Signal contribution of the polar and the inclined pairs in a Bender configuration

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The Bender configuration has two pairs of GRACE-like satellites, one in a polar orbit and the other in an inclined orbit. We are interested in knowing how much do each of these pair of satellites contribute to the monthly time-variable gravity field solutions. In order to answer the contribution question, we simulated the Bender configuration using the static gravity field as the only force model.

Simulation set-up

The Kepler elements used for the simulation are provided in Table 1. These Kepler elements provide data with fairly good spatial coverage, while also minimizing error caused by the spatial and temporal aliasing of ocean tides [1]. The simulation was performed using EGRAFS [2] for 29 days in accordance with the full cycle of the inclined pair. The static gravity field was the only force model used.

Kepler Elements	Polar Satellites	Inclined Satellites
Semi-major axis	6740137 m	6720137 m
Eccentricity	0.0010	0.0010
Inclination	92°	115°
RA of ascending node (Ω)	0°	90°
Argument of Perigee (ω)	0°	0°
β/α	$172/11$	$460/29$

Table 1: Kepler Elements used for simulation of polar and inclined satellite orbits

The gravity field recovery from the simulated data was performed using the acceleration approach [3], where the range accelerations between the pairs of satellites in the Bender configuration are expressed in terms of spherical harmonics. The pseudo-observations vector is denoted y, the design matrix A and the spherical harmonics coefficients vector x. Furthermore, we write the solution in the partitioned model with the polar and inclined pairs being the two partitions that contribute to the spherical harmonic solution. It must be noted that for computing the redundancy contributions only the normal matrices are required, and therefore, the coefficients are not estimated.

$$E\{y\} = E\left\{\begin{pmatrix} y_{\mathrm{P}} \\ y_{\mathrm{I}} \end{pmatrix}\right\} = \begin{pmatrix} A_{\mathrm{P}} \\ A_{\mathrm{I}} \end{pmatrix} x$$

The contributions of each of these partitions to the final solution can be estimated from the redundancy contribution matrices (R_i), which can be calculated as [4]

$$E\{\hat{x}\} = \left(\sum_{j\in \mathrm{P,I}} A_j^{\mathsf{T}} Q_j A_j\right)^{-1} \left(\sum_{i\in \mathrm{P,I}} A_i^T Q_i E\{y_i\}\right)$$

$$= \left(\sum_{j\in \mathrm{P,I}} A_j^{\mathsf{T}} Q_j A_j\right)^{-1} \left(\sum_{i\in \mathrm{P,I}} A_i^T Q_i A_i x\right)$$

$$R_i = \left(\sum_{j\in \mathrm{P,I}}^{n} A_j^T Q_j A_j\right)^{-1} \left(A_i^T Q_i A_i\right)$$

$$\Rightarrow E\{\hat{x}\} = \sum_{i\in \mathrm{P,I}} R_i x \qquad) \sum_{i\in \mathrm{P,I}} R_i = I$$

The diagonal elements of the redundancy contribution matrices (R_i) provide information on how much the particular partition i has contributed to each element of the solution vector. They range from 0 to 1, where 0 indicates no contribution and 1 indicates complete contribution. The sum of the contribution matrices is always an identity matrix I, when the estimator is unbiased.

The redundancy contribution matrices in the above equation provide the spectral contribution of the polar and inclined pairs. This information can be propagated to the spatial domain as follows

$$\mathcal{R}_i = Y\, R_i\, Y^{\mathsf{T}}$$

where Y is the matrix of spherical harmonic functions and $\mathcal{R}_i$ is the square matrix of spatial redundancy contribution. Here again, the diagonal elements are used for discerning the redundancy.

Results and Discussion

Inter-track Separation

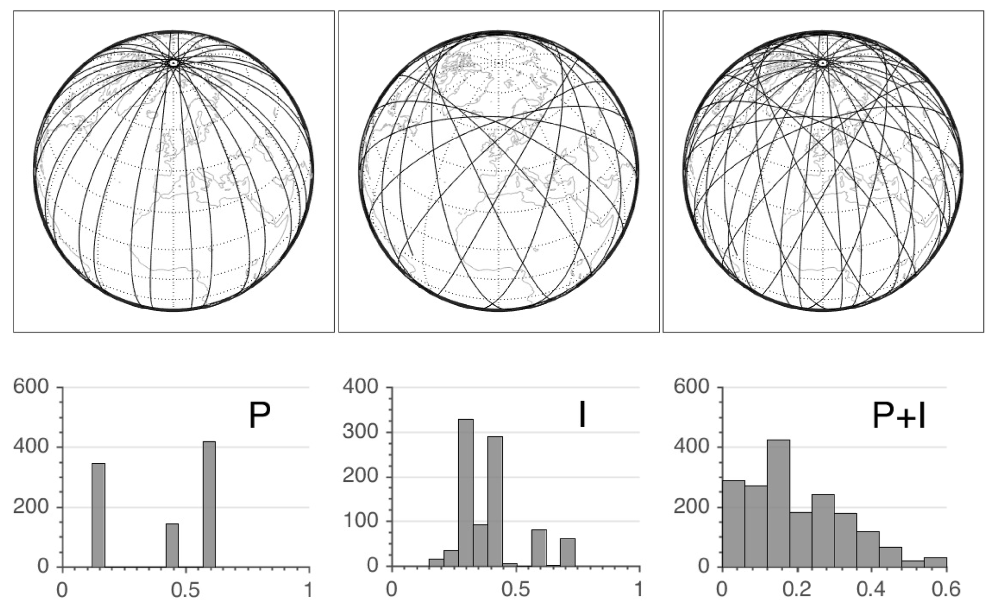

Figure 1: Comparison of spatial sampling at equator for (P) Polar, (I) Inclined and (P+I) Bender Configuration (1-day ground tracks and 29-day histograms)

Spectral Contribution in Bender Configuration

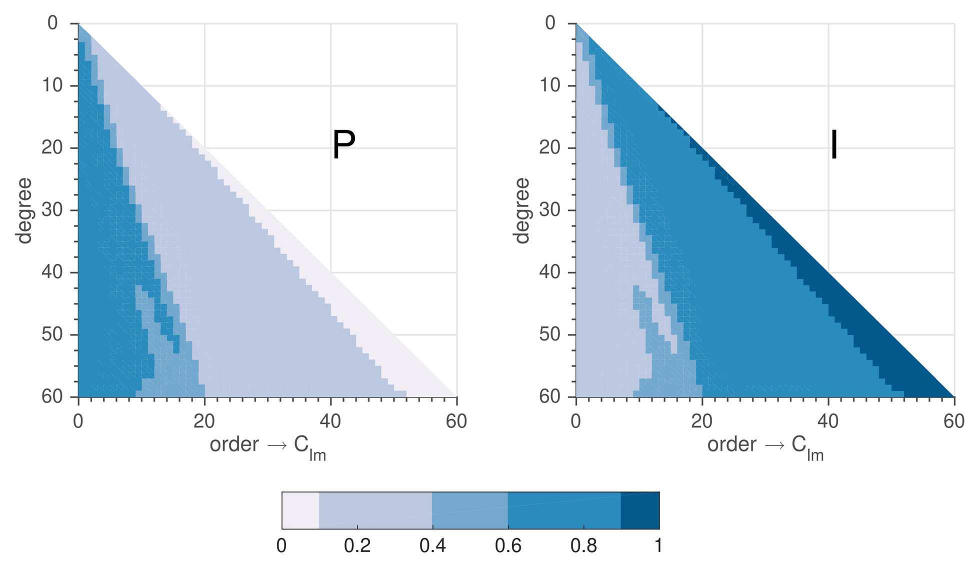

Figure 2: Contribution of (P) polar and (I) inclined pairs to the 29-day SH coefficients

SH Coefficients	Polar Satellites	Inclined Satellites
Zonals	High	Low
Near-Zonals	High	Low
Sectorials	Nil	Very High
Near-Sectorials	Very Low	Very High
Tesserals	Low	High

Table 2: Qualitative contribution of polar and inclined satellites to various classes of SH coefficients

Understanding the variations

The contributions of both the polar and inclined pairs vary predominantly with the order rather than the degree, but the figures invariably show that there is a degree and order combination involved. Here, we analyse the relationship between the redundancy contribution and the difference $l - m$ for a particular degree.

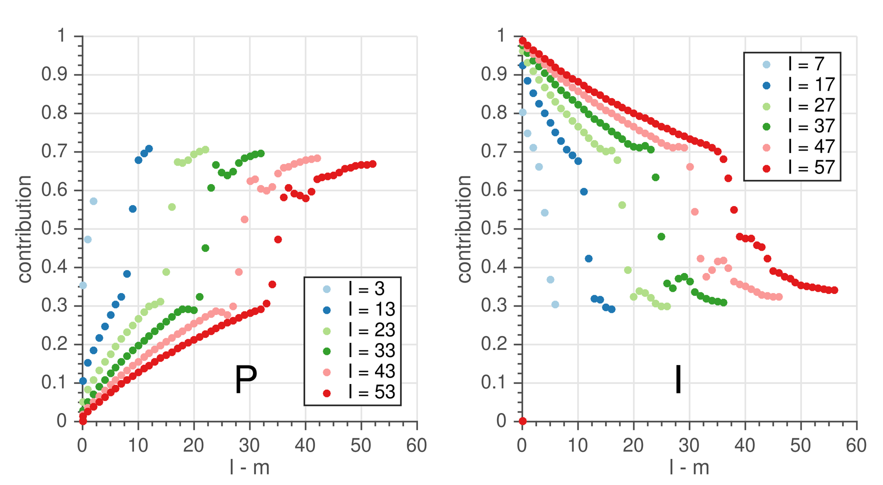

Figure 3: Full cycle contribution as a function of difference between degree and order of SH coefficients

1. Contribution values appear to be linearly dependent on the difference between degree and order (for a fixed degree) below a certain threshold (for this simulation, the value of threshold is 0.3 for polar satellites)
2. Contribution value (polar satellites) increases as the degree-order difference increases for all degrees. The increase however becomes lesser with increasing degree.
3. The contribution values for polar satellites is never equal to unity, contrary to inclined satellites (which reach unity at lower degree-order differences). Hence, it suggests that the sectorials and near sectorials are almost entirely contributed by the inclined satellites.

Daily Evolution of Spectral Contribution

Since the two pairs operate at different repeat periods, we were interested in the evolution of the redundancy contribution on a daily basis. Here we see that the redundancy contribution linearly increases without a saturation point.

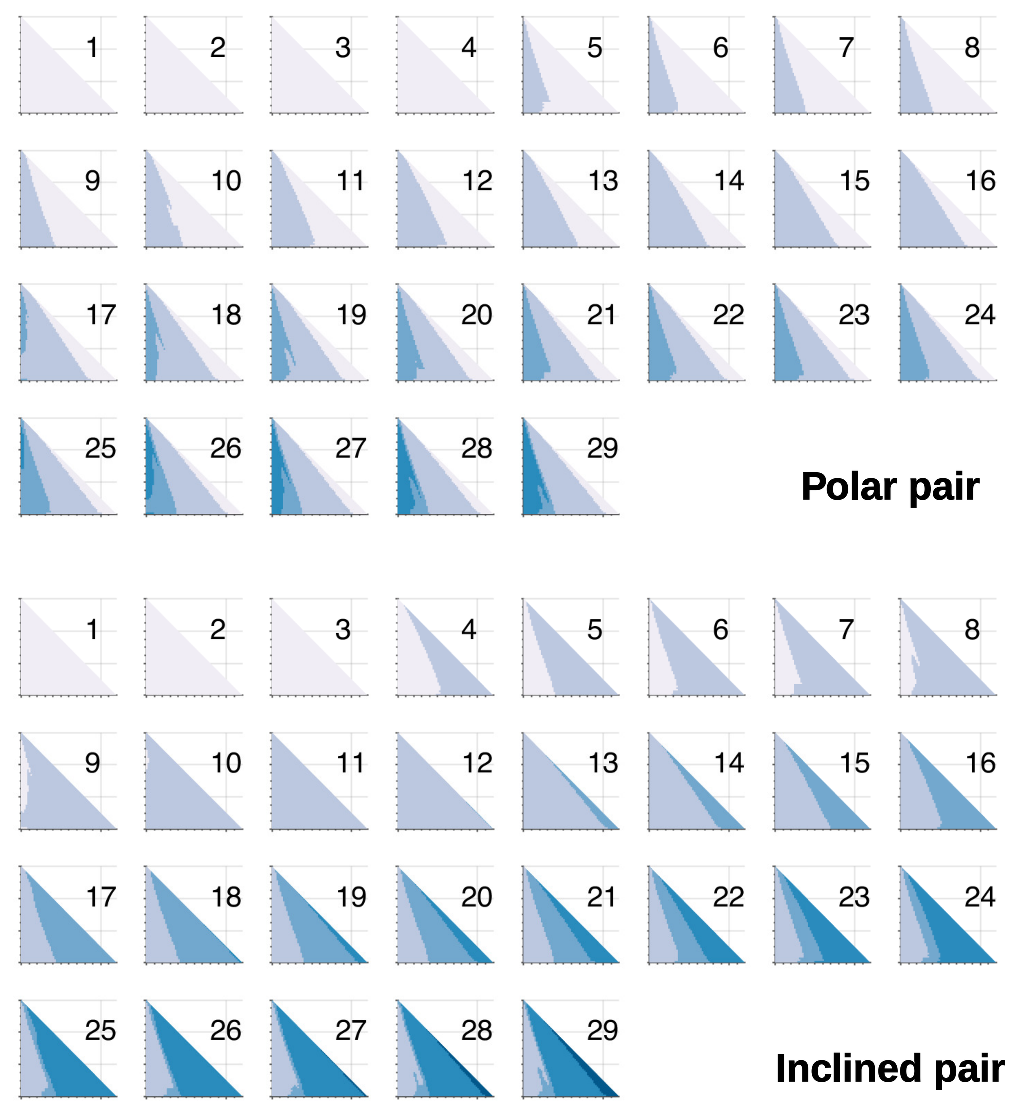

Figure 4: Cumulative day-wise evolution of the contribution of the polar and the inclined pairs to the final SH coefficients

Spatial Contribution

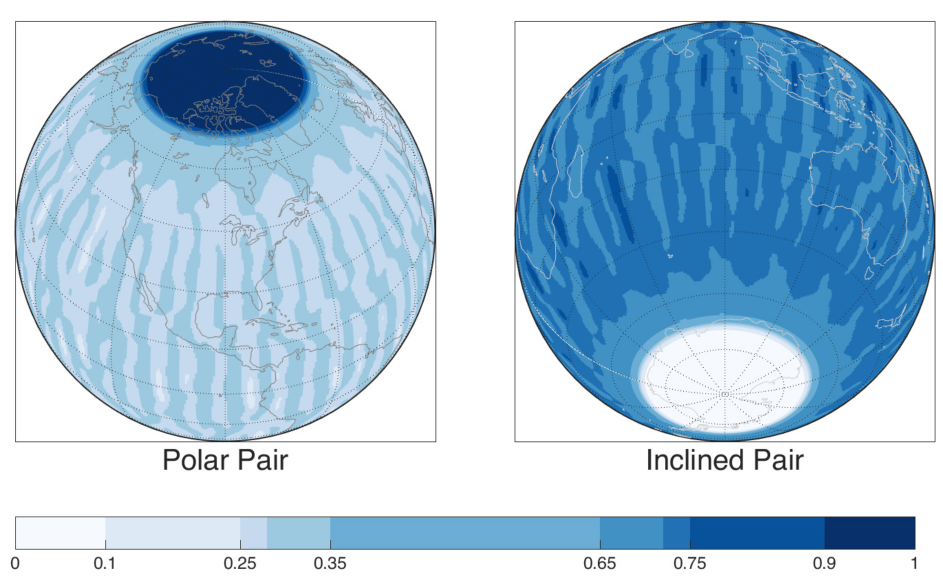

Figure 5: Spatial propagation of the redundancy contribution matrices. It is clear that the inclined pair contributes more than 65 % of the information in the equatorial and mid-latitude region, while the polar pair contributes more than 90 % in the high-latitude (polar) regions

1. Spectrally, the polar pair contributes upto 70 % to the zonals and the near-zonals, while the inclined orbit contributes $\geq$ 60 % to the tesserals and upto 100 % to the sectorials and near-sectorials.
2. Spatially, the contribution of the polar pair is $\leq$ 35 % and that of the inclined pair is $\geq$ 65 % in the equatorial and mid latitude region. However at the poles, as expected, the contribution of the polar pair is $\geq$ 90 %.

References

[1] W. Liu. “Understanding ocean tide aliasing in satellite gravimetry”. PhD thesis. Universität Stuttgart, 2019.

[2] B. Devaraju. *Earth GRAvity Field from Space (EGRAFS)*. 2015. URL: `http://egrafs.tuxfamily.org/`.

[3] M. Weigelt. “The Acceleration Approach”. In: *Global Gravity Field Modeling from Satellite-to-Satellite Tracking Data*. Ed. by M. Naeimi and J. Flury. Cham: Springer International Publishing, 2017, pp. 97–126. DOI: `10.1007/978-3-319-49941-3\_4`.

[4] N. Sneeuw. “A semi-analytical approach to gravity field analysis from satellite observations”. PhD thesis. Technische Universität München, 2000.

[5] J. Ries, S. Bettadpur, R. Eanes, et al. *The development and evaluation of the global gravity model GGM05*. Tech. rep. 2016.