On associating significance levels with temporal changes in empirical orthogonal function analysis: a case study for ENSO

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Outline

Introduction

Time dependence of $\lambda$

Time dependence of EOF
Introduction

Time dependence of $\lambda$

Time dependence of EOF
Temporal changes of what?

- The variability in a field \( T(x; t) \) is described correctly by an ensemble of different realizations \( T_r(x; t) \) at any given fixed time instant \( t \).
- This variability can be decomposed to EOFs:
  \[
  T_r(x) = \sum_k PC_r^{(k)} EOF^{(k)}(x).
  \]
- An amplitude can be associated with each EOF: \( \lambda^{(k)} \).
- The EOFs and the amplitudes can be different in different “instants of time” (years): e.g., we have \( \lambda^{(k)}(t) \) (cf. EGU2020-2894; EGU2020-7527; EGU2020-12061; Maher et al., Geophys. Res. Lett. 45, 11,390, 2018; Haszpra et al., J. Climate 33, 3107, 2020; Haszpra et al., Earth System Dynam. 11, 267, 2020).
- How do we detect this time dependence if we can only estimate these quantities from a finite number of ensemble members?
- The methodology is the same for any \( k \), we thus omit the index \( k \).
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Example: $\lambda^{(1)}$ of the Pacific SST field

MPI-GE historical + RCP8.5

Do we have a trend here?

Linear fit:

$$(2.08 \pm 2.06)^{-5} \text{[K/yr]}$$

$\Longrightarrow$ Yes, we have?
Concept for testing

- A linear fit provides with an upper bound on the significance level for a temporal change, since
  - either the dependence is really linear (unlikely),
  - or not, so that the numerical result is meaningless, but then \( \lambda \) cannot be constant: there is a temporal change!
(Thanks to T. Bodai.)

- However: the errors \( \Delta \lambda \) are heteroskedastic and autocorrelated
  \( \implies \) the traditional significance level is incorrect
  \( \implies \) generalized least squares fit must be used

based on the variance-covariance matrix \( \sigma_{ij}^2 = \langle \Delta \lambda(t_i) \Delta \lambda(t_j) \rangle \).
Methodology

- Estimating the variance-covariance matrix of the errors: between years $t_i$ and $t_j$:

$$\sigma^2_{ij} = \langle \Delta \lambda(t_i) \Delta \lambda(t_j) \rangle = \frac{1}{N} \langle PC(t_i)PC(t_j) \rangle,$$

and the latter can be estimated just by using the ensemble!

- Improving the estimation: fitting a Markovian form to the corresponding correlation matrix:

$$
\begin{pmatrix}
1 & \rho_1 & \rho_1 \rho_2 \\
\rho_1 & 1 & \rho_2 \\
\rho_1 \rho_2 & \rho_2 & 1
\end{pmatrix}
$$

(The fit is performed in a sliding window that is long enough to ensure independence on the window size.)
Do we have a trend here?

Linear fit:

\[
(2.08 \pm 2.06) \times 10^{-5} \text{ [K/yr]}
\]

\[
(0.11 \pm 2.39) \times 10^{-5} \text{ [K/yr]}
\]

\[\Rightarrow\] Not significant.
Subintervals in the example

Grayscale: insignificant, saturated: 5% significant slope.

Yes, there are significant trends! (Warning: multiple testing, see Wilks, BAMS 97, 2263, 2016.)

- Strengthening ENSO before 1900,
- forced fluctuations until 2050
- weakening in the late 21st century.
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Future work...