_B \omega_I^{-2} \]

Mid-frequency, Boussinesq for GW

\[ \omega_I^2 = \frac{k^2 N^2}{m^2} \]

Definition of error squared:

\[ \chi^2(z, t) := \int \int A(m, m^*)B(\omega) \left( \frac{\text{radiative transfer equation}}{A(m, m^*)B(\omega)} \right)^2 dmd\omega \]

Variation of \( \chi^2 \) with respect to \( \partial_t \mathcal{E}_0(z, t) \) and \( \partial_t m^*(z, t) \) yields the best approximate solution to the radiative transfer equation:

\[ \partial_t \mathcal{E}_0 = +C_{\mathcal{E}_0} \partial_z \mathcal{E}_0 + C_{\mathcal{E}_1} \partial_z m^* \mathcal{E}_0 - C_{\mathcal{E}_2} \mathcal{E}_0 \frac{\partial_z U}{N} + C_{\mathcal{E}_3} \mathcal{E}_0 \frac{\partial_z N}{N} - C_{\mathcal{E}_4} D \mathcal{E}_0 + C_{\mathcal{E}_5} S \]  

(2)

\[ \partial_t m^* = +C_{m^*} \frac{\partial_z \mathcal{E}_0}{\mathcal{E}_0} + C_{m^*} \partial_z m^* - C_{m^*} \frac{\partial_z U}{N} + C_{m^*} \frac{\partial_z N}{N} - C_{m^*} D + C_{m^*} S \]  

(3)

\[ C_i = C \int \int f_i(m, m^*, \omega_I) dmd\omega_I \]
The Setting for the offline simulation

Wave field parameters:

Initial energy density:

\[
\varepsilon_0 = 0.0, 0.1, 0.2, 0.3 \text{ kg(ms}^2\text{)}^{-1}
\]

Initial characteristic vertical wave number:

\[
m^*(z, 0) = -\frac{2\pi}{3000} \text{ m}^{-1}
\]

Spectral parameters:

\[
\omega_I \in [0.001826, 0.001916]\text{s}^{-1}
\]

\[
m \in \left[ -\frac{2\pi}{10}, -\frac{2\pi}{80000} \right] \text{ m}^{-1}
\]

Source parameter:

\[
S(z, t) = 0
\]

Background parameters:

Eastward mesospheric wind jet:

\[
U = 0, 5, 10, 15, 20 \text{ ms}^{-1}
\]

\[
\text{height in km}
\]

Dissipation by saturation:

Static instability if:

\[
\frac{1}{\rho N^2} \varepsilon_0 \int \int m^2 A(m) B(\omega_I) dmd\omega_I > \alpha^2
\]

\[
C_D \varepsilon_0 > \alpha^2
\]

When the lhs is bigger than \( \alpha^2 \) the saturation is reached and dissipation sets in:

\[
D = D_0 (C_D \varepsilon_0 - \alpha^2)
\]

case 1:

\[
D_0 = 0
\]

case 2:

\[
D_0 = 100 \text{ m}^2\text{s}^{-1}
\]

\[
\alpha^2 = 0.5
\]
The scheme describes wave refraction and even critical layer (case 1)

Horizontal wave propagation against the windjet, \( k < 0 \)

Horizontal wave propagation in the direction of the windjet, \( k > 0 \) (critical layer)
The scheme describes wave refraction and even critical layer (case 2)

Horizontal wave propagation against the windjet, \( k < 0 \)

Horizontal wave propagation in the direction of the windjet, \( k > 0 \) (critical layer)