A reduced-order model of the zonal jets problem in the Southern Ocean

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INTER RIDGE international cooperation in ridge-crest studies

Hierarchy of the models

HYCOM

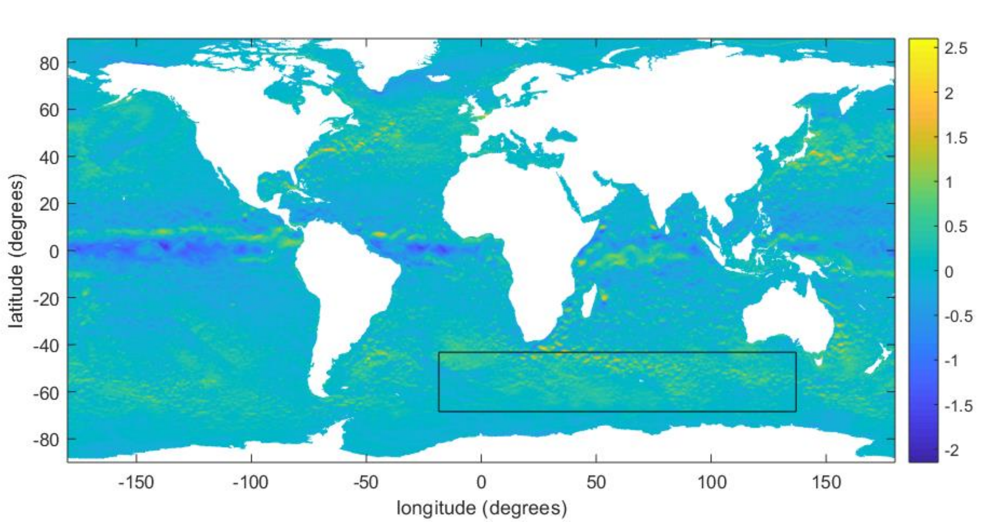
$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\mathbf{v}+2\boldsymbol{\omega}\times\mathbf{v}=-\frac{\nabla M}{\rho}+\frac{\nabla.\boldsymbol{\tau}}{\rho},$$

$$\frac{\partial T}{\partial t}+\nabla.(T\mathbf{v})=\nabla.(\kappa\nabla T)+F^T,$$

$$\frac{\partial S}{\partial t}+\nabla.(S\mathbf{v})=\nabla.(\kappa\nabla S)+F^S,$$

$$\nabla.\mathbf{v}=0,$$

$$\rho=\rho(T,S,P),$$



Quasi-geostrophic Model

$$\partial_t q_i+J(\psi_i,q_i)=a_h\Delta^2\psi_i-\delta_{i3}\frac{a_v}{H_3^2}\Delta\psi_i,\ i=1,2,3,$$

$$q_i=\nabla^2\psi_i+\beta y-(1-\delta_{i1})S_{i1}(\psi_i+U_iy-\psi_{i-1}-U_{i-1}y)-(1-\delta_{i3})S_{i2}(\psi_i+U_iy-\psi_{i+1}-U_{i-1}y),i=1,2,3,$$

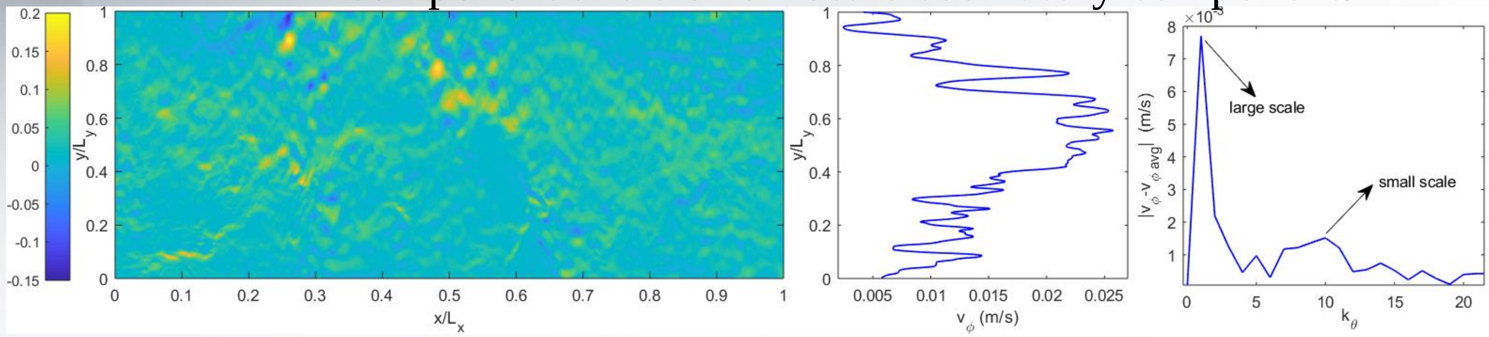
Reduced-order Model

$$\frac{\partial \zeta}{\partial t}+(\mathbf{v}.\nabla)\zeta=\nabla\times\left(\frac{\nabla.\boldsymbol{\tau}}{\rho}+\frac{\mathbf{T}}{h_w}\right)+(\zeta.\nabla)\mathbf{v}+2(\boldsymbol{\omega}.\nabla)\mathbf{v}.$$

$$\frac{\partial \zeta}{\partial t}=\frac{2\Omega}{r}\left[\sin\theta v_\theta+\cos\theta\frac{\partial(\sin\theta v_\theta)}{\partial\theta}\right]+I^{\mathrm{conv}}(v_\theta,v_\phi)+\frac{a_h{}^L}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\zeta}{\partial\theta}\right)+\frac{a_h{}^L}{r^2\sin^2\theta}\frac{\partial^2\zeta}{\partial\phi^2}+\frac{a_v}{r^3}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(r\zeta)\right].$$

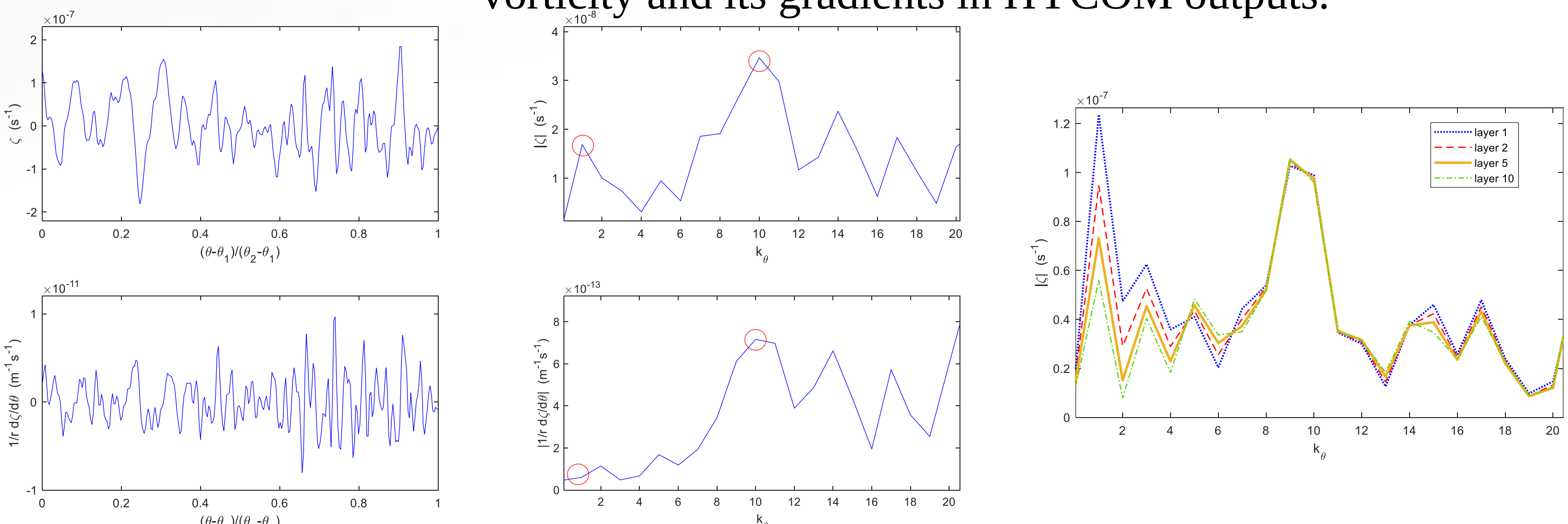
The reduced-order model of the Southern Ocean

The zonal mean flow is dominated by two length scales: a large-scale dome-like component and a small-scale oscillatory component.



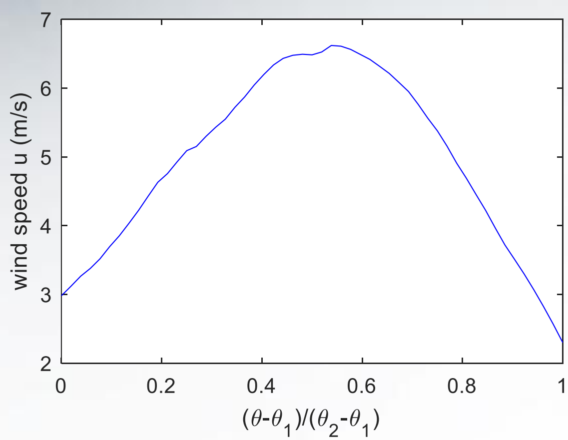
Scale decomposition: $\zeta = \zeta^{(l)} + \zeta^{(L)}.$

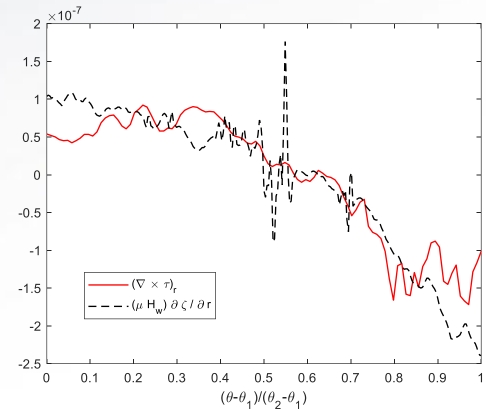
Governing equations for each scale is derived based on order of magnitude analysis of vorticity and its gradients in HYCOM outputs.



Large scale dynamics

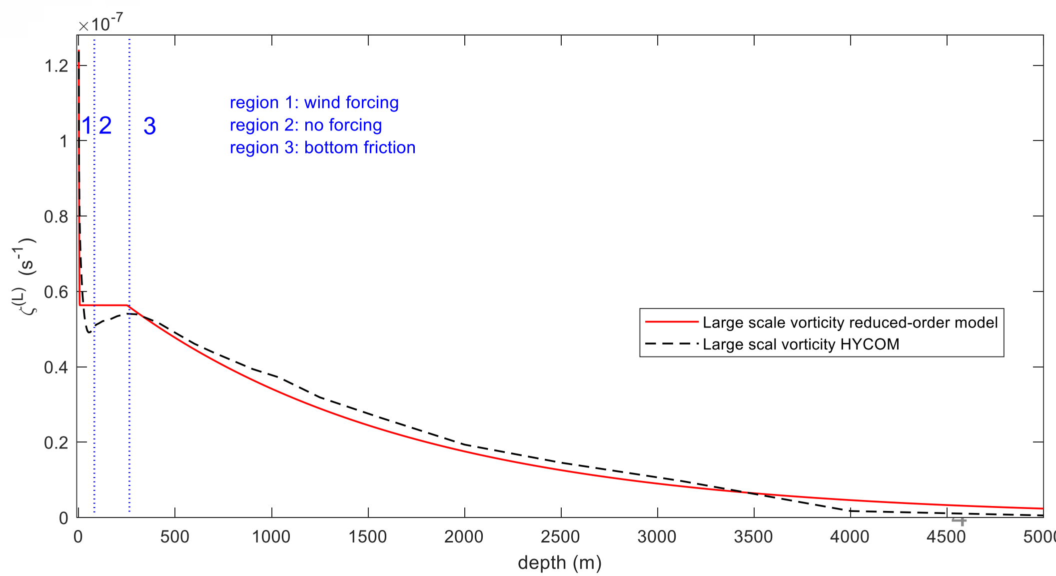
Large scale solution is driven by the wind at the ocean surface and dissipates in the depth by the bottom friction.





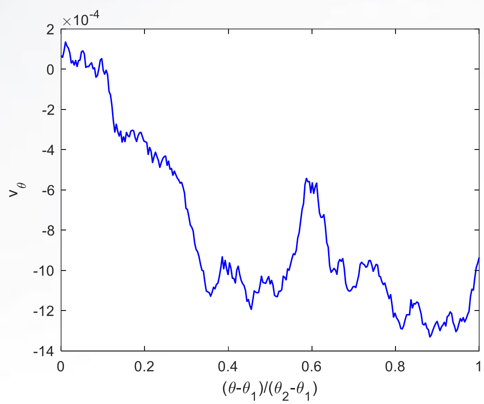
Large-scale equation:

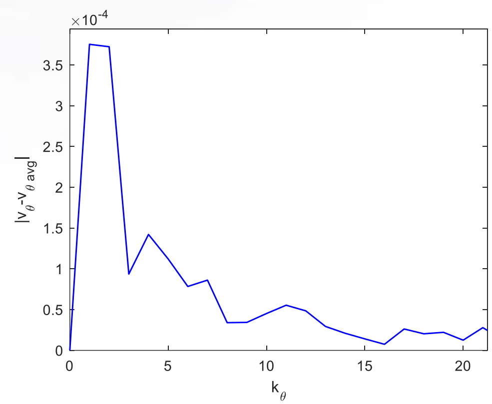
$$a_v \frac{\partial^2 \zeta^{(L)}}{\partial r^2} = \begin{cases} \frac{1}{rH_w}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\tau_\phi\right)\right], & r_w < r < r_{\max}, \\ 0, & r_b < r < r_w, \\ \gamma\zeta^{(L)}, & r_{\min} < r < r_b, \end{cases}$$



Small scale dynamics

- The small-scale solution corresponding to zonal jets is mainly shear-driven.
- The meridional velocity is small compared to zonal velocity and hence, beta term has little contribution in the small-scale solution.



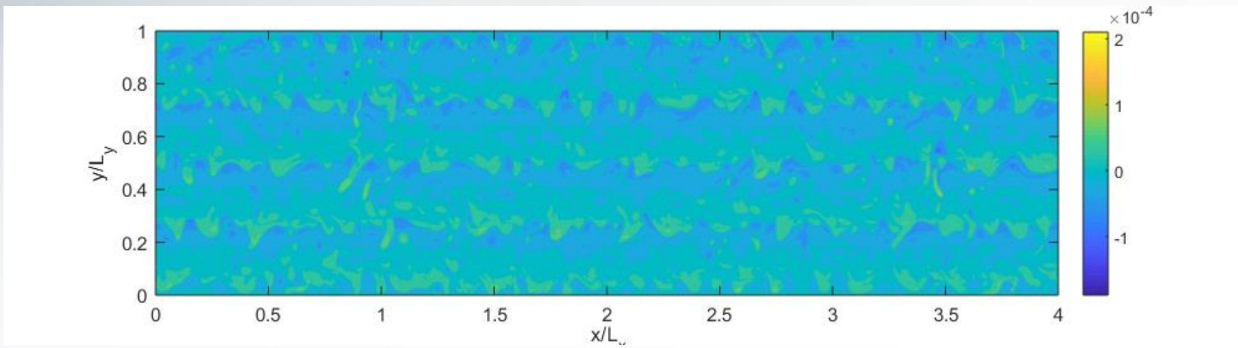


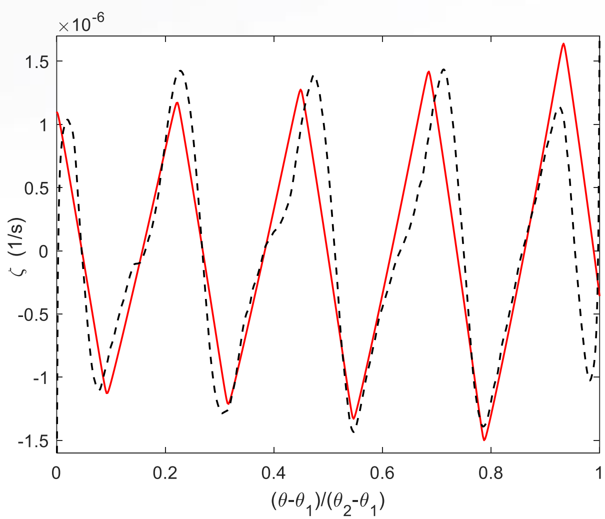
Small-scale equation:

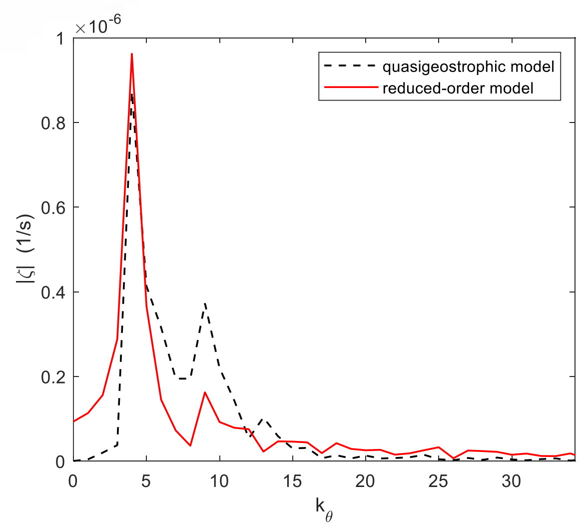
$$\underbrace{\frac{2\Omega}{r}\left[\sin\theta v_\theta + \cos\theta\frac{\partial\left(\sin\theta v_\theta\right)}{\partial\theta}\right]}_{\text{small contribution}} + \frac{a_h}{r^2}\left(\frac{\partial^2\zeta^{(l)}}{\partial\theta^2} + \cot\theta\frac{\partial\zeta^{(l)}}{\partial\theta}\right) + a_v\frac{\partial^2\zeta^{(l)}}{\partial r^2} = 0.$$

Comparisons with quasi-geostrophic model:

Zonal jets in the channel flow configuration



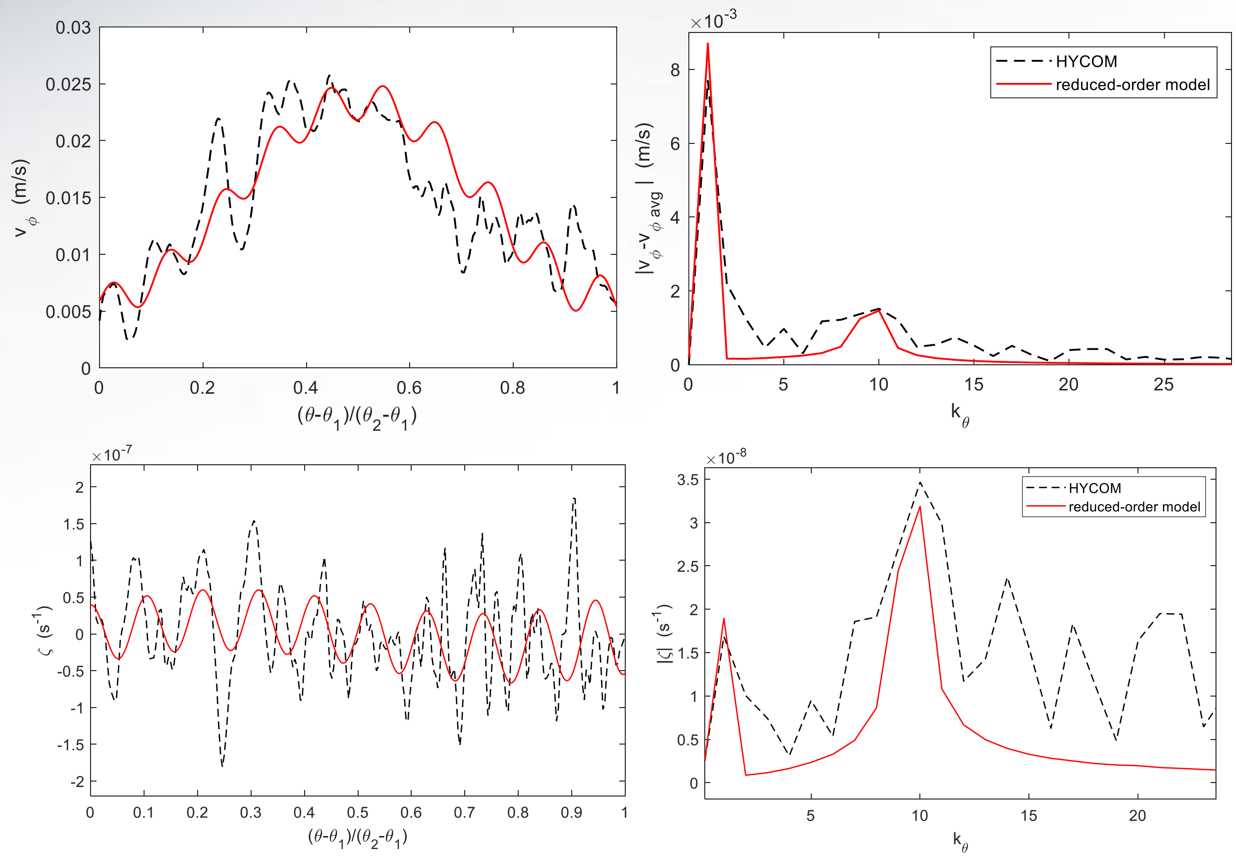




Comparisons with HYCOM outputs:

Time and layer averaged zonal velocity and vorticity

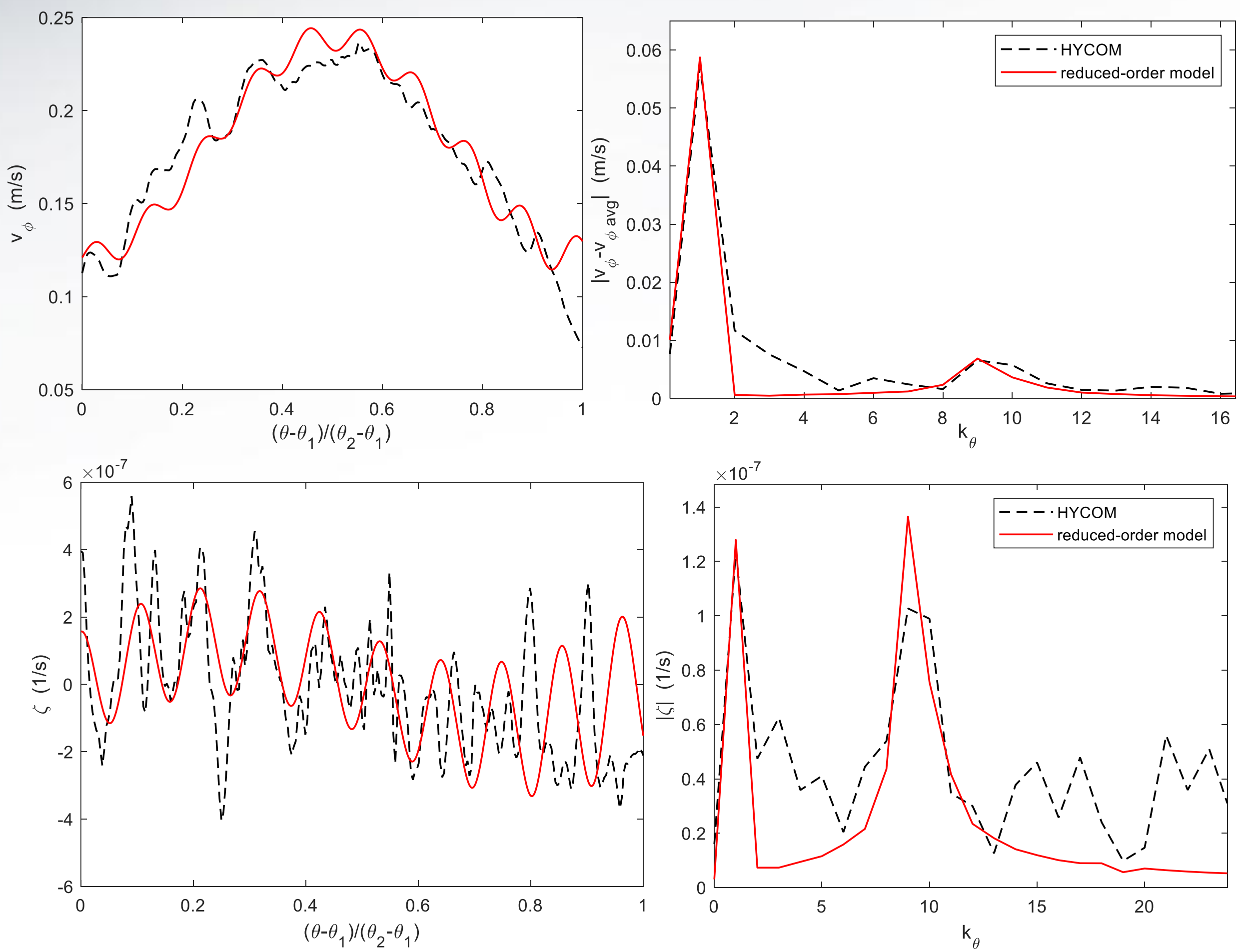
(profiles and space spectra)



Comparisons with HYCOM outputs:

Time averaged zonal velocity and vorticity for the top layer

(profiles and space spectra)



Conclusions:

- The vorticity equation for the Southern Ocean region is simplified using multi-scale perturbation method.
- Zonal jets are derived as a solution to small-scale vorticity equation.
- Contribution of beta term and vertical velocity gradient are negligible in the dynamics of zonal jets.

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Thank you