A simple transport rate relation that unifies aeolian and fluvial nonsuspended sediment transport

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Highlights

- Derivation of simple capacity transport rate relation guided by DEM-based simulations of nonsuspended sediment transport
- Relation agrees with measurements for turbulent bedload and aeolian saltation, both from weak to intense conditions
- Capacity transport rate is controlled by the kinetic fluctuation energy and kinetic energy balances of transported particles
- Capacity transport rate does not depend on the nature of bed sediment entrainment, neither driven by the flow nor by splash

Outline

1. Introduction
2. Motivation
3. Unifying relation
4. Experimental validation
5. Physical origin

Introduction

Environmental parameters:

- Particle density ρ_p [kg/m^3]
- Particle diameter d [m]
- Fluid density ρ_f [kg/m^3]
- Kinematic fluid viscosity ν_f [m^2/s]
- Fluid shear stress τ [N/m^2]
- Sediment transport rate Q [kg/(m.s)]
- Gravitational constant g [m/s^2]

Dimensionless numbers:

Density ratio: $s \equiv \rho_p/\rho_f$

Galileo number: $\mathrm{Ga} \equiv d\sqrt{(s-1)gd}/\nu_f$

Shields number: $\Theta \equiv \tau/[(\rho_p - \rho_f)gd]$

Dimensionless transport rate: $Q_* \equiv Q/\left[\rho_p d\sqrt{(s-1)gd}\right]$

Introduction

Transport rate relations (experiments & DEM simulations):

Viscous bedload (weak)[1]: $Q_* \sim \mathrm{Ga}\Theta(\Theta - \Theta_t)$

Viscous bedload (intense)[2]: $Q_* \sim \mathrm{Ga}\Theta^3$

Turbulent bedload (weak)[3]: $Q_* \sim (\Theta - \Theta_t)^{3/2}$

Turbulent bedload (intense)[4]: $Q_* \sim \Theta^2$

Aeolian saltation (weak)[5]: $Q_* \sim \Theta - \Theta_t$

Aeolian saltation (intense)[6]: $Q_* \sim \Theta^2$

to be unified in this presentation

References (click to open):

(1) Charru et al. (JFM, 2004); Derksen (POF, 2011)
(2) Aussillous et al. (JFM, 2013); Kidanemariam & Uhlmann (IJMF, 2014); Charru et al. (Meccanica, 2016)
(3) Meyer-Peter & Müller (TU Delft, 1948); Wong & Parker (JHE, 2006); Kidanemariam & Uhlmann (JFM, 2017)
(4) Chauchat (JHR, 2018); Maurin et al. (JFM, 2018)
(5) Creyssels et al. (JFM, 2009); Ho et al. (PRL, 2011); Martin & Kok (Science Advances, 2017)
(6) Ralaiarisoa et al. (PRL, 2020)

Motivation: Predicting extraterrestrial morphodynamics

1. There is evidence for aeolian processes on Venus[1], Mars[1], Pluto[2], Saturn's moon Titan[1], Jupiter's moon Io[1], and Neptune's moon Triton[3].
2. However, there are no measurements of $Q_*(\Theta)$ for the atmospheric conditions on these planetary bodies.
3. Hence, for reliable predictions of their morphodynamics, one needs a relation $Q_*(\Theta)$ that captures the essential physics.
4. We find that universal simple physics is behind scaling of Q_*.

References (click to open):

(1) Diniega et al. (Aeolian Research, 2017)
(2) Telfer et al. (Science, 2018)
(3) Sagan & Chyba (Nature, 1990)

Unifying relation (Pähtz & Duran, PRL, 2020)

$$Q_*^\alpha = \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta^\alpha - \Theta_t)\left[1 + \frac{c_M}{\mu_b}(\Theta^\alpha - \Theta_t)\right]$$

Relation parameters:

- $\kappa = 0.4$ (von Kármán constant)
- $\mu_b = 0.63$ (obtained from DEM simulations)
- $c_M = 1.7$ (obtained from DEM simulations)

Correction for slope-driven bedload (i.e., $\tau = \rho_f gh \sin\alpha$, where α is the bed slope angle and h the clear-water depth):

$$(\Theta^\alpha, Q_*^{\alpha 2}) \equiv (\Theta, Q_*^2) \Big/ \left(\cos\alpha - \frac{\sin\alpha}{\mu_b}\frac{s}{s-1}\right)$$

$$Q_*^\alpha = \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta^\alpha - \Theta_t)\left[1 + \frac{c_M}{\mu_b}(\Theta^\alpha - \Theta_t)\right]$$

Relation's validity requires that

- sediment transport is at capacity (i.e., $\Theta/\Theta_t \gtrsim 1.5{-}2$).
- particle trajectories are not much affected by viscous sublayer (i.e., $s^{1/4}\mathrm{Ga} \gtrsim 40$).
- particle inertia dominate viscous drag forcing (i.e., $s^{1/2}\mathrm{Ga} \gtrsim 80{-}200$).
- boundary layer thickness (clear-water depth) is much larger than transport layer thickness.
- bed slope angle is not too close to angle of repose.

explained in next presentation on sediment transport thresholds ("Have we misunderstood the Shields curve?")

Unifying relation (Pähtz & Duran, PRL, 2020)

$$Q_*^\alpha = \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta^\alpha - \Theta_t)\left[1 + \frac{c_M}{\mu_b}(\Theta^\alpha - \Theta_t)\right]$$

Approximate behaviors (and typical conditions where they appear):

$$\frac{\Theta^\alpha}{\Theta_t} - 1 \ll \frac{\mu_b}{c_M\Theta_t} : \quad Q_*^\alpha \simeq \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta^\alpha - \Theta_t) \qquad \text{(aeolian saltation)}$$

$$\frac{\Theta^\alpha}{\Theta_t} - 1 \sim \frac{\mu_b}{c_M\Theta_t} : \quad Q_*^\alpha \simeq \frac{4\sqrt{c_M\Theta_t}}{\kappa\mu_b^{3/2}}(\Theta^\alpha - \Theta_t)^{3/2} \qquad \text{(turbulent bedload)}$$

$$\frac{\Theta^\alpha}{\Theta_t} - 1 \gg \frac{\mu_b}{c_M\Theta_t} : \quad Q_*^\alpha \simeq \frac{2c_M\sqrt{\Theta_t}}{\kappa\mu_b^2}(\Theta^\alpha - \Theta_t)^2 \qquad \text{(sheet flow)}$$

$$\frac{\mu_b}{c_M\Theta_t^{\mathrm{air}}} \gg \frac{\mu_b}{c_M\Theta_t^{\mathrm{water}}}$$

Experimental validation (Pähtz & Duran, PRL, 2020)

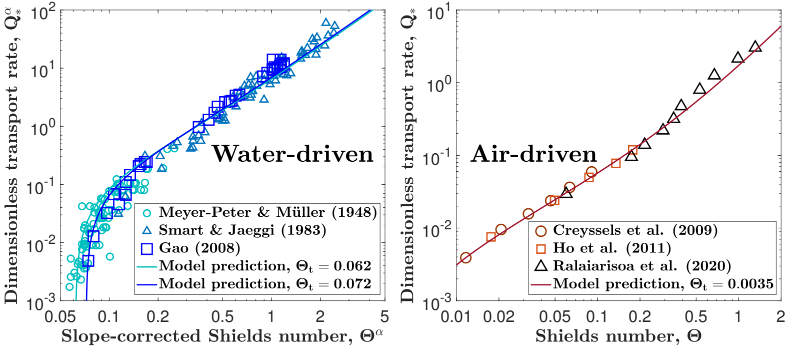


Figure: Modified from Pähtz & Durán[1], relation against measurements[2,3]. Values of Θ_t are close to (water) or equal to (air) predictions from recent threshold model[4] (discussed in next presentation on transport thresholds).

References (click to open):

(1) Pähtz & Durán (PRL, 2020)
(2) Meyer-Peter & Müller (TU Delft, 1948); Smart & Jaeggi (ETH Zürich, 1983); Gao (JHE, 2008)
(3) Creyssels et al. (JFM, 2009); Ho et al. (PRL, 2011); Ralaiarisoa et al. (PRL, 2020)
(4) Pähtz & Durán (JGR: ES, 2018)

Physical origin (Pähtz & Duran, PRL, 2020)

Definitions:

- Transport load (mass of transported particles per unit area) M
  dimensionless: $M_* \equiv M/(\rho_p d)$
- Average particle velocity $\overline{v_x} \equiv Q/M$
  dimensionless: $\overline{v_x}_* \equiv \overline{v_x}/\sqrt{(s-1)gd}$
- Threshold particle velocity $\overline{v_x}_{*t} \equiv \overline{v_x}_*|_{\Theta\to\Theta_t}$

Step 1: Kinetic particle fluctuation energy balance:
$(a_c, a_d, b_c \neq f(M_*))$

$$\frac{1}{2}Q_*\cos\alpha = (a_c + a_d)M_* + b_c M_*^2$$

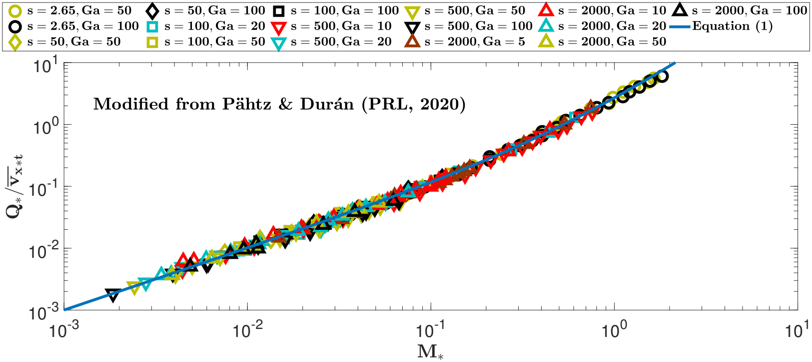
- $\frac{1}{2}Q_*\cos\alpha$ = production rate by mean granular motion
- $a_d M_*$ = dissipation rate by fluid drag
- $a_c M_*$ = dissipation rate by particle-bed collisions
- $b_c M_*^2$ = dissipation rate by binary particle collisions

Step 2: Consistency with definition of $\overline{v_{x*t}}$ allows rewriting as

$$Q_* = M_* \overline{v_{x*t}}(1 + c_M M_*), \quad (1)$$

where $c_M = b_c/(a_c + a_d)$.

Step 3: Comparison with DEM simulations yields $c_M = 1.7$.



Physical origin (Pähtz & Duran, PRL, 2020)

Step 4: Kinetic particle energy balance and optimization principles: (explained in next presentation on sediment transport thresholds)

$$M_* = \frac{1}{\mu_b}(\Theta^\alpha - \Theta_t)$$

$$\overline{v_{x*t}} = 2\kappa^{-1}\sqrt{\Theta_t}\sqrt{\cos\alpha - \frac{\sin\alpha}{\mu_b}\frac{s}{s-1}}$$

Inserting in Eq. (1) and rearranging finally yields

$$Q_*^\alpha = \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta^\alpha - \Theta_t)\left[1 + \frac{c_M}{\mu_b}(\Theta^\alpha - \Theta_t)\right]$$

Relation has been derived without assumptions about the nature of bed sediment entrainment, neither driven by the flow nor by splash.