Langmuir circulation without wind or surface waves

Shear flow interacting with wavy topography

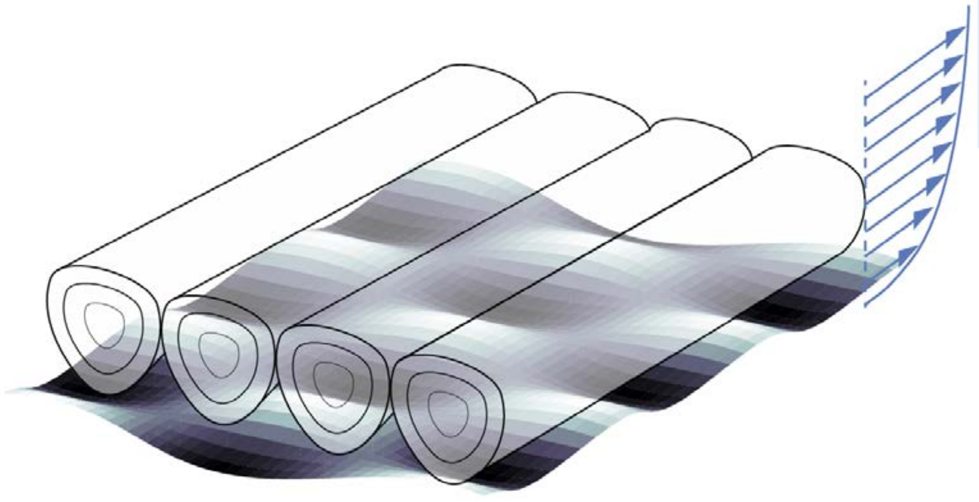
Andreas H. Akselsen, Andreas Brostrøm & Simen Å. Ellingsen

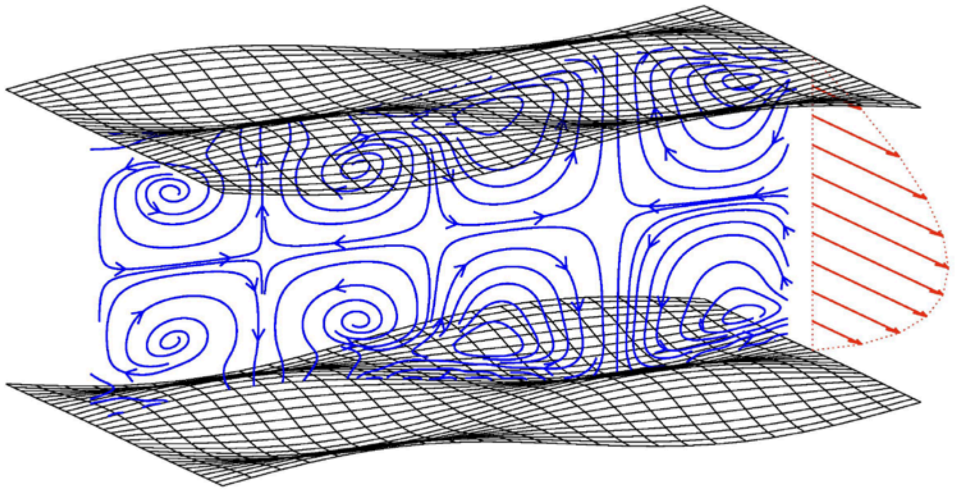
NTNU

Norwegian University of Science and Technology, Trondheim, Norway

One-slide summary

- Streamwise vortices occur in fluid flow over surface with crossing-wave pattern.
- Kinematically equivalent to Langmuir circulation (the «CL1» mechanism, Craik 1970).
- Analytical theory presented for Langmuir rolls during
  - the early onset, and
  - the final, steady-state.
- Explicit expressions when velocity profile is a power law $U(z) = z^q$.
- Confirmed by Lattice-Boltzmann simulations.





NTNU

Introduction

- Full details of this work may be found in the forthcoming paper (arXiv)

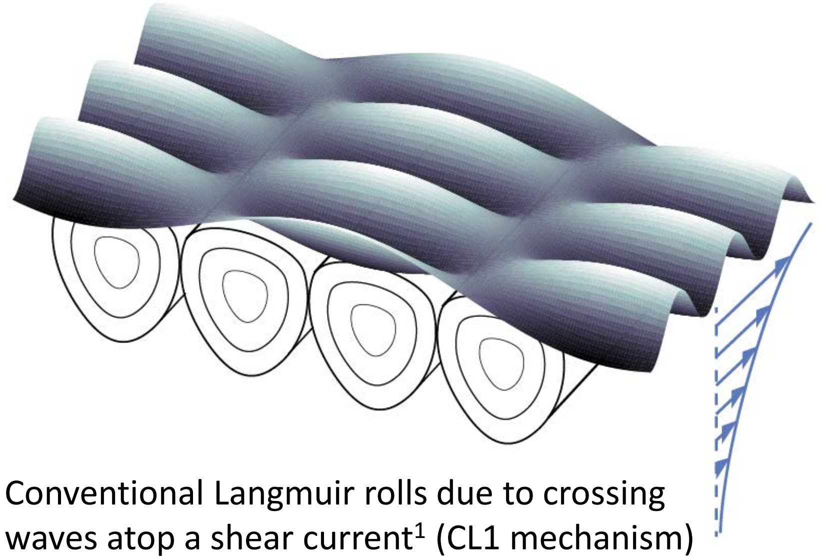
Langmuir-type vortices in boundary layers driven by a criss-cross wavy wall topography

Andreas H. Akselsen[1]†and Simen Å. Ellingsen[1]

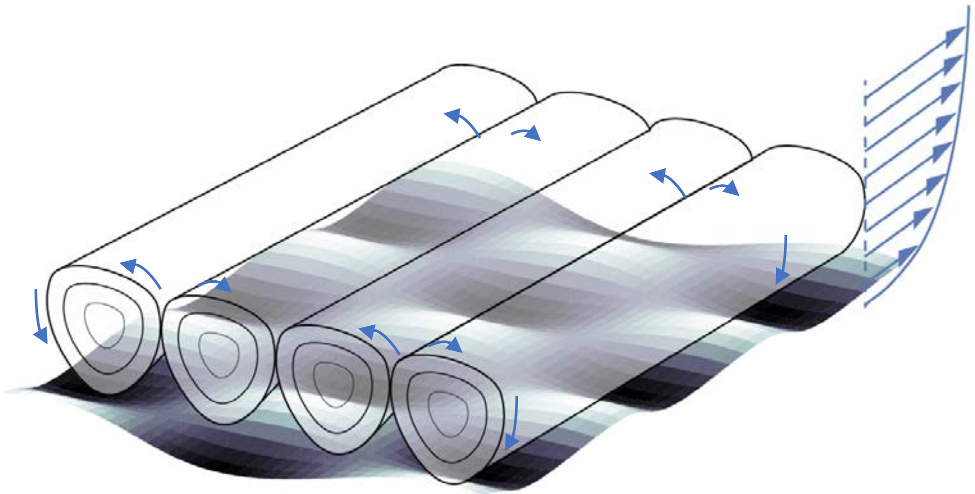
[1]Department of Energy and Process Engineering, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

Introduction

- Conventional Langmuir circulation
  - Occurs due to interaction of waves and near-surface shear[1]
  - Observable as «windrows» on surface, where foam etc gathers in downwelling areas.
  - Important contributor to mixing in the upper ocean[2].
- Neither wind nor surface waves are necessary
  - The Craik-Leibovich mechanisms are purely kinematic interaction between <u>mean shear</u> and <u>wavy fluid motion</u>.



Conventional Langmuir rolls due to crossing waves atop a shear current[1] (CL1 mechanism)



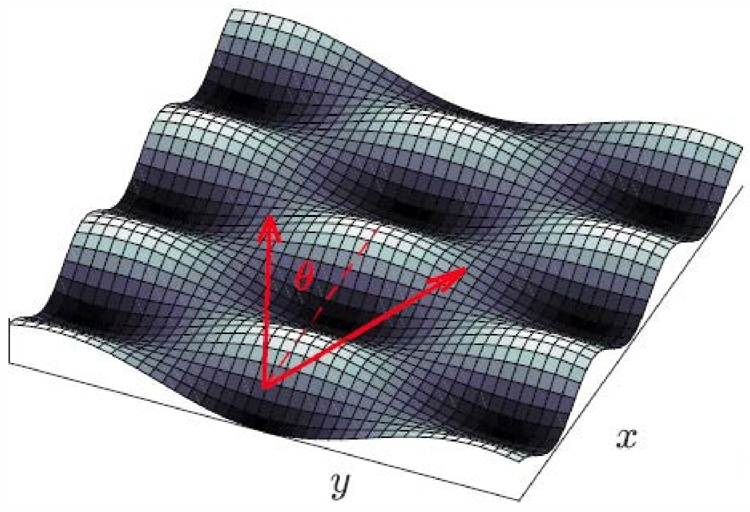
Langmuir rolls due to boundary layer over a criss-cross wavy bottom. (Also CL1 mechanism)

[1] S. Leibovich, *Annu. Rev. Fluid Mech.* **15** 391-427 (1983)
[2] S. E. Belcher *et al.*, *Geophys. Res. Lett.* **39** L18605 (2012)

NTNU

Theory (outline)

- Follows roughly the procedure of Craik 1970[1]. See our manuscript for full details.
- The wavy boundary introduces **perturbations** to the mean shear profile, treated up to **second order** in the wall corrugation steepness.
- Steps:
  - Assume known background flow $U(z)$.
  - Derive (approximate) *linear order* perturbation based on a simplifying model
  - There are 4 second-order modes due to self-advection.
  - One of these modes is resonant, growing linearly with time. It has the form of longitudinal vortices, or «rolls».
- Some more details on the theory and derivation are found on later slides.



Boundary topography with wave-vectors $(k_x, \pm k_y)$ indicated.

[1] A.D.D. Craik, *J. Fluid Mech.* **41** 801-821 (1970)

Theory step 1: approximate linear solution

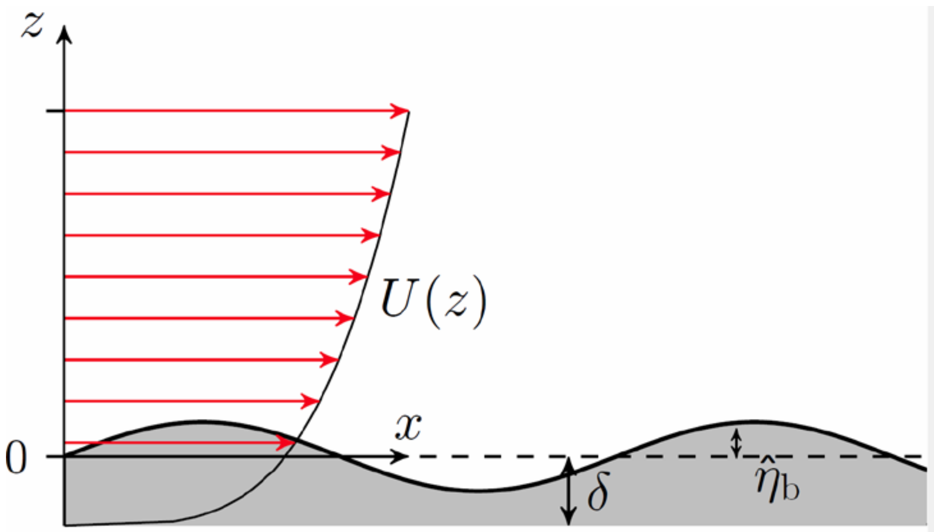
- Model: model the real (no-slip) wall by a *displacement thickness* of the same shape, creating a free slip, impermeable boundary.
  - Captures all essentials of this kinematic effect
- Treat first-order perturbation as *steady* and *inviscid*
  - Viscosity's primary effect is to create mean shear *U*(*z*) and displacement.
- Linearising Euler equation w.r.t. perturbations and eliminating velocities gives a Rayleigh equation for the first-order pressure perturbation $p_1(\boldsymbol{k}, z)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$:

$$p_1'' - 2\frac{U'}{U}p_1' - k^2 p_1 = 0$$

and boundary conditions at bottom/top of domain.

- Easily solved numerically. Analytical solution for power law case $U = z^q$
- 1st order velocities given by

$$u = -\frac{p_1}{U} - \frac{U'p_1'}{k_x^2 U^2}, \qquad v = -\frac{k_y p_1}{k_x U}, \qquad w = \frac{\mathrm{i}p_1'}{k_x U}.$$



Theory step 2: resonant 2nd order mode

- Navier-Stokes equation at 2nd order reduces to

$$(\partial_t - Re^{-1}\nabla^2)\nabla^2 w = \mathcal{R}(z)\,;$$

Reynolds number (based on characteristic velocity and depth)

2nd order vertical velocity perturbation

Convective term, products of 1st order quantities.

with

Found p_1 numerically or analytically on previous slide.

$$\mathcal{R}(z) = 8\frac{k_{1y}^2}{k_{1x}^2}\frac{U'}{U^3}\left[(k_{1x}^2 - k_{1y}^2)p_1^2 + (p_1')^2\right].$$

- Second order harmonics from sums & differences of wave vectors ($\pm k_x$, $\pm k_y$).
- Modes with purely spanwise wave number

  are resonant. Other 2nd order harmonics are negligible.

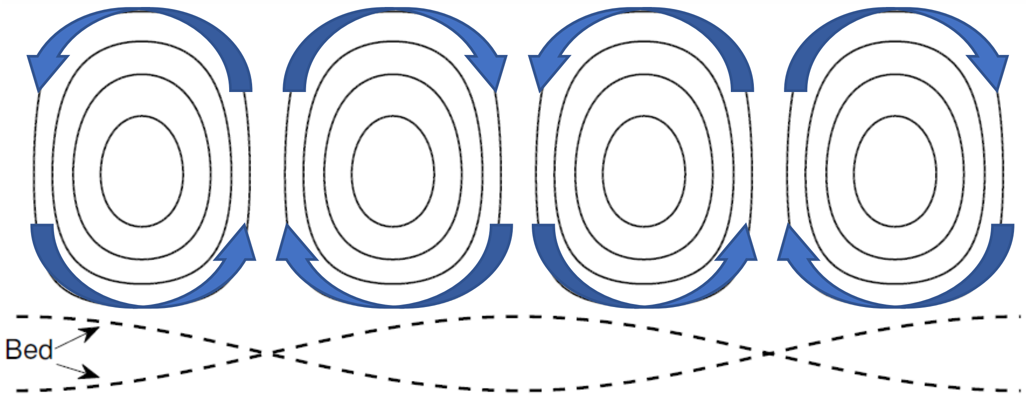
- Solutions can be found in two cases:

1. **Early onset, transient growth**
   - Initially the 2nd order motion is <u>transient</u> and <u>inviscid</u>.
2. **Ultimate steady state**
   - Eventually vortices are stabilised by viscosity, reaching a <u>viscous</u> and <u>steady</u> state.

Theory result: early onset transient growth

- Initial growth assumed essentially inviscid; set $Re^{-1}=0$.
- Results in
$$w = \sum_{\pm} d^{\pm} e^{\pm\kappa z} + w_{\times}(z);$$
with particular solution growing <u>linearly</u> in time*:*
- $$\boxed{w_{\times}(z) = \frac{t}{\kappa}\int_0^z d\xi\, \mathcal{R}(\xi)\, \sinh\kappa(z-\xi).}$$
($d^{\pm}$ chosen to satisfy boundary conditions)
- Spanwise velocity:
$$v = \mathrm{i}w'/\kappa_y$$

- Streamwise velocity increases <u>quadratically</u> with *t*:
$$u(z,t) = -\frac{t}{2}U'(z)w(z,t),$$



Downwelling towards crest/trough-line, upwelling from saddlepoint-line

Theory result: ultimate, steady state solution

- Set transient term in Navier-Stokes to zero. Again a simple solution:

$$w = \sum_{\pm} (d_0^{\pm} + z d_1^{\pm}) \mathrm{e}^{\pm\kappa z} + w_\times(z)\,;$$

$$w_\times(z) = \frac{Re}{2\kappa^3}\int_0^z \mathrm{d}\xi\, \mathcal{R}(\xi)\, G[\kappa(z-\xi)];$$

$$G(Z) = \sinh(Z) - Z\cosh(Z).$$

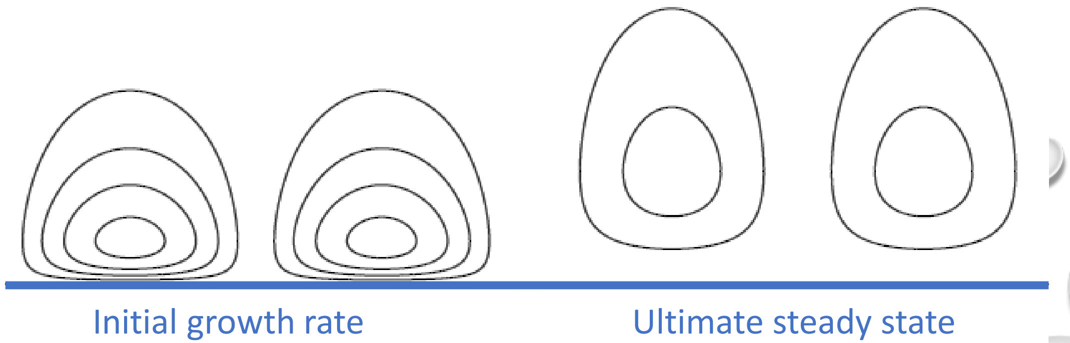
- As before, $v = \mathrm{i}\,w'/\kappa_y$, and we find the streamwise 2nd order velocity*

$$u(z) = \sum_{\pm} d_u^{\pm}\mathrm{e}^{\pm\kappa z} + \frac{Re}{\kappa}\int_0^z \mathrm{d}\xi\, U'(\xi) w(\xi) \sinh\kappa(z-\xi).$$

Note that spanwise and vertical velocities *u,w* scale as *Re*, but the streamwise velocity *u* scales as *Re*^{2}.

- Initial growth closer to the wall; vortices moving towards the bulk before steady state.

Example: «deep» water (upper boundary far away):

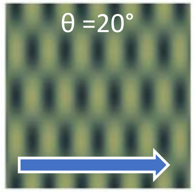


NTNU

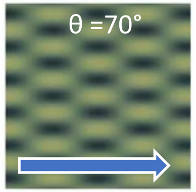
* see manuscript for explicit expressions for the *d*-coefficients

Theory result: dependence on crossing angle θ

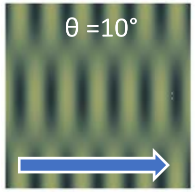
- Circulation strongest for «protracted eggcarton», θ ~ 10° - 20°



Initial growth rate greatest around 20°

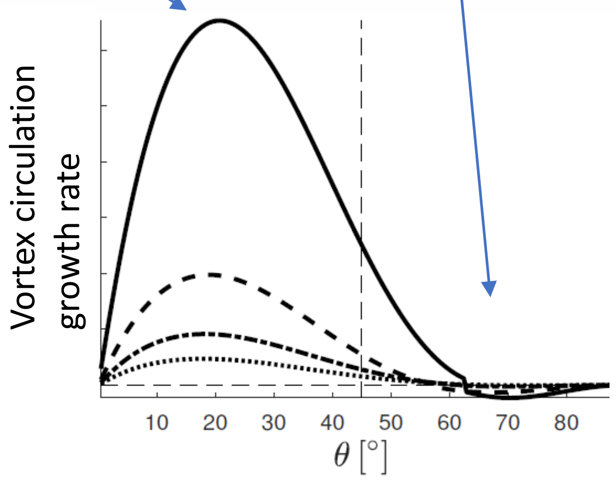


Vortices can change sign at large θ, but are then weaker

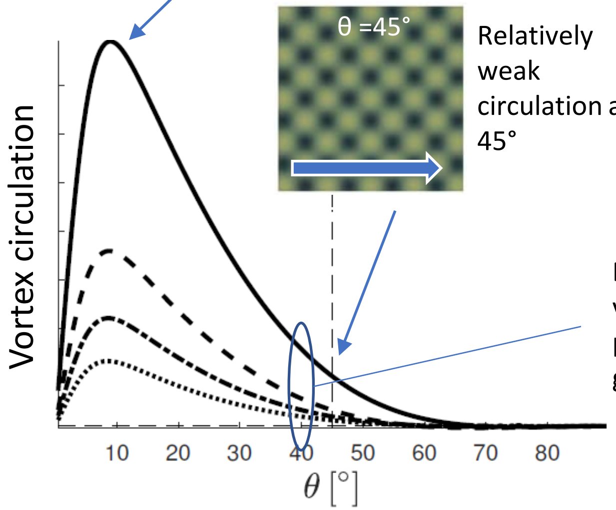


Ultimate vortex strength peaks around 10°

Relatively weak circulation at 45°



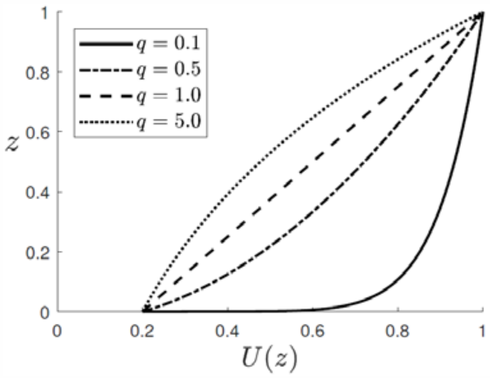
Initial growth rate



Ultimate steady state

Different velocity profiles, same general trend.

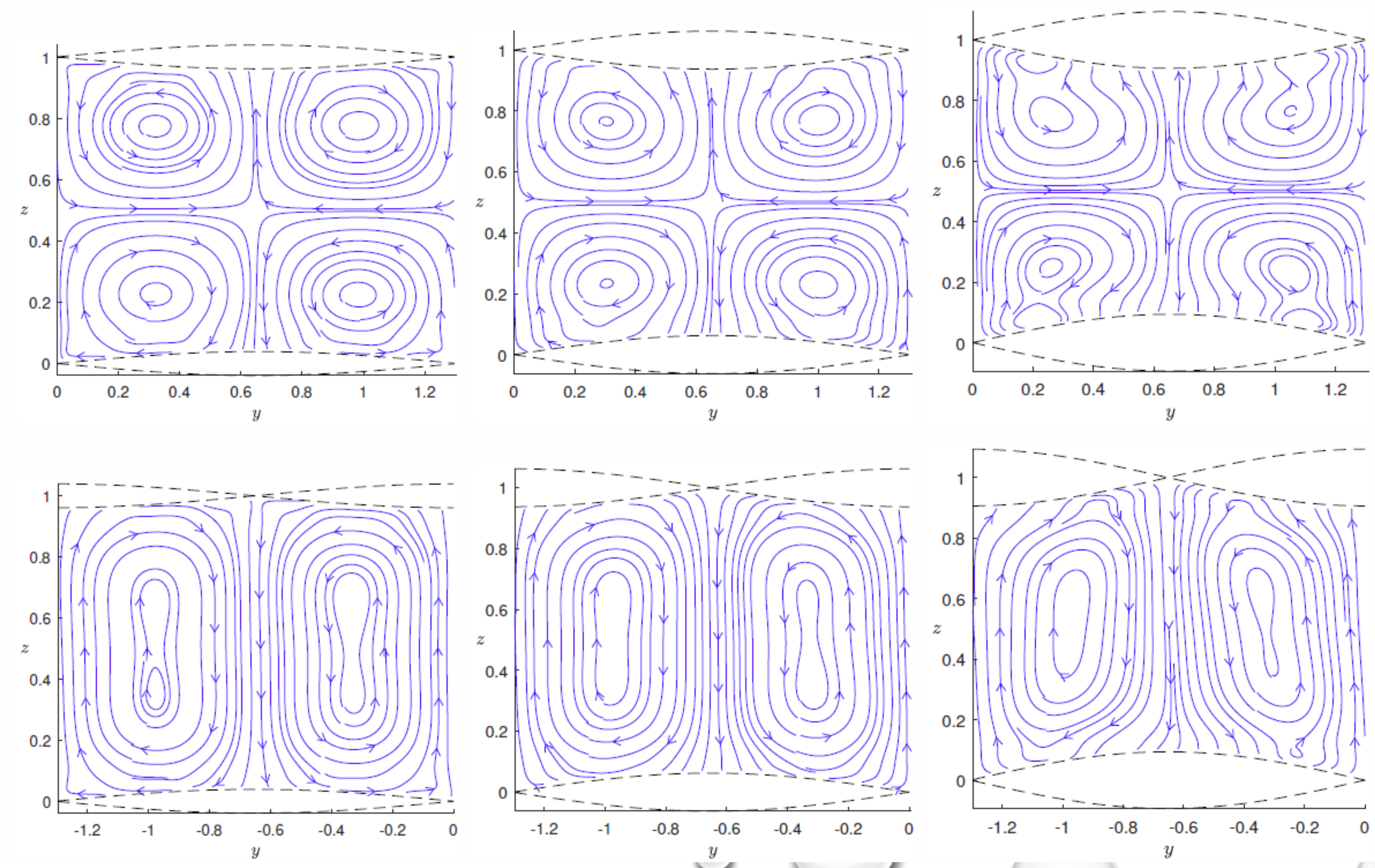
Velocity profiles z^q



Confirmation by numerical simulation

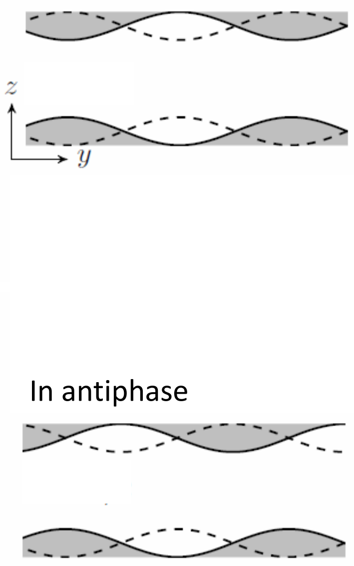
- The phenomenon is confirmed by **simulation** (Lattice-Boltzmann method) of laminar flow between two plates with crossing-wave pattern.
- Tested effect of amplitude, Re and spanwise phase shift.

Increasing amplitude of «wall waves»



In phase

In antiphase

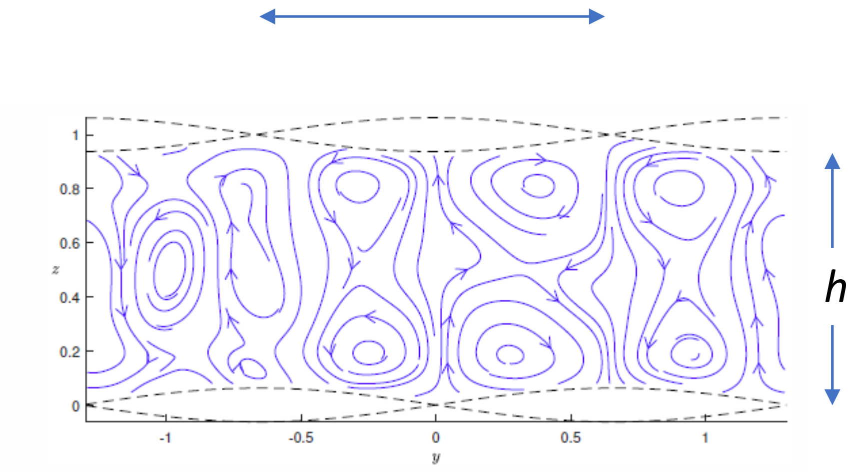


θ = 22.5° , Re ≈ 400.

Confirmation by numerical simulation

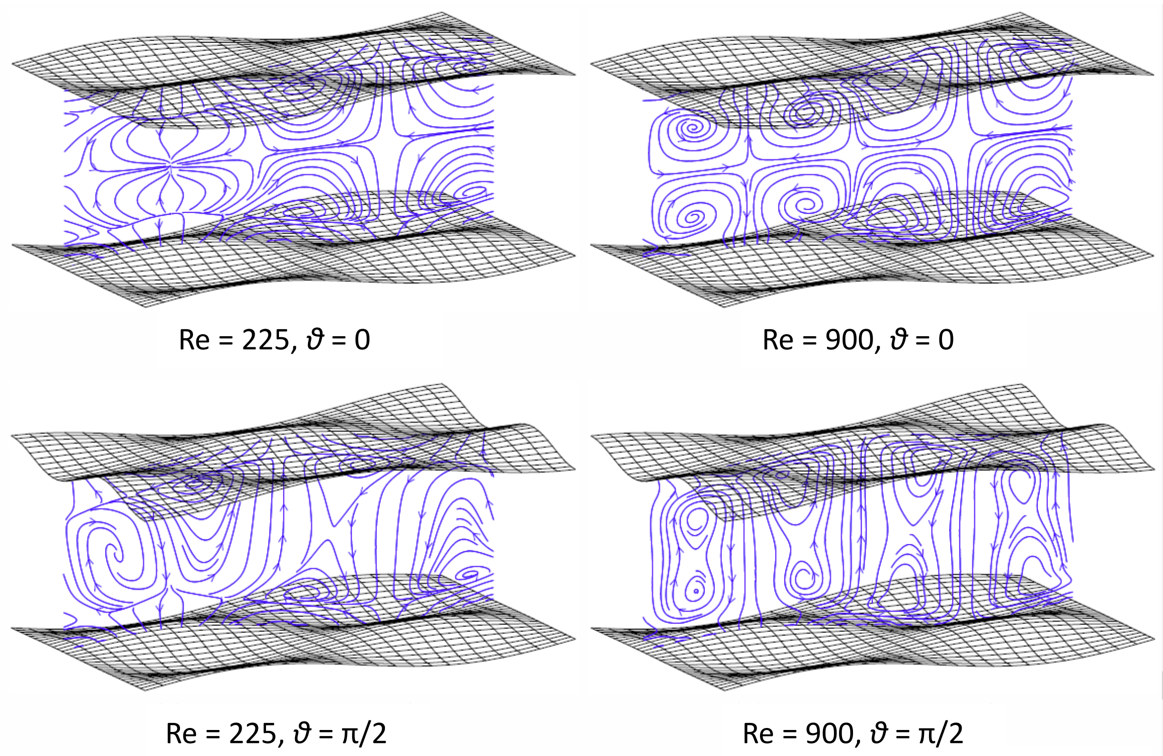
- The phenomenon is confirmed by **simulation** (Lattice-Boltzmann method) of laminar flow between two plates with crossing-wave pattern.

- Tested effect of amplitude, Re and spanwise phase shift.



Vortices become unstable at high Reynolds numbers.
Here Re = 1600; Geometry: , $\vartheta = \pi/2$, θ = 22.5°, h = $\lambda/2$.

Flow snapshots: cross-flow plane



Streamlines slices of full three-dimensional flow field. $a \approx 0.0625$, h = $\lambda/2$
$\theta \approx \pi/8$. Slice at $x = \pi/6k_x$.

NTNU