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Role of non-diagonal pressure tensor components in the balance of magnetopause current sheet

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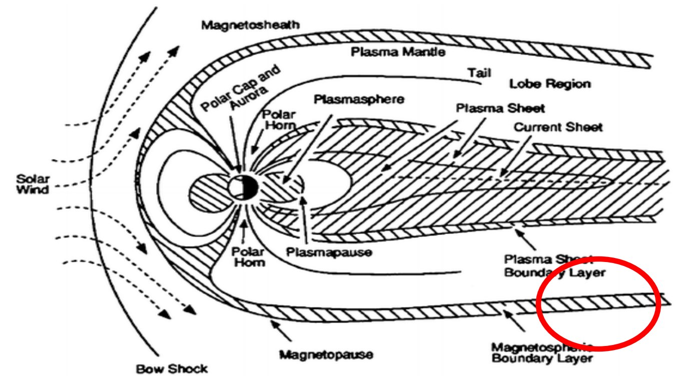
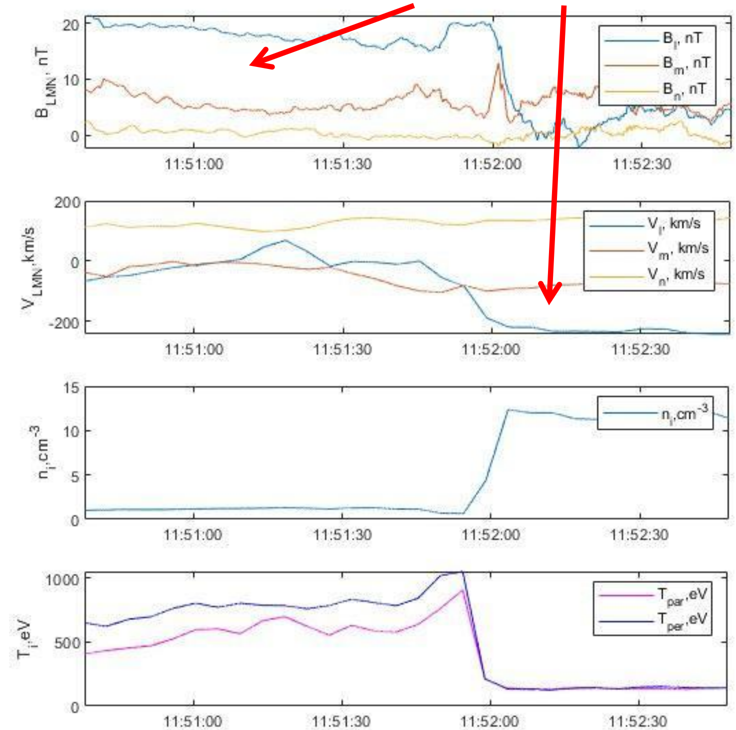
In this study we construct a stationary mhd-model for describing a current sheet, separating regions with different plasma flows. The main idea of the method is to reduce the incompressible mhd-system to the well-known Grad-Shafranov equation by using a modified magnetic field potential and modified pressure. Here we develop the idea supposed by Wiegelmann and Nickeler with generalization to the 3D field structure and nonparallel velocities. This development is supposed to be used for night-side Earth magnetopause description. Therefore, one of the obtained result is interesting because it can us help with speculation about the role of non-diagonal pressure tensor components, which in our model stabilize nonparallel plasma flows.

Problem formulation and general assumptions

Spacecraft observations demonstrate the existence of quasi-stationary current sheets separating regions with plasma flows propagating in different directions, as well as current sheets separating fast fluxes from strong magnetic fields. We want to construct mhd-model for such current sheets, trying to make it as simple as possible. However, we suppose that this model should describe night-side structure of Earth magnetopause.



Magnetopause current sheet between strong magnetic field and fast flow.



Assume that the model:

1. is defined by ideal incompressible mhd-system;

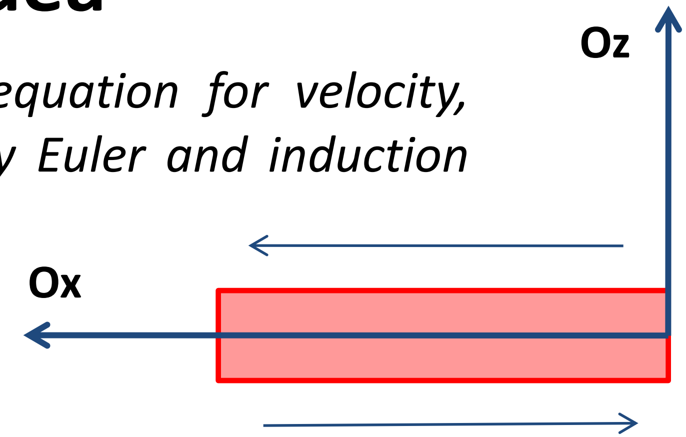
2. is stationary and stable;

3. includes plasma flows parallel or quasi-parallel to magnetic field;

4. has known relation between filed and plasma velocities (Mach function).

Well-known solution and main idea

Our ideal mhd-system consists of the continuity equation for velocity, solenoidality equation for magnetic field, stationary Euler and induction equations, Ampere's law:



$$\operatorname{div}(\rho \mathbf{v}) = 0 \qquad \operatorname{div} \mathbf{B} = 0$$

$$\rho(\mathbf{v}, \nabla)\mathbf{v} = [\mathbf{j}, \mathbf{B}] - \nabla P$$

$$\operatorname{rot}\,[\mathbf{v}, \mathbf{B}] = 0$$

$$\operatorname{rot} \mathbf{B} = \mu_0\, \mathbf{j}$$

For example, in traditional coordinate system for exponential pressure behavior well-known 1D Hariss sheet can be obtained

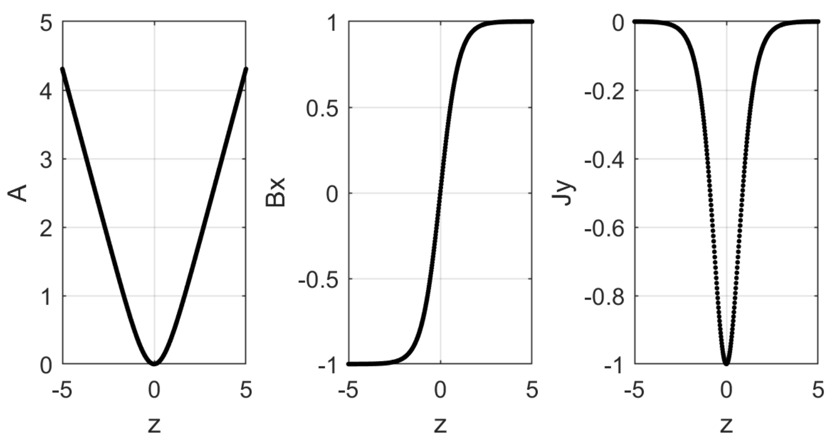
$$P = \exp(-A)$$

$$\frac{dP}{dA} = -\frac{1}{\mu_0}\Delta A$$

For zero plasma velocity its solution is well-known: by vector potential the system can be reduced to Grad-Shafranov equation

$$\nabla P = -\frac{1}{\mu_0}\Delta A \nabla A$$

which was analyzed for many particular cases. Nickeler and Wiegelmann main ideal was to reduce in the same way the system with fluxes by using modified functions.



Plasma flow and modified functions

Non-zero plasma flow can be taken into account if we assume that we know the function connecting the velocity and the field. Then substituting magnetic field in the system and excluding current density we obtain equation similar to GS:

$$\mathrm{div}(\rho\mathbf{v}) = 0 \qquad \mathrm{div}\,\mathbf{B} = 0$$
$$\rho(\mathbf{v},\nabla)\mathbf{v} = [\mathbf{j},\mathbf{B}] - \nabla P$$
$$\mathrm{rot}\,[\mathbf{v},\mathbf{B}] = 0$$
$$\mathrm{rot}\,\mathbf{B} = \mu_0\,\mathbf{j}$$

$$\mathbf{v} = M_0(|\mathbf{B}|)\cdot\mathbf{B}$$
$$\mathbf{B} = (\alpha_z, 0, -\alpha_x)$$

$$\nabla\left(P + \frac{|\mathbf{v}|^2}{2}\right) = (1-M_0^2)[\mathrm{rot}\,\mathbf{B},\mathbf{B}] - \frac{|\mathbf{B}|^2}{2}\nabla(1-M_0^2) =$$
$$= -(1-M_0^2)\Delta\alpha\nabla\alpha - \frac{|\nabla\alpha|^2}{2}\nabla(1-M_0^2)$$

Wiegelmann and Nickeler in 2010 noted that using modified pressure and modified potential, it is possible to reduce this equation into classical GS and build CS like in classical Harris case.

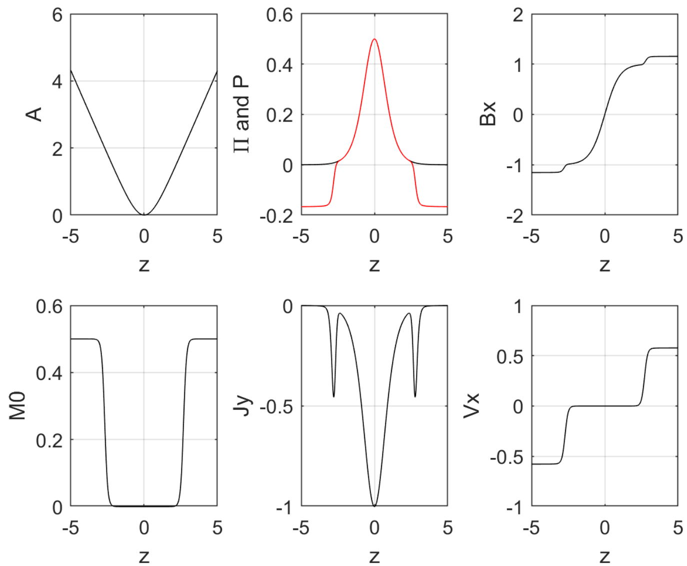
$$\Pi = P + \frac{|\mathbf{v}|^2}{2} \qquad \alpha = \alpha(A)$$

$$\nabla\Pi = -(1-M_0^2)\alpha'^2\Delta A\nabla A - \frac{|\nabla A|^2}{2}\nabla A((1-M_0^2)\alpha'^2)'$$

Modified potential in this case is defined by condition

$$(1-M_0^2)\alpha'^2 = 1 \quad \Rightarrow \quad \nabla\Pi = -\Delta A\nabla A$$

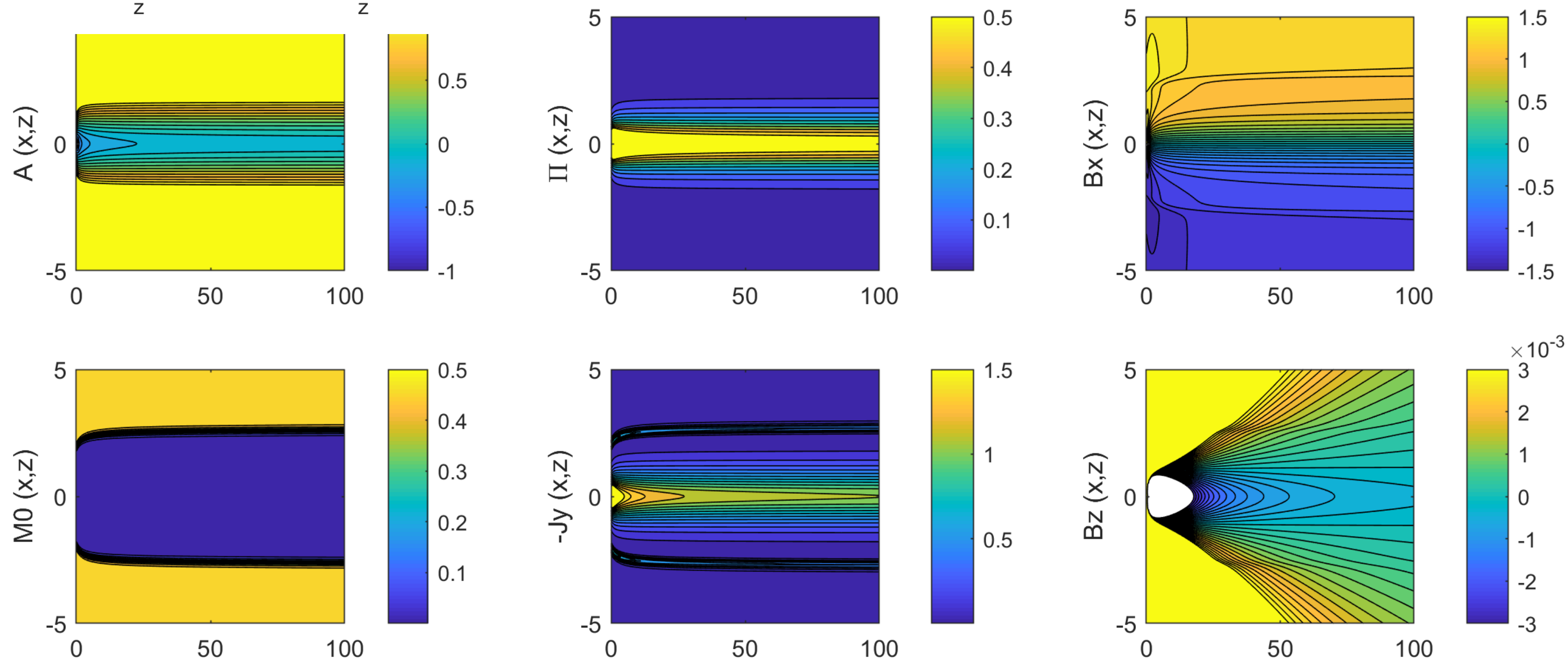
1D and 2D Harris CS with flows



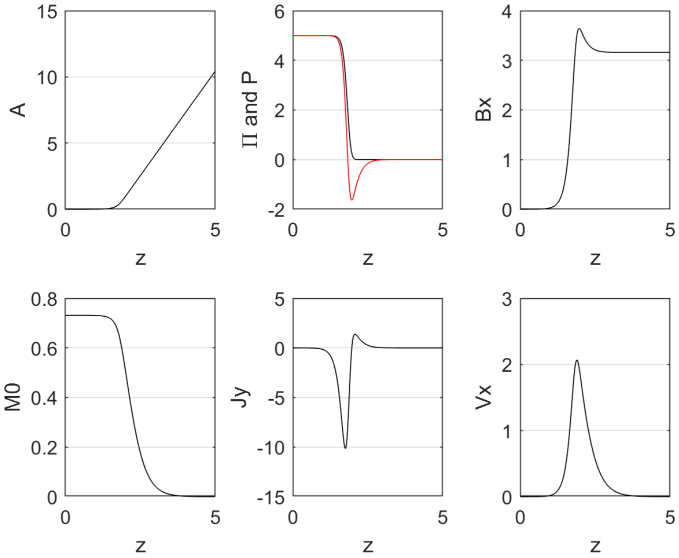
Step 1. Define modified pressure as a function of the modified potential (from general consideration or spacecraft data);
Step 2. Solving GS equation, restore the spatial structure of the modified functions;
Step 3. Define Mach function as a function of modified potential;
Step 4. Using Mach function, restore real vector potential, magnetic field and current density.

Examples for Harris exponential behavior of mod. pressure from mod. potential show spatial structure of current sheets, separating symmetrical oncoming flows

$$\Pi(A) = \frac{\Pi_0}{2}\exp\left(\frac{-2A}{A_0}\right)$$

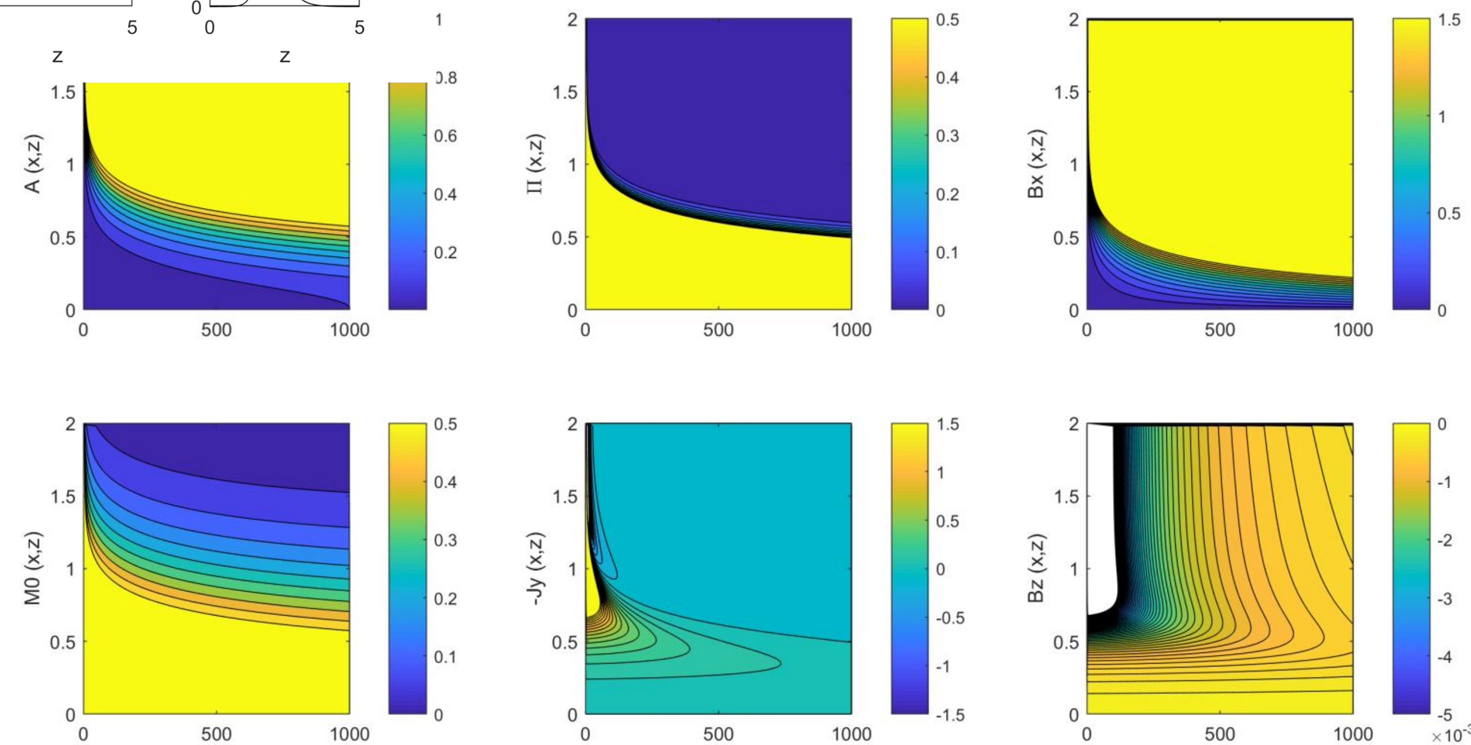


1D and 2D Nickholson CS with flows



.. however, for magnetopause description it is more reasonable to use Nickholson behavior (gauss type), which define boundary structure with current sheet, separating areas with strong magnetic field and strong plasma pressure…

$$\Pi = \overline{\Pi} * \exp\left(-\frac{A^2}{\overline{A}^2}\right)$$



Generalization on 3D-field and nonparallel flows

Modified functions allow us to generalize model. For example, if we assume quasi-3D structure, we can replace field in the system

$$\mathrm{div}(\rho\mathbf{v}) = 0 \qquad \mathrm{div}\,\mathbf{B} = 0$$

$$\rho(\mathbf{v},\nabla)\mathbf{v} = [\mathbf{j},\mathbf{B}] - \nabla P$$

$$\mathrm{rot}\,[\mathbf{v},\mathbf{B}] = 0$$

$$\mathrm{rot}\,\mathbf{B} = \mu_0\mathbf{j}$$

$$\mathbf{B} = (\alpha_z, I(\alpha), -\alpha_x), \quad \partial/\partial y = 0 \qquad \mathbf{v} = M_0(\alpha)\cdot\mathbf{B}$$

and reduce obtained equation

$$\nabla\left(P + \frac{\mathbf{v}^2}{2}\right) = [rot(\mathbf{B}),\mathbf{B}] - [rot(\mathbf{v}),\mathbf{v}]$$

$$= -(1-M_0^2)\Delta\alpha\nabla\alpha - \frac{|\nabla\alpha|^2}{2}\nabla(1-M_0^2) - \nabla\left((1-M_0^2)\frac{I^2}{2}\right)$$

to GS solution for the following modified pressure

$$\Pi = P + \frac{\mathbf{v}^2}{2} + (1-M_0^2)\frac{I^2}{2} = P + \frac{M_0^2\mathbf{B}_x^2 + \mathbf{B}_y^2 + M_0^2\mathbf{B}_z^2}{2}$$

More interesting that we can consider nonparallel plasma flows, demanding

$$\mathbf{v} = M_0\mathbf{B} + M_1\mathbf{B} + \frac{[\mathbf{E},\mathbf{B}]}{B^2}$$

$$div(\mathbf{v}) = div(\mathbf{v}_0) + div(\mathbf{v}_1) = 0$$

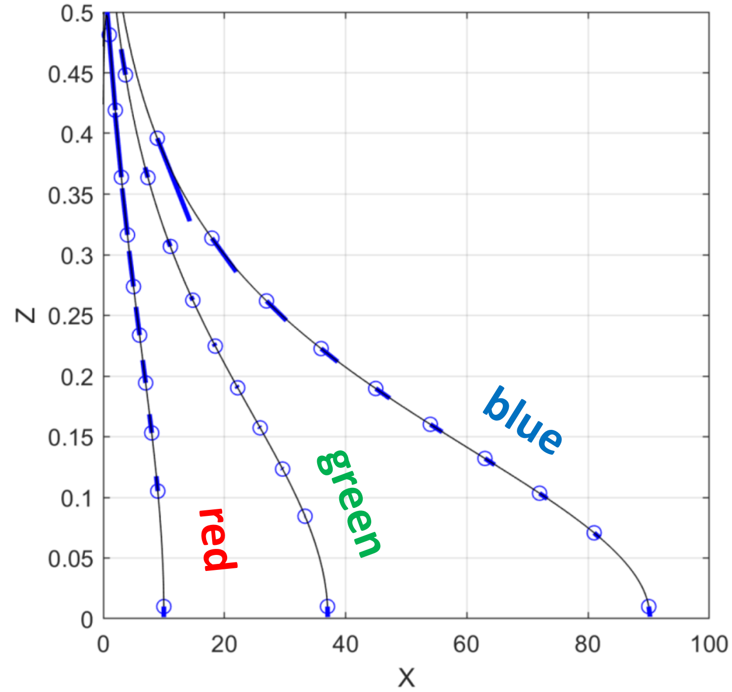
$$rot[\mathbf{v},\mathbf{B}] = rot\left(\frac{\mathbf{B}(\mathbf{B},\mathbf{E}) - \mathbf{E}B^2}{B^2}\right) = rot(\mathbf{E}) = 0$$

Nonparallel corrections

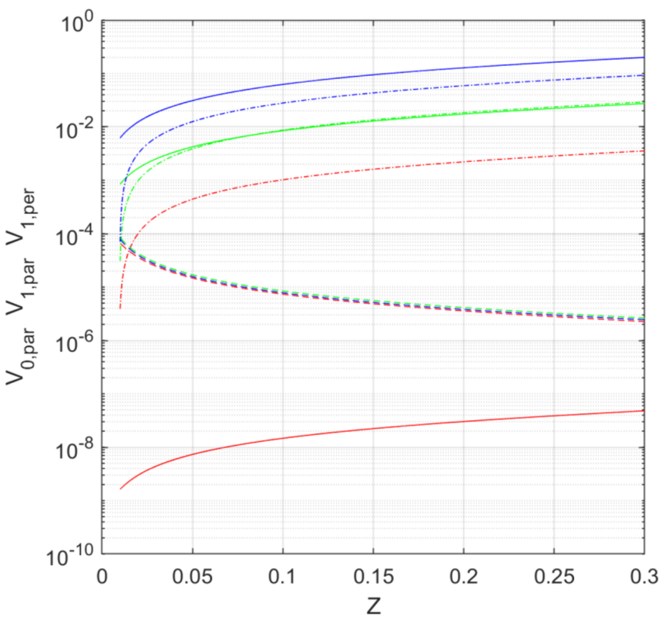
without transforming of magnetic field CS structure, discontinuity condition leads to additional correction on parallel velocity, that can be restored along the field lines

$$div(\mathbf{v}_1) = (\nabla M_1, \mathbf{B}) + div\left(\frac{[\mathbf{E},\mathbf{B}]}{B^2}\right)$$

$$\frac{dM_1}{dl} = -\frac{1}{B} div\left(\frac{[\mathbf{E},\mathbf{B}]}{B^2}\right)$$



and Euler equation leads to modified pressure corrections



$$\Pi_1^{xz} = -v_1^z v_0^x - v_1^x v_0^z - v_1^z v_1^x \qquad \Pi_1^{zz} = -2v_0^z v_1^z - (v_1^z)^2$$

$$\Pi_1^{yx} = -v_1^x v_0^y - v_0^x v_1^y - v_1^x v_1^y \qquad \Pi_1^{xx} = -2v_1^x v_0^x - (v_1^x)^2$$

$$\Pi_1^{yz} = -v_1^z v_0^y - v_0^z v_1^y - v_1^z v_1^y$$

Here you can see three examples of non-disturbed velocities (solid lines), parallel corrections (dash-dot lines) and perpendicular corrections (dashed lines) for three magnetic field lines of Hariss type CS. It is well seen that perpendicular correction is larger than parallel one near the neutral plane, while far from equator it is sufficiently negligible.

Key points of the presentation:

1. We consider a method of building an ideal mhd-model for describing current sheets separating plasma flows with different velocities.
2. The main idea of this method was sugested in 2010 Nickeler & Wiegelmann. It is possible to use modified pressure functions and modified vector potential to reduce mhd-system into Grad-Shafranov type equation.
3. We show that in 1D and 2D Harris-type structures plasma flow parallel to the field can be balanced in a current sheet with symmetrical current density peaks, in 1-D and 2-D Nicholson-type current sheets the boundary between string field and strong pressure can be balanced by asymmetric current profiles with different sign components.
4. The method allows generalization of mhd-model to the quasi-three-dimensional case (by changing the modified pressure function), as well as to the case with a stream that is not parallel to the field without transforming of magnetic field CS structure.
5. The components of the flow that is not parallel to the field in the stationary approximation can be balanced by the off-diagonal components of the pressure tensor and the additive parallel components, which can be restored along the field lines.

Thank you for your attention!

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