Modelling earthquake rupture dynamics across diffuse deforming fault zones

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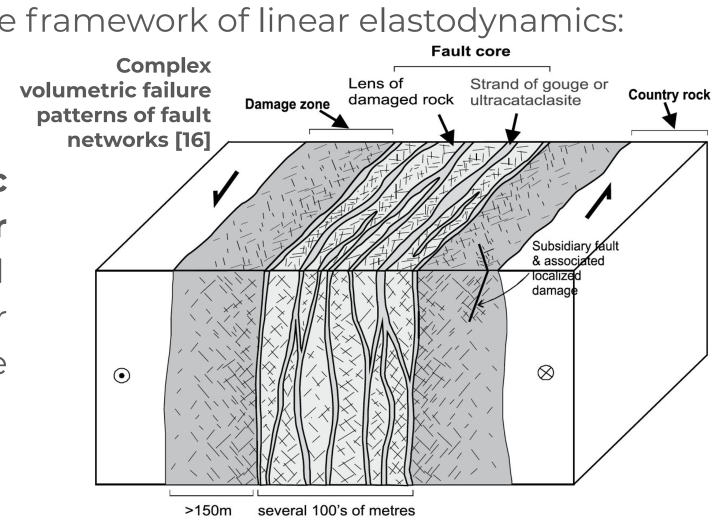
1. Introduction

Complex volumetric failure patterns are observed from well-recorded large and small earthquakes [1,2] as well as in laboratory experiments [4]. To understand the mechanics of slip in extended fault zones, the TEAR project (https://www.tear-erc.eu) aims to model **how faults slip** based on the conservation of mass, momentum, and energy using rheological models of **generalized visco-elasto-plastic materials**. We here explore two diffuse fault zone approaches extending the modeling of dynamic earthquake rupture beyond treatment as a discontinuity in the framework of linear elastodynamics:

(i) a **PETSc** [15] **spectral element adaptation of the stress-glut method** [3]

(ii) the **GPR unified first order hyperbolic formulation of hyperelasticity for continuum mechanics** [7, 8, 9] extended for dynamic rupture using a high order Discontinuous Galerkin scheme and the **ExaHyPE** PDE engine [5].



2. Reference Problem

With both methods, we solve the reference problem of a kinematic self-similar shear crack [10]. The 2D problem [12] consists of a homogeneous and isotropic elastic medium, and the crack propagating along the x axis as depicted in Fig 1. The initial conditions are [10]:

• Density (ρ) = 2500 kg/m³	• μ_s = 0.5, μ_d = 0.25
• P-wave velocity (V_p) = 4000 m/s	• S-wave velocity (V_s) = 2309 m/s
• Normal stress (S_{yy}) = -40 MPa	• Shear stress (S_{xy}) = 20 MPa
• Characteristic distance (L) = 250 m	• Sliding speed (V) = 2000 m/s

This reference uses an externally imposed traction, and while it does not include the fully dynamic behaviour, it allows to verify the relation between slip, slip rate and traction [10]. We perform numerical refinement analysis based on fault receivers located at 0, 2, 4, 6, 8 km along-strike and a set of receivers perpendicular to the fault at 0, 50, 150, 200, 220m.

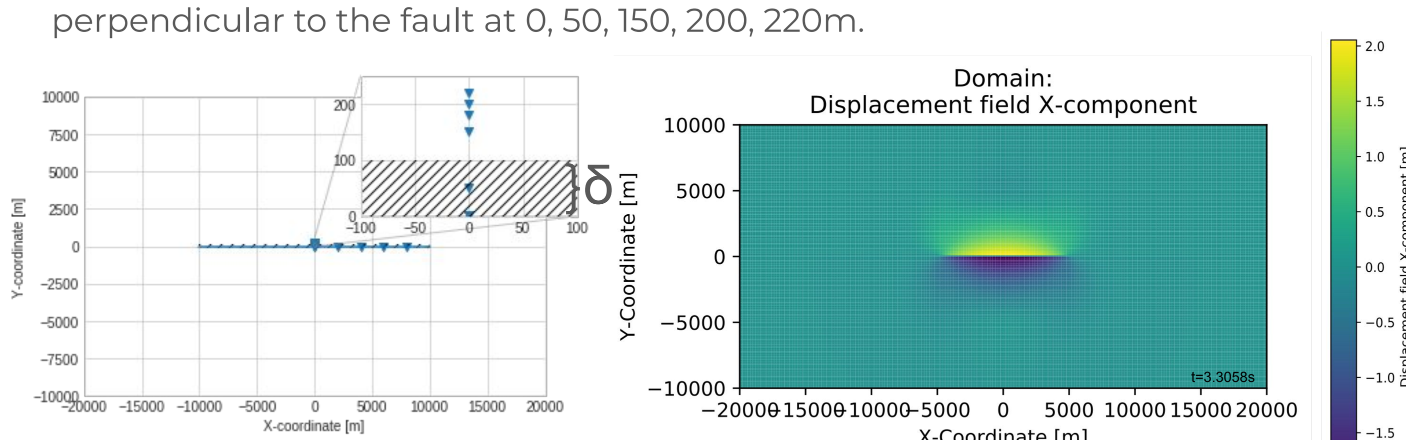


Fig 1. (Left) Fault geometry, model domain, receiver locations (along and perpendicular the fault), and fuzzy fault zone in a signed distance function (SDF) representation. (Right) x component of the displacement from our FE stress-glut method.

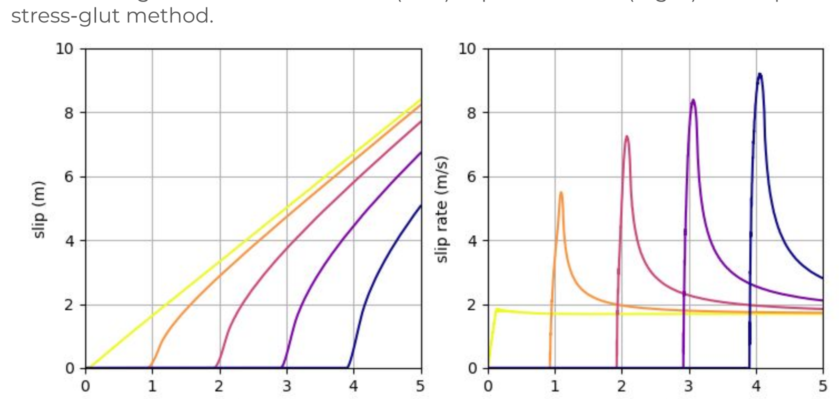


Fig 2. Reference solution using SEM2DPACK (https://github.com/jpampuero/sem2dpack) for (left) slip and (right) slip rate at the five along-fault stations shown in Fig. 1 . Spatial resolution is 100 meter and polynomial order 6 with Kelvin-Voigt viscous damping.

3. A Spectral Element Stress Glut Approach

3.1 Method

The stress glut (SG) method was originally developed for the finite difference method [13, 14]. There, the SG approximates the fault-jump conditions through inelastic increments to the stress components in a one grid step width inelastic zone. Here we extend the SG method to spectral elements (SESG) using PETSc [15]. The fault zone is represented using a signed distance function (Fig. 1) SESG has the potential to allow for arbitrary fault orientation with respect to the mesh. This includes the use of non-planar faults and time dependent fault geometries. Our method exploits the fact that the stress can be defined locally within each element using local (element-wise) defined quantities.

3.2 Fault representation

The signed distance function (SDF) represents the fault zone independently of the mesh discretization and provides a straightforward manner to compute the normal of the fault. The extended fault is defined by all coordinates for which the absolute value of SDF is <= δ. Thus, the fault thickness is given by 2δ.

3.3 Refinement Study

The spectral element method allows us to refine the spatial resolution of the mesh (h) and the polynomial degree (p) of the basis functions. Additional to this, we can refine the inelastic fault zone thickness (δ). Our stress glut implementation conditions the stress to the critical value of elastic stress yielding inside the inelastic zone. Then, the residual strain is translated into slip rates on the velocity plane, located on the outer limits of the inelastic fault zone.

Depending on the reference solution that we want to compare against, we can vary h, p, δ as refinement parameters.

h-, p- and δ- Refinement

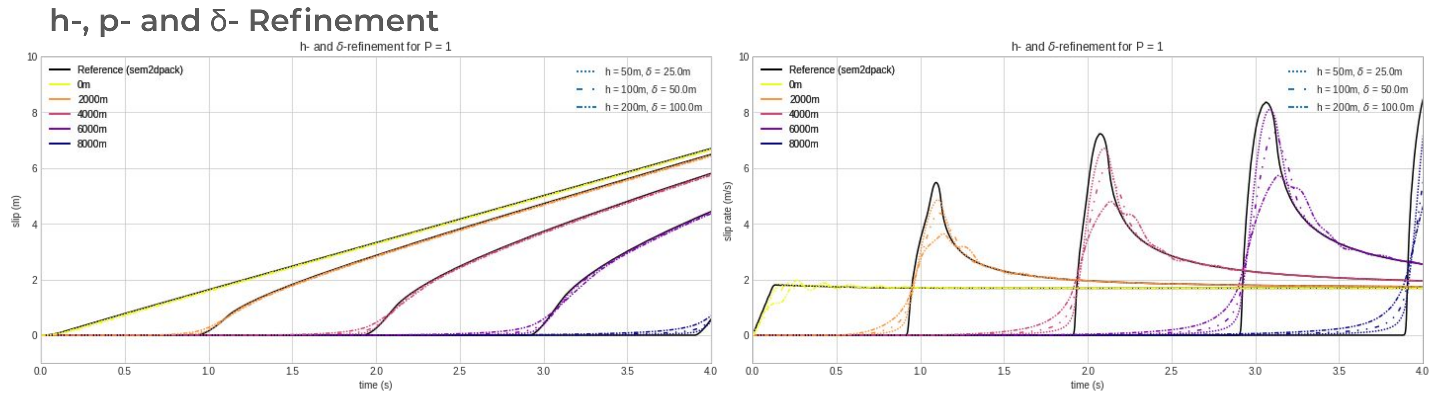


Fig 3 Slip (left) and slip rate (right) for different cell sizes (h) and fault thickness (δ). Solid lines represent the reference using SEM2DPACK. Non-continuous lines are the result of the simulations. All simulations used p = 1.

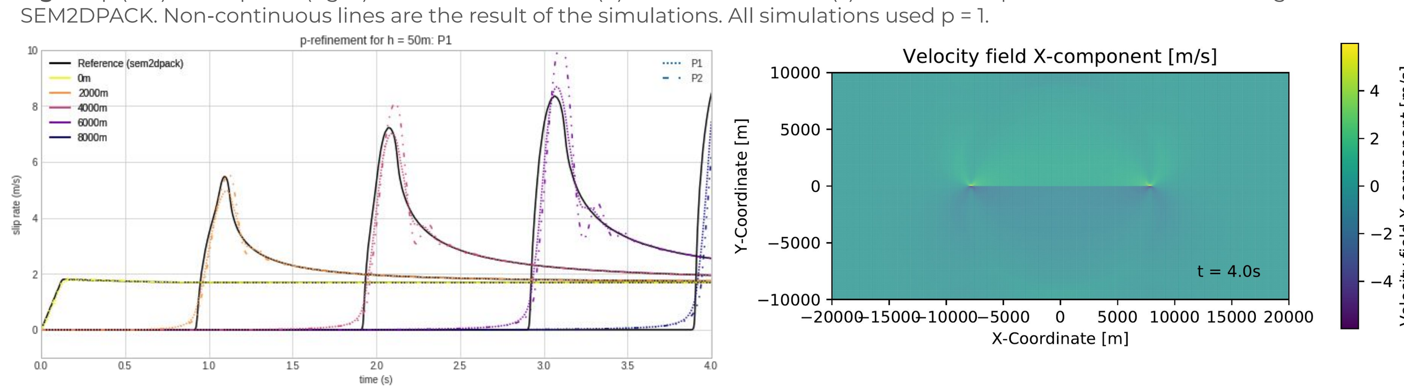


Fig 4. (left) Slip rate under p-refinement. Solid lines are the reference using SEM2DPACK. Dotted lines: h = 50 m, p = 1, δ = 25 m. Dashed and dotted lines: h = 50 m, p = 2, δ = 25 m. With the latter conditions, we plot the x component of the velocity field from our FE stress-glut method (right).

Receivers locations perpendicular to fault orientation

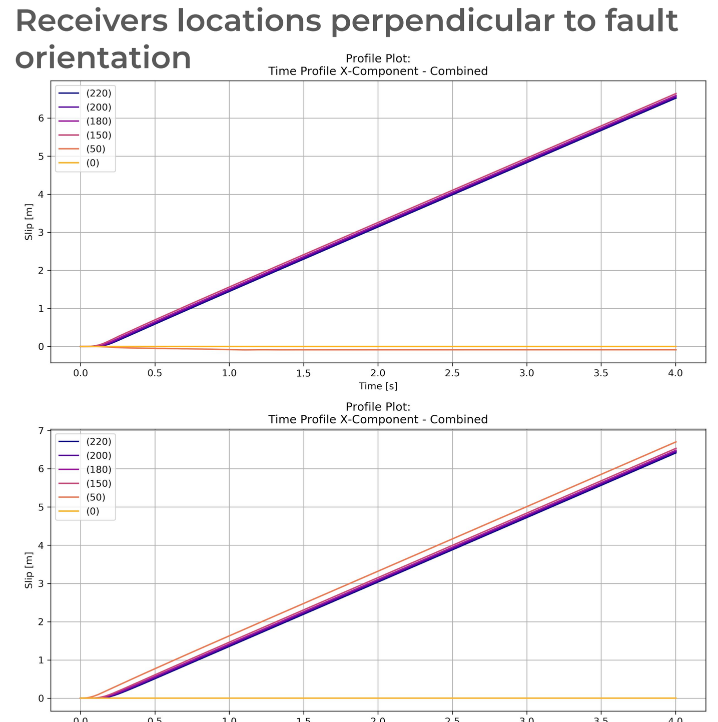


Fig 5. Slip profiles across the fault. Element size (h) = 100 m, p = 2 with δ =100 m (top) and δ = 50 m (bottom). Here, the receivers are located at 0, 50, 150, 200, 220m, perpendicular to the fault. The slip values for the receiver at 50 m become comparable with near off-fault receivers after reducing the inelastic fault thickness to 50 m.

4. Nonlinear hyperelasticity for dynamic rupture in nonlinear elasto-plastic material with a high order accurate discontinuous Galerkin (DG) method

4.1 The Godunov-Peshkov-Romenski (GPR) framework

Instead of an infinitesimal thin fault interface, we model a diffuse fault zone embedded in a continuum visco-plasto-elastic material. We use a unified first order hyperbolic formulation of continuum mechanics, the GPR model [7, 8], which obeys the first and second law of thermodynamics. The GPR model is an extension of nonlinear hyperelasticity, which is able to describe simultaneously nonlinear elasto-plastic solids at large strain, as well as viscous and ideal fluids. The GPR model can account for nonlinear dynamic rupture via an additional scalar describing material damage governed by an advection-reaction equation [9]. We here adopt this method for laboratory derived friction laws of compressional shear cracks in fault zones.

We solve the hyperbolic PDE system using the high performance computing (HPC) toolkit ExaHyPE (https://exahype.eu/), which employs the Arbitrary derivative (ADER) high order discontinuous Galerkin (DG) finite element method [5]. To achieve numerical stability, the numerical scheme employs an a-posteriori sub-cell finite volume limiter on space time adaptive Cartesian meshes [9, 11].

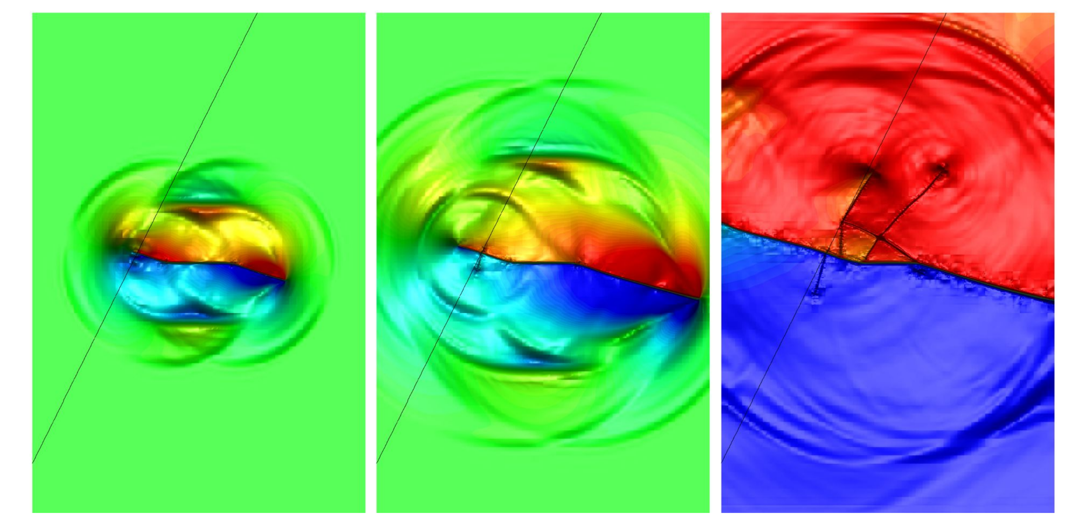


Fig. 6. Evolution of spontaneously growing dynamic rupture and secondary off-plane rupture in a heterogeneous material at 0.5, 0.9, 5.0 sec simulated using the GPR model [Fig. 15 of reference 9]. Here, a von Mises yielding criterion is applied for the off-fault plastic damage. The two types of materials (separated by the black line) are characterized by the critical stress of 180 MPa and 240 MPa, respectively. Supershear rupture can be observed around at the crack front tip. .

GPR formalism:

1) Conservation of mass, momentum and energy:

$$\frac{\partial \alpha}{\partial t} + v_k \cdot \frac{\partial \alpha}{\partial x_k} = 0 \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_k)}{\partial x_k} = 0 \qquad (2)$$

$$\frac{\partial (\alpha \rho v_i)}{\partial t} + \frac{\partial (\alpha \rho v_i v_k + \alpha p \delta_{ik} - \alpha \sigma_{ik})}{\partial x_k} = 0 \qquad (3)$$

$$\frac{\partial A_{ik}}{\partial t} + \frac{\partial A_{im} v_m}{\partial x_k} + v_m \left(\frac{\partial A_{ik}}{\partial x_m} - \frac{\partial A_{im}}{\partial x_k}\right) = -\frac{1}{\theta_1(\tau_1)} E_{A_{ik}} \qquad (4)$$

$$\frac{\partial J_k}{\partial t} + \frac{\partial (J_m v_m + T)}{\partial x_k} + v_m \left(\frac{\partial J_k}{\partial x_m} - \frac{\partial J_m}{\partial x_k}\right) = -\frac{1}{\theta_2(\tau_2)} E_{J_k} \qquad (5)$$

$$\frac{\partial \xi}{\partial t} + v_k \cdot \frac{\partial \xi}{\partial x_k} = -\theta E_{\xi} \qquad (6)$$

$$\frac{\partial \rho S}{\partial t} + \frac{\partial (\rho v_k S + \rho E_{J_k})}{\partial x_k} = \frac{\rho}{T}\left(\frac{1}{\theta_1} E_{A_{ik}} E_{A_{ik}} + \frac{1}{\theta_2} E_{J_k} E_{J_k} + \theta E_{\xi} E_{\xi}\right) \geq 0 \qquad (7)$$

$$\frac{\partial (\rho E)}{\partial t} + \frac{\partial (v_k \rho E + v_i (p \delta_{ik} - \sigma_{ik}))}{\partial x_k} = 0 \qquad (8)$$

α is the diffuse interface coefficient, v_k is velocity, ρ is density, p is hydrostatic pressure, σ_{ik} is total stress tensor; A_{ik} is distortion field, J_k is thermal impulse, T is temperate, S is the entropy, θ is a function of the state parameters determining the rate of damage, ξ is damage coefficient, the total energy potential $E = E(\rho, S, v, A, J, \xi)$, τ_1 and τ_2 is shear or thermal relaxation time.

2) Constitutive laws:

$$\sigma_{ij} = \sigma'_{ij} + p \delta_{ij} \qquad (9)$$

$$\sigma'_{ij} = \rho A_{jk} E_{A_{jk}} + \rho J_i E_{J_k} \qquad (10)$$

$$P = \rho^2 E_{\rho} \qquad (11)$$

σ_{ij} is total stress tensor; σ'_{ij} is deviator stress tensor; P isotropic pressure.

3) Damage evolution:

$$\tau_1 = \frac{1}{(1-\xi)/\tau_I + \xi/\tau_D} \qquad (12)$$

$$\tau_D = \tau_{D0} \exp(\alpha_1 - \beta_1 (1-\xi) Y) \qquad (13)$$

$$\tau_I = \tau_{I0} \exp(\alpha_2 - \beta_2 \xi Y) \qquad (14)$$

$$\theta = (1-\xi)(\xi + \xi_\epsilon)[(1-\xi)(\frac{Y}{Y_0})^a + \xi(\frac{Y}{Y_1})] \qquad (15)$$

τ_D and τ_I is shear relaxation time for damaged and intact materials. $\tau_{D0}, \tau_{I0}, \alpha_1, \beta_1, \alpha_2, \beta_2$ are constant parameters for various rock materials. Y is critical stress from von Mises criterion. a, Y_0, Y_1 are constants.

(Details can be found in Tavelli. et al., 2020 arXiv:2003.02760)

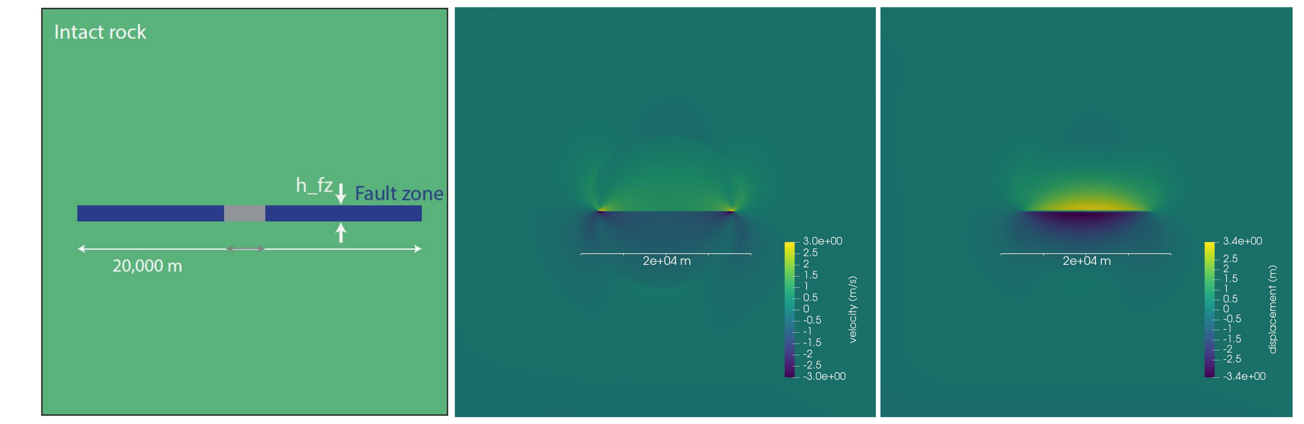


Fig. 7. (Left) Sketch of the fault zone setup (not to scale). The initial condition and material properties are stated in Sec. 2. (Middle) Velocity field (x-component) at 4 sec in the rupture model. Length scale is 20,000 m. The width of the diffuse fault zone is h_fz=100 m. (Right) Displacement field (x-component) at 4 sec. The emanated wave field is shown as horizontal particle velocity.

4.3 Refinement analysis

➢ **p-refinement: order 3 vs. order 6**

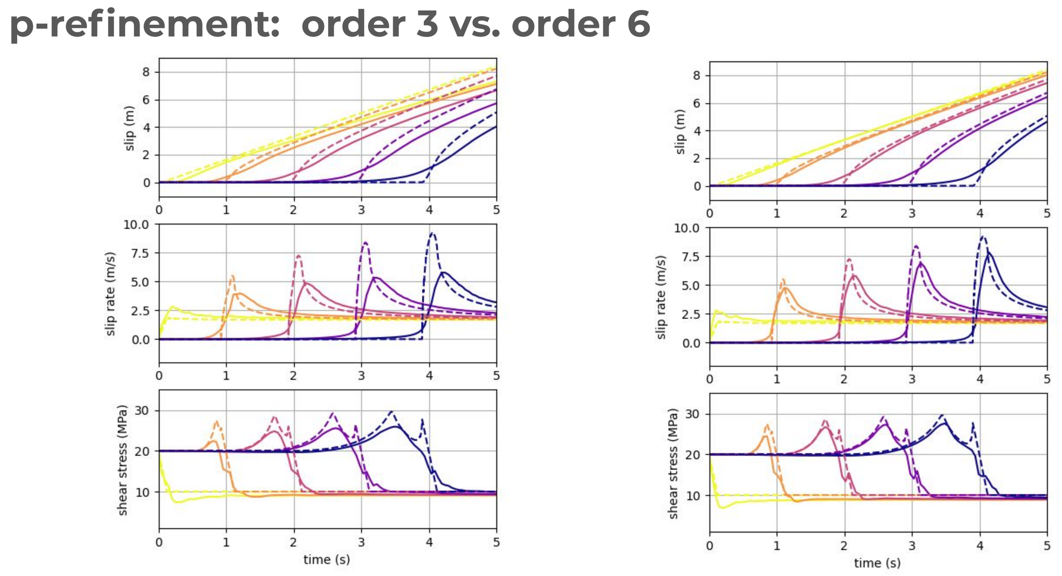


Fig. 8. On-fault results of a self-similar crack model (solid line) with reference model (dashed line). Each line represents an on-fault station at a certain distance away from the fault center. Fault zone width is 100 m. Spatial resolution is 220 m and polynomial order 3 (left) and order 6 (right). The equivalent FV sub-cell sizes are ~31 and ~17 m, respectively. Slip rate, slip and stress drop will increase with polynomial order when the fault zone width is constant.

➢ **static Adaptive Mesh Refinement (AMR) feature**

Fig 9. (left) Static adaptive mesh refinement (AMR) in ExaHyPE engine. Refined region with Finite Volume limiter domain (red) and coarse ADER-DG domain (blue). A layer of help cell (light blue) outlines FV domain which enable numerical stability. (Right) Shear stress distribution at 4 sec demonstrates AMR feature.

5. Summary and Outlook

- ➢ We extend the classical **stress glut** method to **spectral elements**. The implementation aims to permit modeling of **non-planar faults and time-dependent fault geometries.**
- ➢ **Refinement analysis** for a self-similar crack problem shows slip and slip rate approaching the reference solution when increasing polynomial order, or decreasing mesh size and fault zone thickness simultaneously.
- ➢ Shrinking the thickness of the fault zone implies formation of a discontinuity in slip and slip rate, as in classical dynamic rupture modeling.
- ➢ We extend the **GPR model of continuum mechanics** with **an additional dynamic process that describes material failure** during dynamic earthquake rupture. Damage is governed by an advection-reaction equation, and the reaction source term can resemble friction laws.
- ➢ Refinement analysis for a self-similar crack problem shows, that on-fault slip rate and slip increase with polynomial order when the fault zone width is kept constant. Slip rate and slip will decrease with fault zone width when polynomial order is constant.
- ➢ On-fault slip rate as well as wavefields of both methods are comparable to the reference.
- ➢ We will next extend both methods to full dynamic rupture benchmarks, specifically including off-fault plasticity.

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