1. Introduction

Complex volumetric failure patterns are observed from well-recorded large and small earthquakes [1,2] as well as in laboratory experiments [3]. To understand the mechanics of slip in extended fault zones, the TEAR project aims to model how faults slip based on the conservation of mass, momentum, and energy using rheological models of generalized visco-elastic-plastic materials. We here explore two diffuse fault zone approaches extending the modeling of dynamic earthquake rupture beyond treatment as a discontinuity in the framework of linear elastodynamics (i) a PETSc [15] spectral element adaptation of the stress-glut method [3] and (ii) the CPR unified first order hyperbolic formulation of hyperelasticity for continuum mechanics [7,8,9] for dynamic rupture using a high order Discontinuous Galerkin scheme and the ExaHyPE PDE engine [5].

2. Reference Problem

With both methods, we solve the reference problem of a self-similar shear crack problem [10,12]. The 2D problem [12] consists of a homogeneous and isotropic elastic medium, a linear slip undergoing friction law, and the crack propagating along the x-axis as depicted in Fig.1. The initial conditions are [10]

\[ \begin{align*}
\text{Density } (\rho) &= 2500 \text{ kg/m}^3 \\
\text{P-wave velocity } (V_p) &= 6000 \text{ m/s} \\
\text{P-wave speed } (V_s) &= 400 \text{ MHz} \\
\text{Shear stress } (\gamma) &= 2500 \text{ m/s} \\
\text{Sliding speed } (V) &= 2000 \text{ m/s} \\
\text{Density } (\rho) &= 2500 \text{ Kg/m}^3 \\
\text{P-wave speed } (V_p) &= 2309 \text{ m/s} \\
\text{Shear speed } (V_s) &= 20 \text{ m/s} \\
\text{Shear stress } (\gamma) &= 20 \text{ MPa} \\
\text{Characteristic distance } (L) &= 250 \text{ m} \\
\text{Sliding speed } (V) &= 2000 \text{ m/s} \\
\end{align*} \]

This reference uses an externally imposed traction, and while it does not include material failure. Instead of an infinitesimal thin fault interface, we model a diffuse fault zone with non-planar faults and time-dependent fault geometries. In addition to this, we can refine the inelastic fault zone thickness [8]. Our stress glut implementation conditions: the stress to the critical value of elastic stress yielding inside the inelastic zone. Then, the residual strain is translated into slip rates on the velocity plane, located on the outer limits of the inelastic fault zone. Depending on the reference solution that we want to compare against, we can vary h, p as refinement parameters.

\[ h, p \text{- Refinement} \]

3. A Spectral Element Stress Glut Approach

3.1 Method

The stress glut (SG) method was originally developed for the finite difference method [13,14]. There, the SG approximates the fault jump conditions through metastable movements in a zone grid step width inelastic zone. Here we extend the SG method to spectral elements (SESE) using PETSc [15]. The fault zone is represented using a spectral distance function (Fig. | SESE). The SG method has the potential to allow for arbitrary fault orientation with respect to the mesh. This includes the use of non-planar faults and time dependent fault geometries. Our method exploits the fact that the stress can be described locally within each element using (local-element-wise) defined quantities.

3.2 Fault representation

The spectral distance function (SDF) represents the fault zone independently of the mesh discretization and provides a straightforward manner to compute the normal of the fault. The extended fault is defined by all coordinates for which the absolute value of SDF is < ε. Thus, the fault thickness is given by 2ε.

3.3 Refinement Study

The spectral element method allows us to refine the spectral resolution of the mesh (h) and the polynomial degree (p) of the basis functions. Additionally to this, we can refine the inelastic fault zone thickness (δ). Our stress glut implementation conditions: the stress to the critical value of elastic stress yielding inside the inelastic zone. Then, the residual strain is translated into slip rates on the velocity plane, located on the outer limits of the inelastic fault zone. Depending on the reference solution that we want to compare against, we can vary h, p as refinement parameters.

\[ h, p \text{- Refinement} \]

4. Nonlinear hyperelasticity for dynamic rupture in nonlinear elasto-plastic material with a high order accurate discontinuous Galerkin (DG) method

4.1 The Codinov-Peshkov-Romenski (CPR) framework

Instead of an infinitesimal thin fault interface, we model a diffuse fault zone embedded in a continuum visco-elastic-plastic material. We use a unified first order hyperbolic formulation of continuum mechanics, the CPR model [7,8], which obeys the first and second law of thermodynamics. The CPR model is an extension of nonlinear hyperelasticity, which is able to describe simultaneously nonlinear-elastic solids at large strain, as well as viscous and ideal fluids. The CPR model can account for nonlinear dynamic rupture via an additional scalar describing material damage governed by an advection-reaction equation [9]. We here adapt this method for laboratory derived friction laws of compressional shear cracks in fault zones. We solve the hyperbolic PDE system using the high order method for laboratory derived friction laws of compressional shear cracks in fault zones. We solve the hyperbolic PDE system using the high order method for laboratory derived friction laws of compressional shear cracks in fault zones. We solve the high order method for laboratory derived friction laws of compressional shear cracks in fault zones.

4.3 Refinement analysis

Refinement analysis for a self-similar crack model. Each line represents an off-fault station at a certain distance away from the fault center. Fault width is 102 m, spatial resolution is 102 m, 102 m, 102 m, and 102 m. The width of the diffuse fault zone is 5, 20, 50, and 100 m. The equivalent FV sub-cell size is 5, 102 m, 102 m, 102 m, and 102 m. Slip rate, slip, and stress drop decrease with fault zone when the fault zone width is constant. Fault zone width decrease: 200 m vs. 71 m. Slip rate and stress drop decrease with fault zone when the fault zone width is constant.

5. Summary and Outlook

We extend the classical stress glut method to spectral elements. The implementation aims to permit modeling of non-planar faults and time-dependent fault geometries. Refinement analysis for a self-similar crack problem shows slip and slip rate approaching the reference solution when increasing polynomial order, or decreasing mesh size and fault zone thickness simultaneously.

Shrinking the thickness of the fault zone implies formation of a discontinuity in slip and slip rate, as in classical dynamic rupture modeling.

We extend the CPR model of continuum mechanics with an additional dynamic process that describes material failure during dynamic earthquake rupture. Damage is governed by an advection-reaction equation, and the reaction source term can resemble friction laws.

Refinement analysis for a self-similar crack problem shows, that on-fault slip rate and slip increase with polynomial order when the fault zone width is kept constant. On-fault slip and stress drop decrease with fault zone width when polynomial order is constant.

We next extend both methods to fully dynamic rupture benchmarks, specifically including off-fault plasticity.