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Multi-fluid single-column modelling of Rayleigh-Bénard convection

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Modern NWP and climate models are run at resolutions too fine for the assumptions underlying traditional convection schemes to be valid. However, resolutions are still orders of magnitude too coarse to resolve all dynamically relevant convective processes. To gain insight into developing parametrisations at these intermediate scales, we consider the grey zone of an idealised model: 2D dry Rayleigh-Bénard convection.

The grey zone of Rayleigh-Bénard convection

- The problem: viscous Boussinesq fluid confined between two smooth horizontal plates separated by a distance H, with a fixed buoyancy difference ΔB between bottom and top. Convection develops if:

$$\mathrm{Ra} \equiv \frac{\Delta B\, H^3}{\nu\, \kappa} > \mathrm{Ra}_c \simeq 1700$$

- The circulation generates a vertical heat flux (normalised by the heat flux of the stationary reference state):

$$\mathrm{Nu} \equiv \frac{\langle wb - \kappa\, \partial b/\partial z \rangle}{\kappa \Delta B / H}$$

- What happens if we systematically coarsen the horizontal resolution?

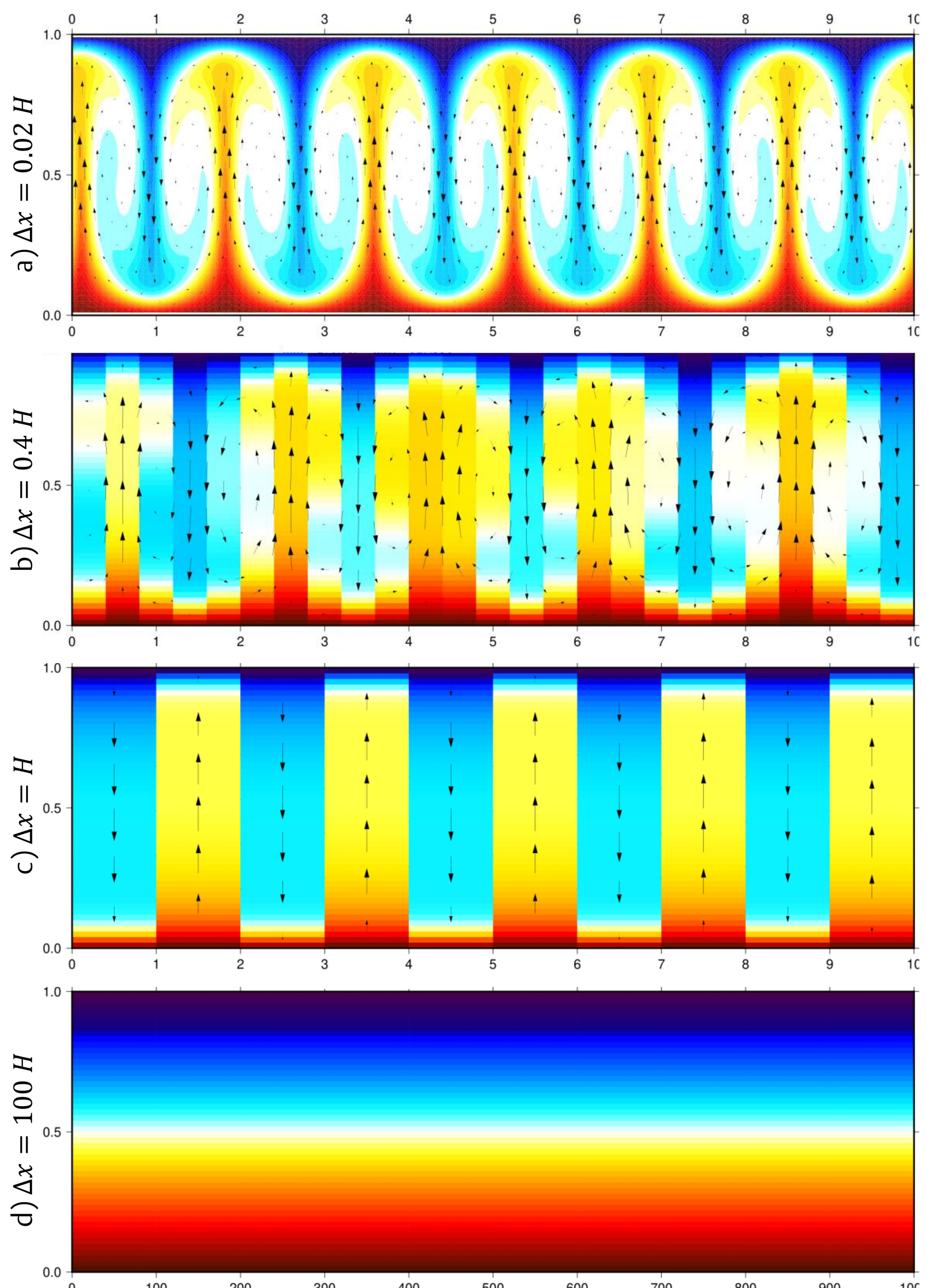


Figure 1: Rayleigh-Bénard convection at Ra $\approx 10^5$ for a range of horizontal resolutions, after a steady state has been reached. The colours are buoyancy, the arrows are velocity vectors.
The convection is fully resolved in a), showing clear convective rolls. As the resolution decreases, less of this structure is picked up (b). When the resolution becomes similar to the length scale of the rolls, λ_c, the convection pattern projects strongly onto the grid (c). This is reminiscent of grid-point storms in NWP. Decreasing the resolution even further, and convection does not develop at all (d). The implicit filter of the grid has reduced Ra to below Ra_c.

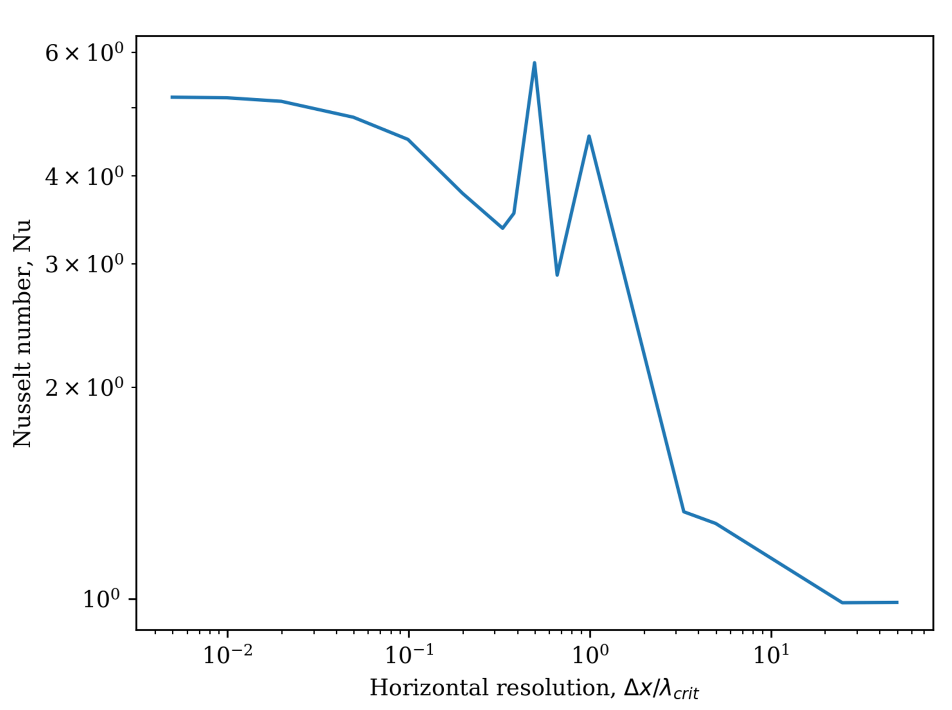
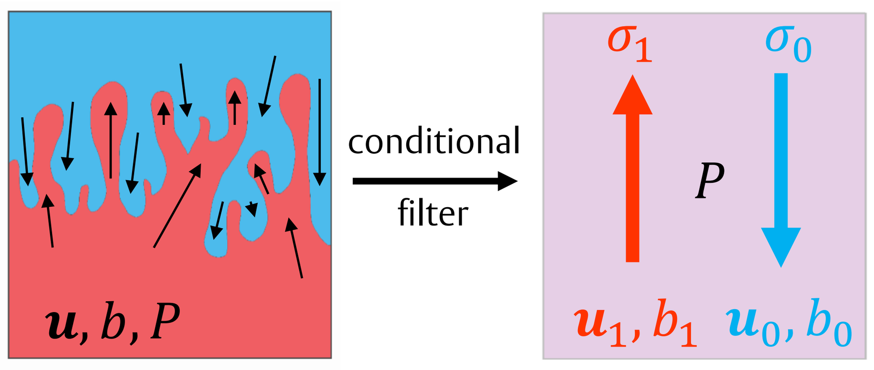


Figure 2: Normalised heat transport as a function of horizontal resolution. Generally the heat transport decreases with resolution, as we would expect due to less of the advective motion being resolved. However, close to λ_c the heat transport can be overestimated due to projection onto the grid. This is one of the problems which makes the grey zone so difficult to work in. At the coarsest resolutions there is no motion so the heat transport is entirely diffusive.

The multi-fluid approach

- The multi-fluid approach applies a conditional filter to the high-resolution governing equations, giving a set of prognostic equations for n fluid partitions. These allow for e.g. net mass transport, and horizontal propagation of convection.



- Each fluid has its own volume fraction (σ_i), velocity, and buoyancy. This means the fluids can move through each other.
- The filtering introduces terms involving the exchange of mass, momentum, and buoyancy between the fluids; these need to be parametrised. They are the equivalent of entrainment/detrainment.
- To first order, the fluids share a single pressure; deviations from this pressure within each fluid also need to be parametrised.

Two-fluid single column results

- We construct reference two-fluid single column solutions from the fully resolved solution (Fig. 1a) by assigning $w \leq 0$ fluid to partition 0, $w > 0$ fluid to partition 1, and horizontally averaging. This choice forces the transferred vertical velocity to be $w_{ij}^T = 0$.
- We make the closure assumptions:
  - Buoyancy transferred from fluid i to fluid j: $b_{ij}^T = b_i \pm C|b_i|$
  - Difference of pressure in fluid i from mean pressure P: $p_i = -\gamma \frac{d\sigma_i w_i}{dz}$
  - Volume fraction transfer rate from i to j: $S_{ij} = \max\left(-\frac{dw_i}{dz}, 0\right)$

This approach allows a strong circulation to develop even at the coarsest possible resolution (fig. 3).

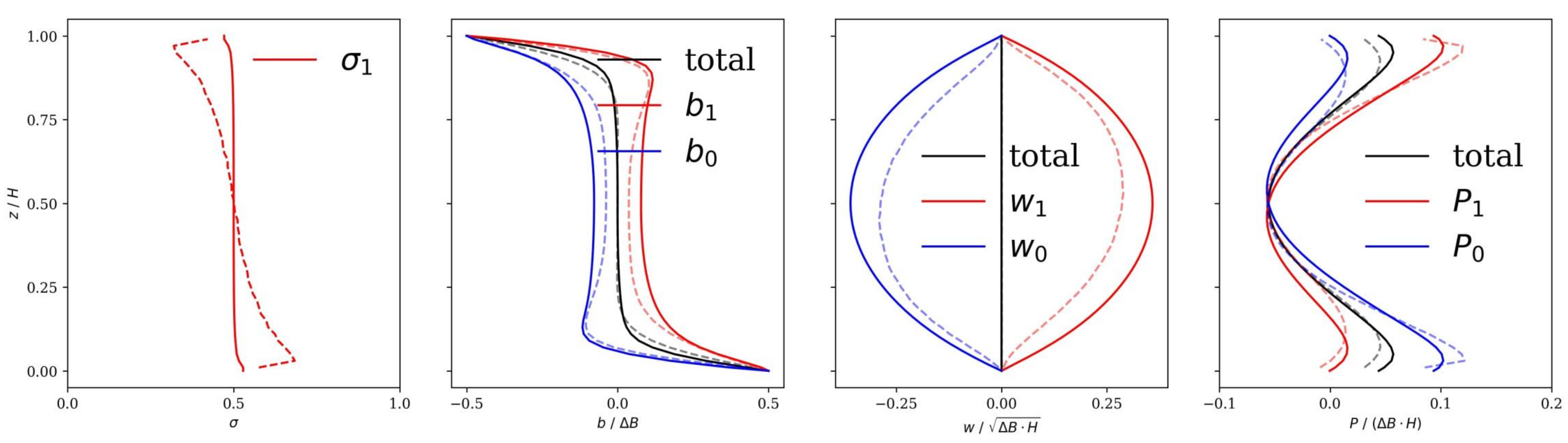


Figure 3: Results of two-fluid single-column model with $C = 0.5, \gamma/\kappa = 15$ (solid lines) versus high resolution reference (dashed lines). The two-fluid model is initialised from a resting state. The model captures the profiles of total buoyancy, vertical velocity and pressure (black lines) very well, as well as the pressure and buoyancy within the individual fluids (coloured lines). This leads to a strong heat transport of Nu ≈ 6.

Future work

- Find a better closure for the volume fraction transfer; prognosing σ well is at the heart of the convection parametrisation problem.
- Base the transferred buoyancy and momentum on some knowledge of higher-order subfilter moments (e.g. variances).
- Move towards higher Ra, as the true atmosphere is turbulent.
- Move towards the grey zone from the coarse-resolution limit.

Multi-fluid Boussinesq equation set

$$\frac{\partial \sigma_i}{\partial t} + \nabla \cdot (\sigma_i \boldsymbol{u}_i) = \sum_j (\sigma_j S_{ji} - \sigma_i S_{ij}) \tag{1}$$

$$\frac{\partial \sigma_i \boldsymbol{u}_i}{\partial t} + \nabla \cdot (\sigma_i \boldsymbol{u}_i \otimes \boldsymbol{u}_i) = \sigma_i b_i \hat{\boldsymbol{k}} - \sigma_i \nabla (P + p_i) + \sigma_i \nu \nabla^2 \boldsymbol{u}_i + \sum_j \left(\sigma_j \boldsymbol{u}_{ji}^T S_{ji} - \sigma_i \boldsymbol{u}_{ij}^T S_{ij}\right) \tag{2}$$

$$\frac{\partial \sigma_i b_i}{\partial t} + \nabla \cdot (\sigma_i \boldsymbol{u}_i b_i) = \sigma_i \alpha \nabla^2 b_i + \sum_j \left(\sigma_j b_{ji}^T S_{ji} - \sigma_i b_{ij}^T S_{ij}\right) \tag{3}$$

$$\sum_i \sigma_i = 1 \tag{4}$$

$$\sum_i \nabla \cdot (\sigma_i \boldsymbol{u}_i) = 0. \tag{5}$$

Here $\sigma_i, \boldsymbol{u}_i, b_i, p_i$ respectively denote the volume fraction, velocity, buoyancy, and difference from the mean pressure, within fluid partition i; S_{ij} denotes the rate of volume fraction transfer from fluid i to fluid j; b_{ij}^T denotes the buoyancy transferred from fluid i to fluid j when mass is exchanged; and $\boldsymbol{u}_{ij}^T$ denotes the velocity transferred from fluid i to fluid j when mass is exchanged. Subfilter viscous terms, Reynolds stresses, and the unresolved part of pressure drag have been omitted.

Multi-fluid convection papers

J. Thuburn et al. (2018). "A Framework for Convection and Boundary Layer Parameterization Derived from Conditional Filtering". In: *Journal of the Atmospheric Sciences* 75.3, pp. 965–981

J. Thuburn and G. K. Vallis (2018). "Properties of conditionally filtered equations: Conservation, normal modes, and variational formulation". In: *Quarterly Journal of the Royal Meteorological Society* 144.714, pp. 1555–1571

J. Thuburn, G. A. Efstathiou, and R. J. Beare (2019). "A two-fluid single-column model of the dry, shear-free, convective boundary layer". In: *Quarterly Journal of the Royal Meteorological Society* 145.721, pp. 1535–1550

H. Weller and W. A. McIntyre (2019). "Numerical solution of the conditionally averaged equations for representing net mass flux due to convection". In: *Quarterly Journal of the Royal Meteorological Society* 145.721, pp. 1337–1353

W. A. McIntyre, H. Weller, and C. E. Holloway (2020). "Numerical methods for entrainment and detrainment in the multi-fluid Euler equations for convection". In: *Quarterly Journal of the Royal Meteorological Society* 146.728, pp. 1106–1120

H. Weller, W. McIntyre, D. Shipley, and P. Clark (2020). "Multi-fluids for Representing Sub-filter-scale Convection". In: *Journal of Advances in Modeling Earth Systems (in review)*