Approximate Convection-Diffusion Wave Method to Compute Discharge Using only Stage

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Overview

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- Problem Statement
- Objectives
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Introduction

- Water resources management is about solving problems to secure water for people, based on a sound scientific understanding of hydrologic and hydraulic processes. This includes protection from excess water and water shortage, as well as providing sufficient water for a sustainable environment.

- To achieve sustainable water resource management, the first thing is to quantify water resources.

Introduction

- There exist a number of different methods to measure discharge.

- The ACDW Method can be classified as purely hydro-dynamic based as the main governing equation is derived from Approximate Convection Diffusion, which is a simplified version of SV equation and the Hayami's Diffusion equation.

- The Hydraulic approach was initiated with established of Jones Formula in 1916. Since its publication, the Jones formula has been the subject of many research works, either as the starting point for obtaining more accurate equations or for establishing a general applicability criterion (Dottori et al., 2009).

Introduction

- Study on purely hydraulic approach for computing discharge has lead to a plethora of analytical methods.

  - ❖ Jones formula
    - ❖ Hendersons Formula
    - ❖ Modified Jones
    - ❖ Refined Jones
  - ❖ Marchi's formula
  - ❖ Fenton's formula
  - ❖ Fread's Formula
    - ❖ Modified Freads's Formula

Objectives

1

- To develop a methodology using Approximate Convection Diffusion Wave (ACDW) equation to approximate discharge using only stage hydrograph.

2

- To evaluate the practical applicability of the method and identify its limitation using hypothetical data and actual field data.

Problem Statement

- Use of advanced technology is expensive, time and resource consuming. Requires highly skilled and trained personals

- The rating curve method is based on statistical fitting of observed stage and discharge. It does not take into consideration the change in the hydraulic parameters such as Manning’s n and change of cross-section.

- The hydraulic methods have not been extensively tested for natural river systems and its suitability/applicability is not researched.

Literature Review

- It is a classical problem in river hydraulics to predict downstream discharge based on channel property when upstream discharge hydrograph is known(Dooge, 1973). Many methods have been proposed to solve the problem (Yevdjevich, 1964).

- For almost all the unsteady flow problems including overland flow (Kazezyilmaz-Alhan et al., 2005), river routing(Ponce et al., 1978), converting stage to discharge(Fenton, 1999), and many more, SV equations served as the fundamental governing equations.

Literature Review

- SVE is associated with practical difficulties as no analytical solutions are available. Moreover, numerical methods do not always guarantee an accurate solution.
- Kinematic wave(KW) is perhaps the simplest form among available models for modelling GVF flow (Lighthill & Whitham, 1955; Miller, 1983; Singh, 2002; Singh & Lima, 2018; Singh, 2017; Woolhiser & Liggett, 1967). This model is widely used by field engineers and scientists for hydrodynamics modelling.
- It is reported that term dy/dx is responsible for diffusion leading to attenuation of the flood wave(Perumal & Raju, 1999a). Since the term is neglected, the KW is not capable to describe diffusion and hence attenuation. Furthermore, it was developed with an assumption that stage and discharge bears a unique one-to-one relationship, thus its use to model a flood wave with looping rating curve is not suitable.

Literature Review

- Recently Perumal & Raju (1999a) have derived an equation that has a similar form as KW based on simplified momentum equation governing the flood waves in the transition between diffusive and KW, including KW.
- It was revealed that the ACD is applicable for modelling unsteady flow in a prismatic channel with the ability to describe a loop-rating curve.
- Since its inception, it has been widely used to develop overland flow model(Kale & Perumal, 2015; Kemble, Perumal, & Jain, 2012; Saxena & Perumal, 2014), routing model in stage formulation(Perumal & Raju, 1998a, 1998b), routing model in discharge formulation (Perumal & Price, 2013) and for the development of rating curves(Perumal et al., 2010; Perumal,et al., 2004; Perumal & Moramarco, 2005: lee & Muste 2017)
- Along with SVE and ACD, Hayami's Diffusive wave analogy equation is also used to solve routing problems (Hayami, 1951; Kuhnle & Bowie, 1992; Moussa, 1996; Fenton, 1999).

ACDW Method Development

$$\frac{\partial A}{\partial t} + c\frac{\partial A}{\partial x} = D\frac{\partial^2 A}{\partial x^2}$$

Hayami's Diffusive Equation in stage formulation

$$\frac{\partial y}{\partial x} + \frac{1}{c}\frac{\partial y}{\partial t} = 0$$

Approximate Convection Diffusive Method in stage formulation

$$\frac{\partial y}{\partial x} = \frac{1}{Tc^2}\frac{\partial Q}{\partial t} - \frac{2}{Tc}\frac{\partial A}{\partial t}$$

Analytically derived using Diffusive wave model and ACD equation

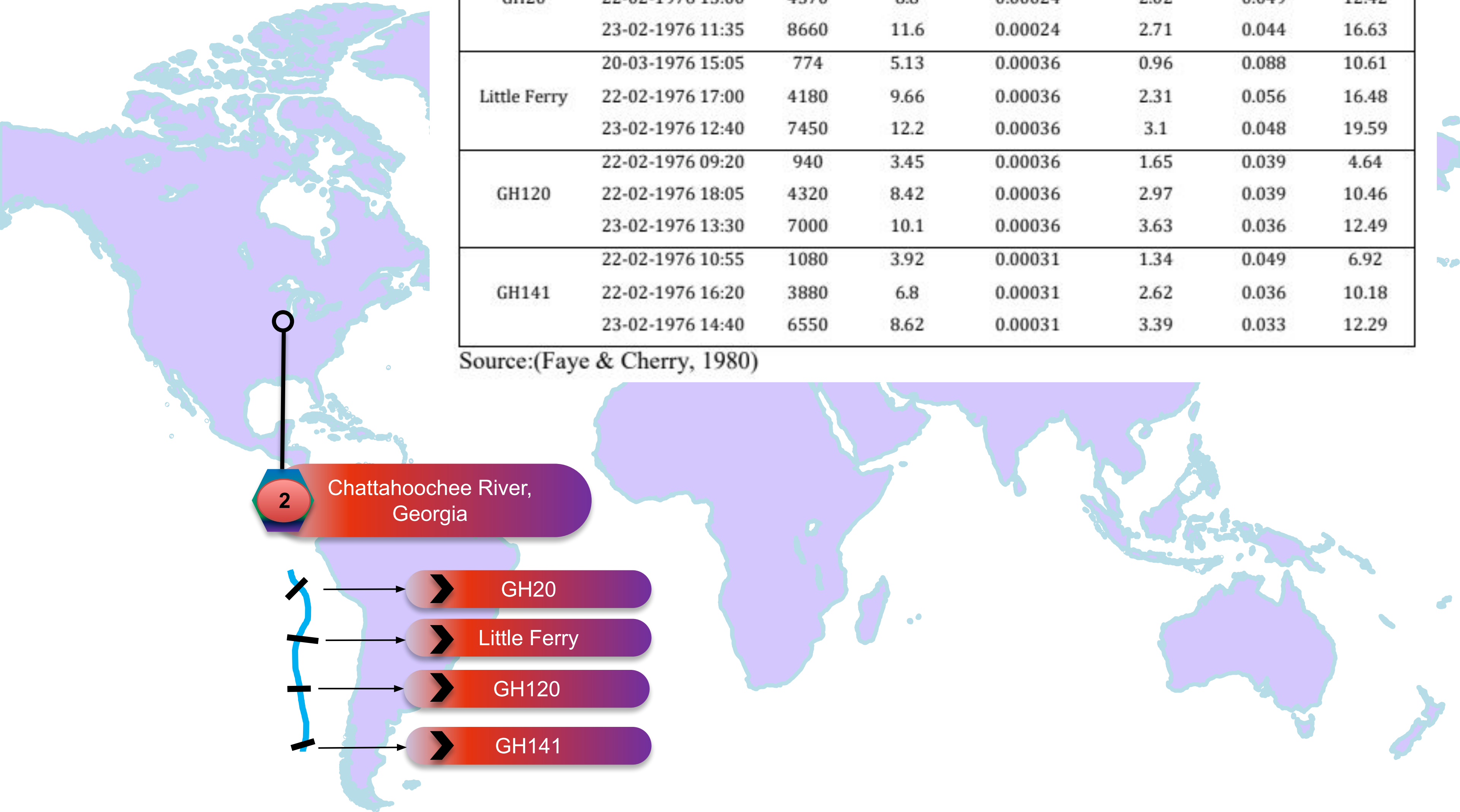
$$Q_2 = -\frac{{Q_o}^2}{2s_oT_2c_2^2\Delta t} + \frac{1}{2}\sqrt{\left(\frac{{Q_o}^2}{s_oT_2c_2^2\Delta t}\right)^2 + 4\left({Q_o}^2 + \frac{{Q_o}^2}{s_oT_2c_2^2}\frac{Q_1}{\Delta t} + \frac{2{Q_o}^2}{s_oT_2c_2}\left[\frac{A_2 - A_1}{\Delta t}\right]\right)}$$

Assumptions

1. The channel is assumed to be nearly prismatic,
2. The entire channel cross-section is conveying discharge,
3. The flow velocity and, consequently, the wave celerity can be considered approximately constant over a small stretch of the river where the flow depth measurement cross-section is located.
4. The section control prevails at the section where the discharge is estimated for the observed stage.

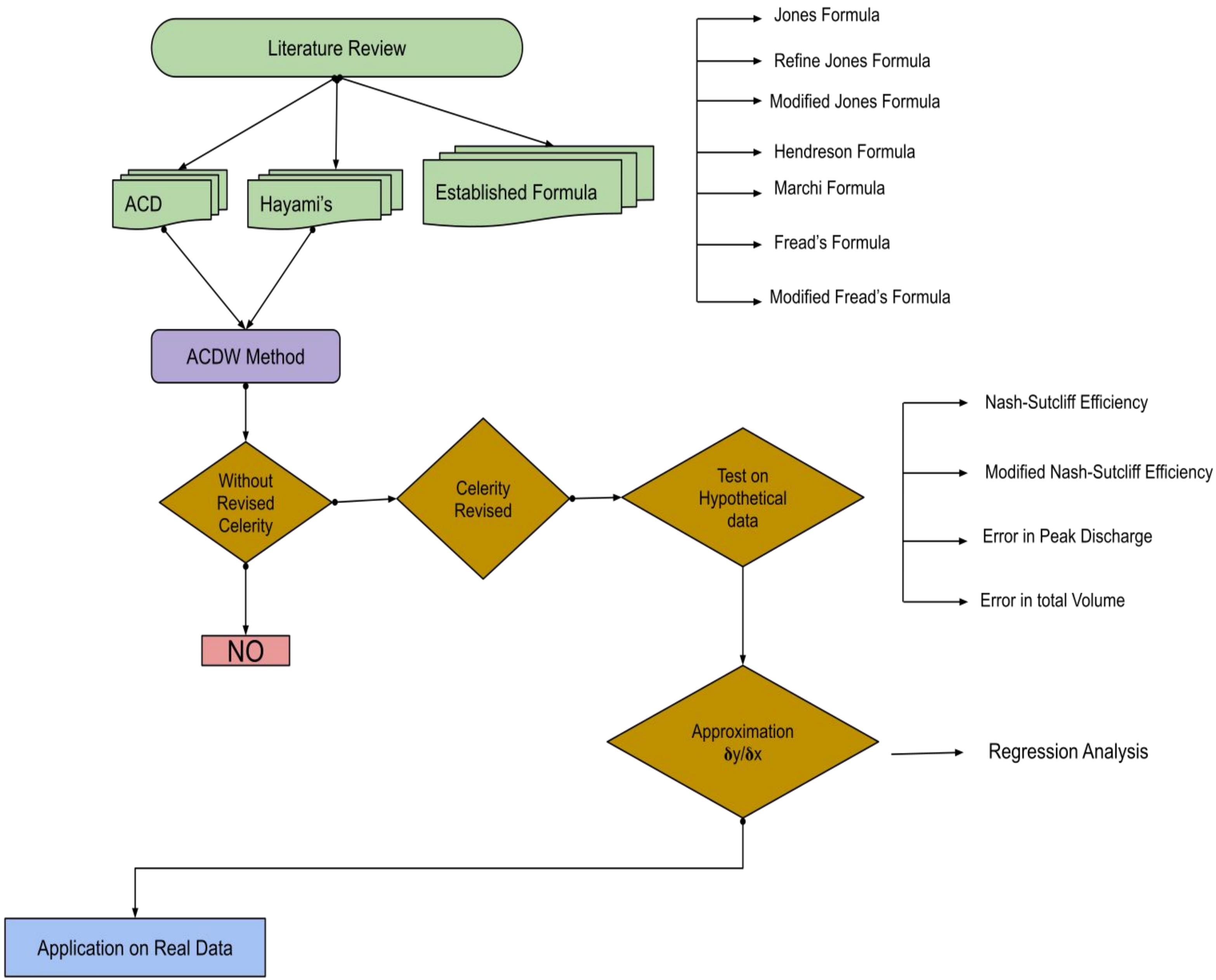
Gauging Stations	Date	Discharge	R	Effective Channel slope	Mean Flow Velocity	Manning's N	Max Flow Depth
GH20	22-02-1976 07:00	577	4.26	0.00024	0.68	0.089	6.59
	22-02-1976 15:00	4370	8.8	0.00024	2.02	0.049	12.42
	23-02-1976 11:35	8660	11.6	0.00024	2.71	0.044	16.63
Little Ferry	20-03-1976 15:05	774	5.13	0.00036	0.96	0.088	10.61
	22-02-1976 17:00	4180	9.66	0.00036	2.31	0.056	16.48
	23-02-1976 12:40	7450	12.2	0.00036	3.1	0.048	19.59
GH120	22-02-1976 09:20	940	3.45	0.00036	1.65	0.039	4.64
	22-02-1976 18:05	4320	8.42	0.00036	2.97	0.039	10.46
	23-02-1976 13:30	7000	10.1	0.00036	3.63	0.036	12.49
GH141	22-02-1976 10:55	1080	3.92	0.00031	1.34	0.049	6.92
	22-02-1976 16:20	3880	6.8	0.00031	2.62	0.036	10.18
	23-02-1976 14:40	6550	8.62	0.00031	3.39	0.033	12.29

Source:(Faye & Cherry, 1980)

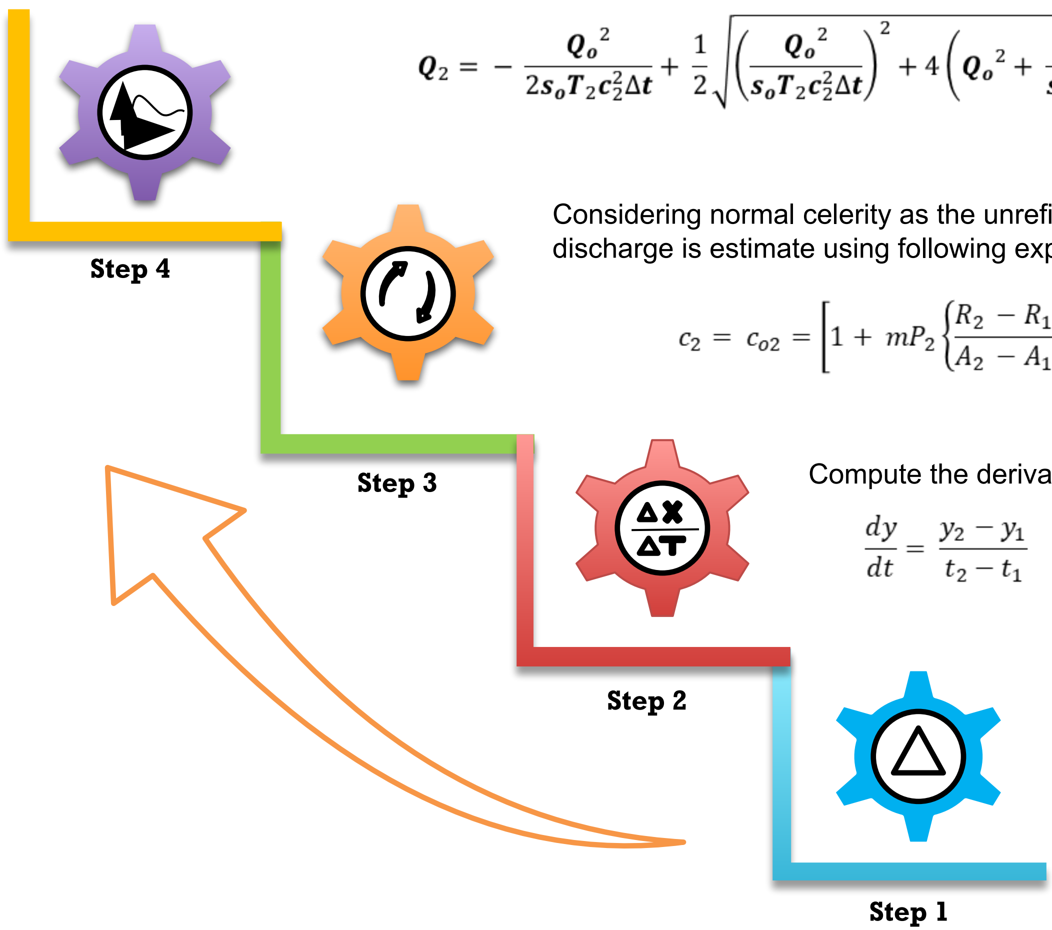


The Method is tested on these sites

Methodology Flow Chart



Steps in ACDW Method



$$Q_2 = -\frac{Q_o^{\,2}}{2 s_o T_2 c_2^2 \Delta t} + \frac{1}{2}\sqrt{\left(\frac{Q_o^{\,2}}{s_o T_2 c_2^2 \Delta t}\right)^2 + 4\left(Q_o^{\,2} + \frac{Q_o^{\,2}}{s_o T_2 c_2^2}\frac{Q_1}{\Delta t} + \frac{2 Q_o^{\,2}}{s_o T_2 c_2}\left[\frac{A_2 - A_1}{\Delta t}\right]\right)}$$

Considering normal celerity as the unrefined celerity at current time step, unrefined discharge is estimate using following expressions

$$c_2 = c_{o2} = \left[1 + m P_2 \left\{\frac{(R_2 - R_1)}{(A_2 - A_1)}\right\}\right] v_{o2}$$

Compute the derivatives terms using back-ward scheme

$$\frac{dy}{dt} = \frac{y_2 - y_1}{t_2 - t_1} \qquad \frac{dR}{dy} = \frac{R_2 - R_1}{y_2 - y_1}$$

Compute geometric properties corresponding to stage

Numerical Experimentation

Test 1 on Prismatic Rectangular Channels (adopted from Perumal, 2004)

Set of hypothetical rectangular channels with each of the considered channel reaches being characterized by a given unique sets of constant Manning's Coefficient and a constant bed slope.

Test 2 on prismatic Trapezoidal Channels (Adopted from Todini, 2007)

Input Stage Hydrograph is based on pearson Type III Distribution

$$y_t = y_o + (y_p - y_o)\left(\frac{t}{t_p}\right)^{\frac{1}{\gamma-1}} e^{\left(\frac{1-\frac{t}{t_p}}{\gamma-1}\right)}$$

Time to peak = 10 hrs, Peak Stage = 12 m and γ= 1.15;
Initial Stage = stage corresponding to 100 m^3/s

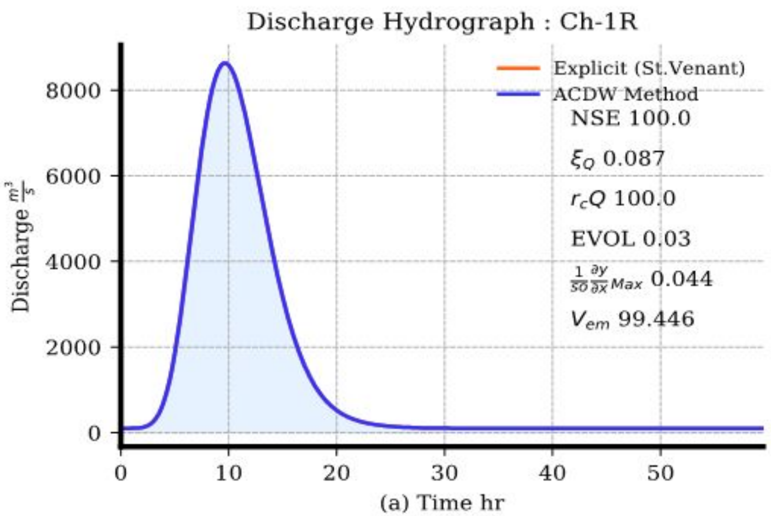
Results and Discussion

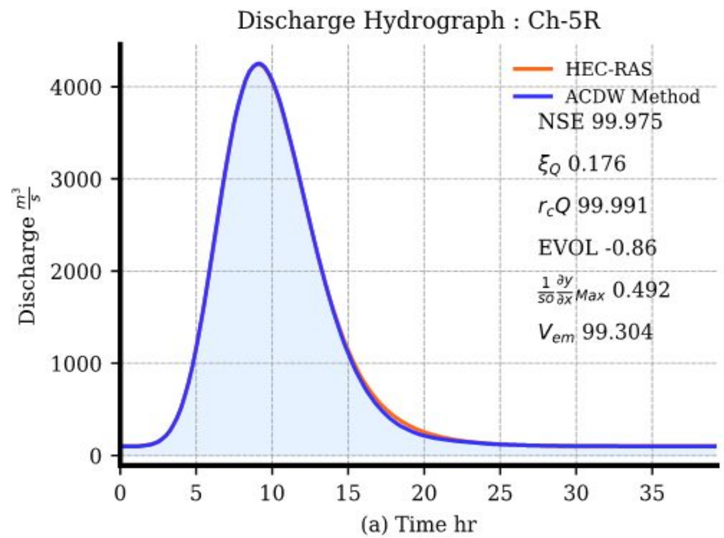
Test 1 - Hypothetical Dataset

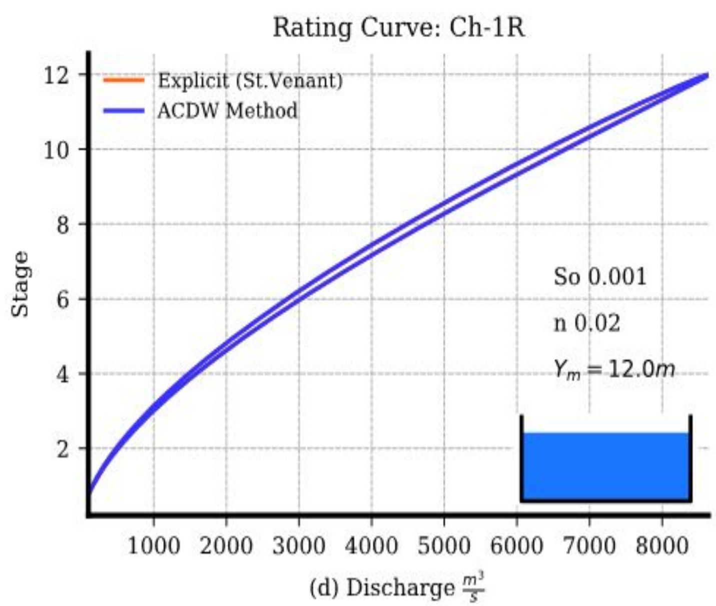
Channel Type	n	so	Jones	Refined Jones	Fenton's	Marchi's	ACDW Method	NSE
1 R	0.02	0.001	98.25	98.4	95.32	98.87	99.45	100
2 R		0.0008	99.13	99.51	97.13	99.36	99.49	100
3 R		0.0006	98.51	99.45	97.53	99.45	99.62	100
4 R		0.0004	96.5	99.23	96.5	99.23	99.61	100
5 R		0.0002	82.87	94.42	86.79	94.43	99.3	99.98
6 R	0.03	0.001	99.49	99.89	98.12	99.89	99.45	100
7 R		0.0008	99.04	99.84	98.18	99.84	99.66	100
8 R		0.0006	97.99	99.69	97.62	99.69	99.79	100
10 R		0.0004	94.44	98.99	94.97	98.99	99.77	100
13 R		0.0002	70.97	**Error**	**Error**	**Error**	**98.35**	**99.9**
14 R	0.04	0.001	99.21	99.9	98.44	99.9	99.78	100
15 R		0.0008	98.57	99.82	98.08	99.82	99.84	100
16 R		0.0006	96.93	99.57	96.9	99.57	99.86	100
17 R		0.0004	91.8	98.25	92.83	98.25	99.63	99.99
18 R		0.0002	52.07	**Error**	**Error**	**Error**	**97.15**	**99.73**

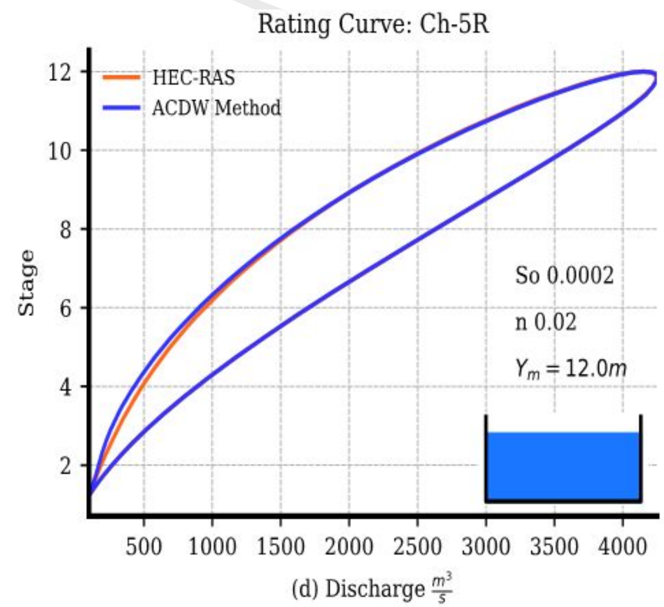
Results and Discussion

Test 1 - Hypothetical Dataset









Results and Discussion

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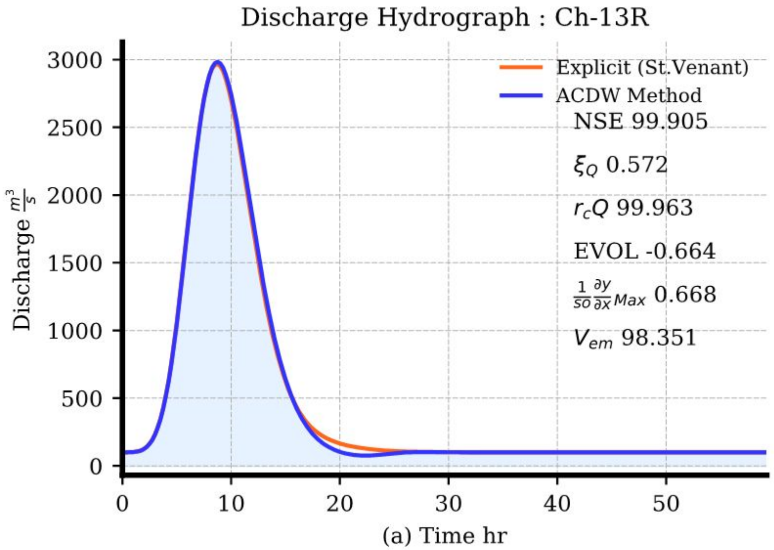

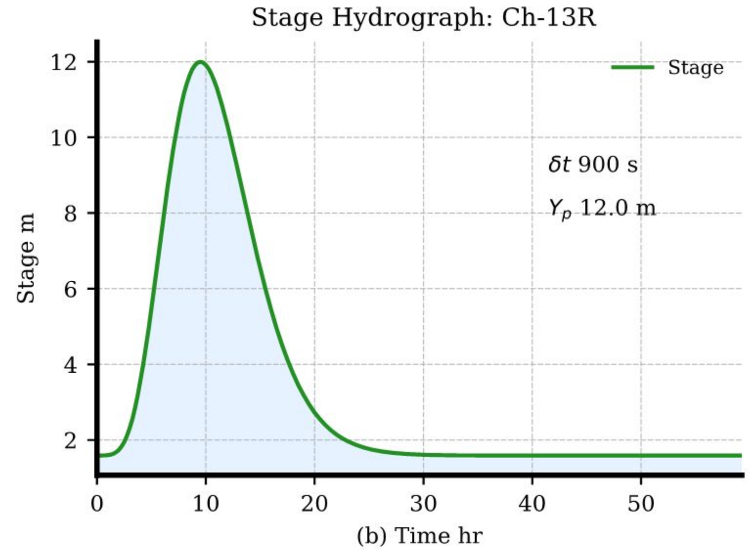

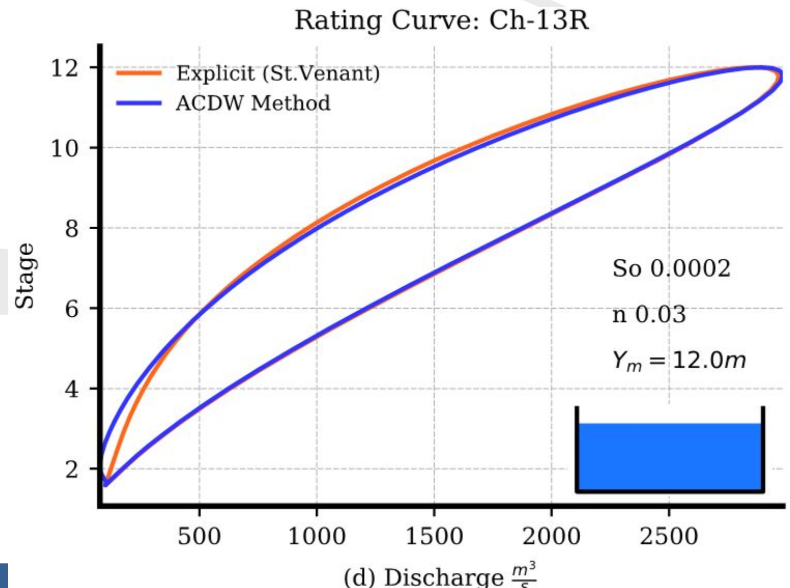

Results and Discussion

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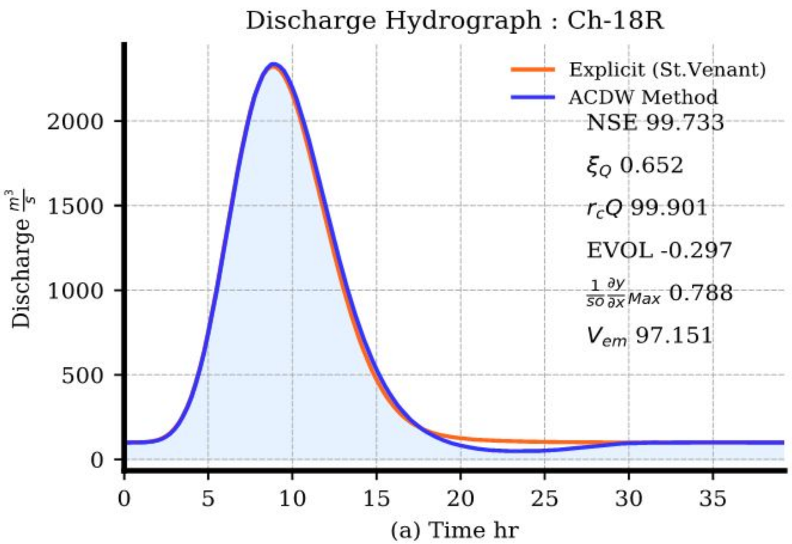
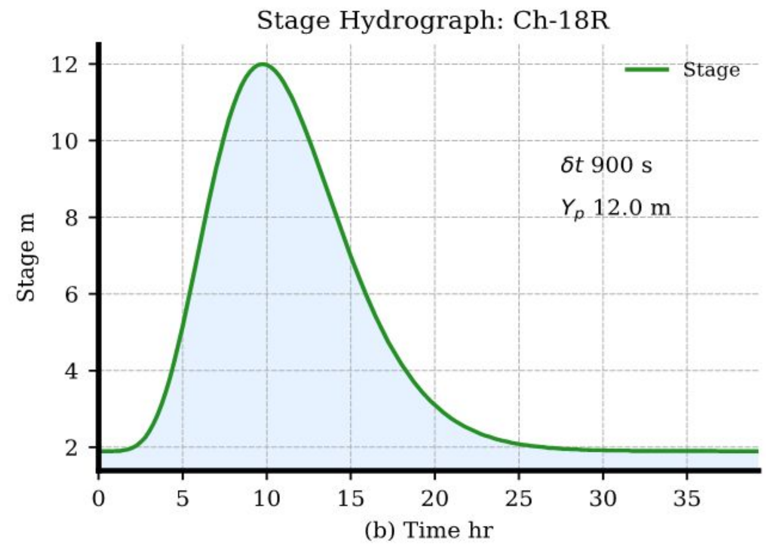
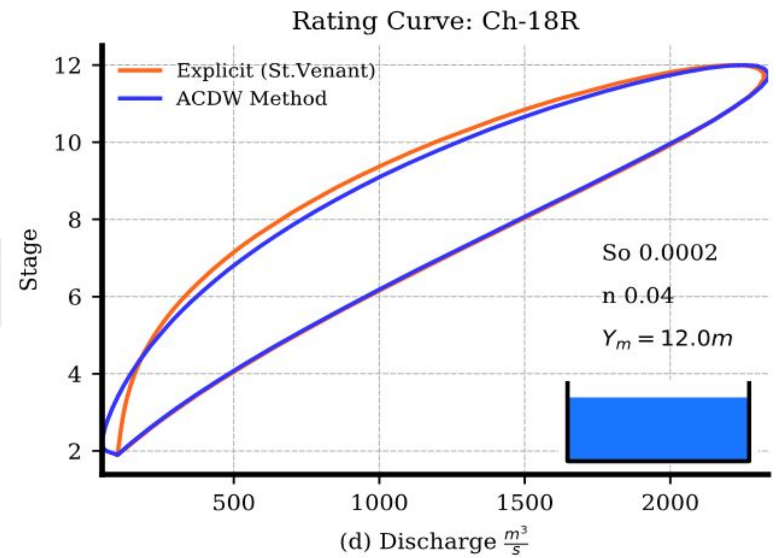






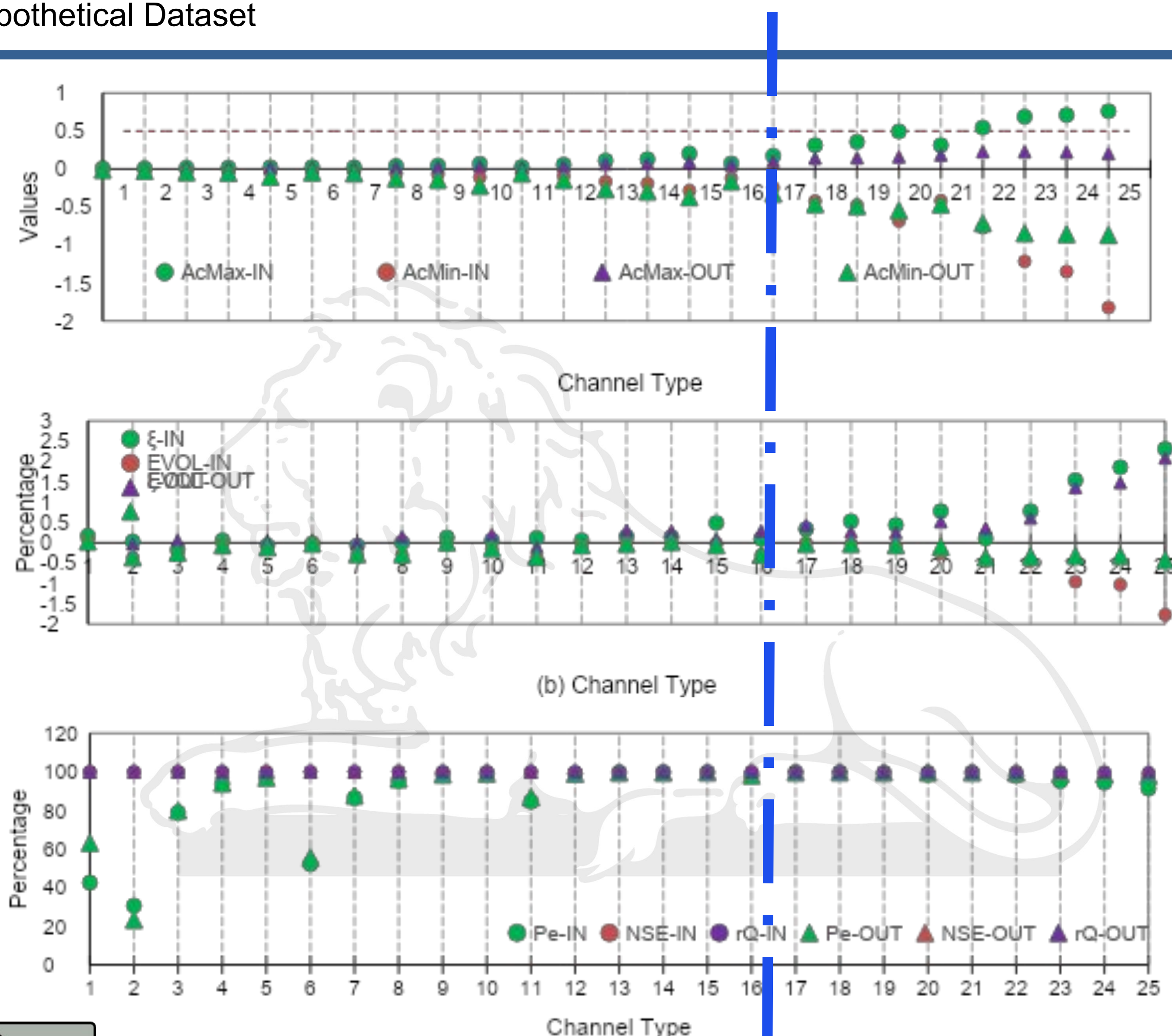
Results and Discussion

2. Test 2 - Hypothetical Dataset

Channel Configuration			Inlet section							Outlet section						
Channel Type	so	n	Pe	NSE	ξ	rQ	EVOL	AcMax	AcMin	Pe	NSE	ξ	rQ	EVOL	AcMax	AcMin
1 T	0.002	0.01	42.57	100	0.17	100	0.06	0.0092	-0.0186	63.18	100	0	100	0.03	0.0064	-0.0123
2 T	0.002	0.02	30.65	100	0.03	100	-0.37	0.009	-0.0262	23.22	100	-0.03	100	-0.36	0.0074	-0.0252
3 T	0.002	0.035	78.91	100	-0.17	100	-0.27	0.0151	-0.0265	79.92	100	0.06	100	-0.24	0.0126	-0.0431
4 T	0.002	0.04	93.8	100	0.06	100	-0.03	0.0157	-0.0302	94.03	100	-0.11	100	-0.06	0.0131	-0.0519
5 T	0.002	0.06	97.2	100	-0.07	100	-0.09	0.0227	-0.0388	96.99	100	0.05	100	-0.11	0.0181	-0.1015
6 T	0.001	0.01	52.23	99.99	-0.02	100	0.01	0.0253	-0.0395	55.62	99.99	-0.03	100	-0.02	0.021	-0.0459
7 T	0.001	0.02	86.46	100	-0.07	100	-0.3	0.021	-0.0395	87.69	100	0	100	-0.28	0.0164	-0.0572
8 T	0.001	0.035	95.39	100	0	100	-0.3	0.0382	-0.0644	96.24	100	0.15	100	-0.28	0.0277	-0.1249
9 T	0.001	0.04	98.97	100	0.13	100	0.02	0.0442	-0.0705	98.63	100	0	100	0.01	0.0308	-0.1422
10 T	0.001	0.06	99.18	100	0.06	100	-0.15	0.068	-0.1093	99.11	100	0.2	100	-0.13	0.0415	-0.221
11 T	0.0005	0.01	84.52	100	0.12	100	-0.36	0.0274	-0.0491	86.83	100	-0.1	100	-0.35	0.0226	-0.0594
12 T	0.0005	0.02	99.03	100	0.06	100	-0.04	0.0577	-0.0924	99.05	100	-0.01	100	-0.06	0.0421	-0.1439
13 T	0.0005	0.035	99.65	100	0.15	100	0	0.1103	-0.166	99.62	100	0.31	100	-0.03	0.0675	-0.2665
14 T	0.0005	0.04	99.71	100	0.14	100	0.02	0.1281	-0.1916	99.61	100	0.29	100	0.02	0.0745	-0.2959
15 T	0.0005	0.06	99.63	100	0.49	100	-0.02	0.2025	-0.2853	99.74	100	0.09	100	-0.06	0.0955	-0.3707
16 T	0.00025	0.01	98.38	100	0.1	100	-0.3	0.0737	-0.1204	98.2	100	0.29	100	-0.29	0.0558	-0.1638
17 T	0.00025	0.02	99.69	100	0.34	100	0	0.1722	-0.2476	99.69	100	0.43	100	-0.03	0.1059	-0.3249
18 T	0.00025	0.035	99.53	99.99	0.53	100	-0.03	0.3134	-0.4229	99.77	100	0.27	100	-0.03	0.1429	-0.4607
19 T	0.00025	0.04	99[illegible]3	99.99	0.44	99.99	-0.08	0.3564	-0.4764	99[illegible]71	100	0.25	100	-0.06	0.148	-0.4871
20 T	0.00025	0.06	98.[illegible]6	99.94	0.78	99.98	-0.25	0.494	-0.6837	99[illegible]68	99.99	0.52	100	-0.1	0.1599	-0.5444
21 T	0.0001	0.01	99.[illegible]5	99.99	0.11	100	-0.45	0.3129	-0.418	99[illegible]66	100	0.36	100	-0.36	0.1846	-0.466
22 T	0.0001	0.02	98.[illegible]8	99.9[illegible]	0.78	99.96	-0.45	0.5445	-0.773	99[illegible]54	99.99	0.61	99.99	-0.33	0.2291	-0.7128
23 T	0.0001	0.035	95.[illegible]1	99.51	1.54	99.83	-0.96	0.688	-1.2153	9[illegible]9	99.95	1.36	99.98	-0.32	0.2273	-0.8458
24 T	0.0001	0.04	94.[illegible]5	99.34	1.86	99.77	-1.03	0.7106	-1.3496	98[illegible]76	99.93	1.48	99.98	-0.32	0.2225	-0.857
[illegible]	[illegible]	0.06	91.34	98.56	2.31	99.5	-1.77	0.7614	-1.8223	98.11	99.87	2.09	99.96	-0.42	0.2031	-0.8678

Results and Discussion

Test 2 - Hypothetical Dataset



Results and Discussion

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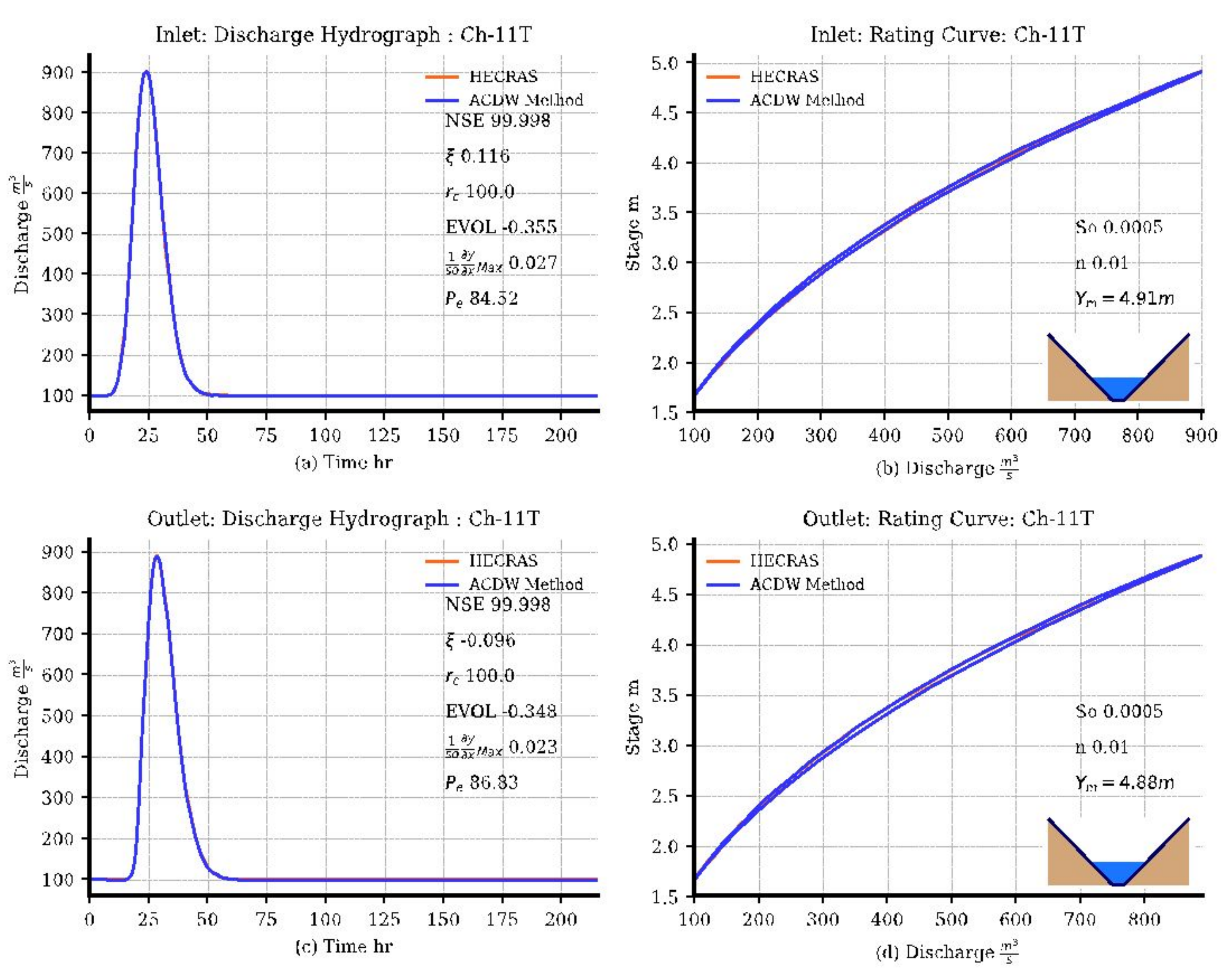

Results and Discussion

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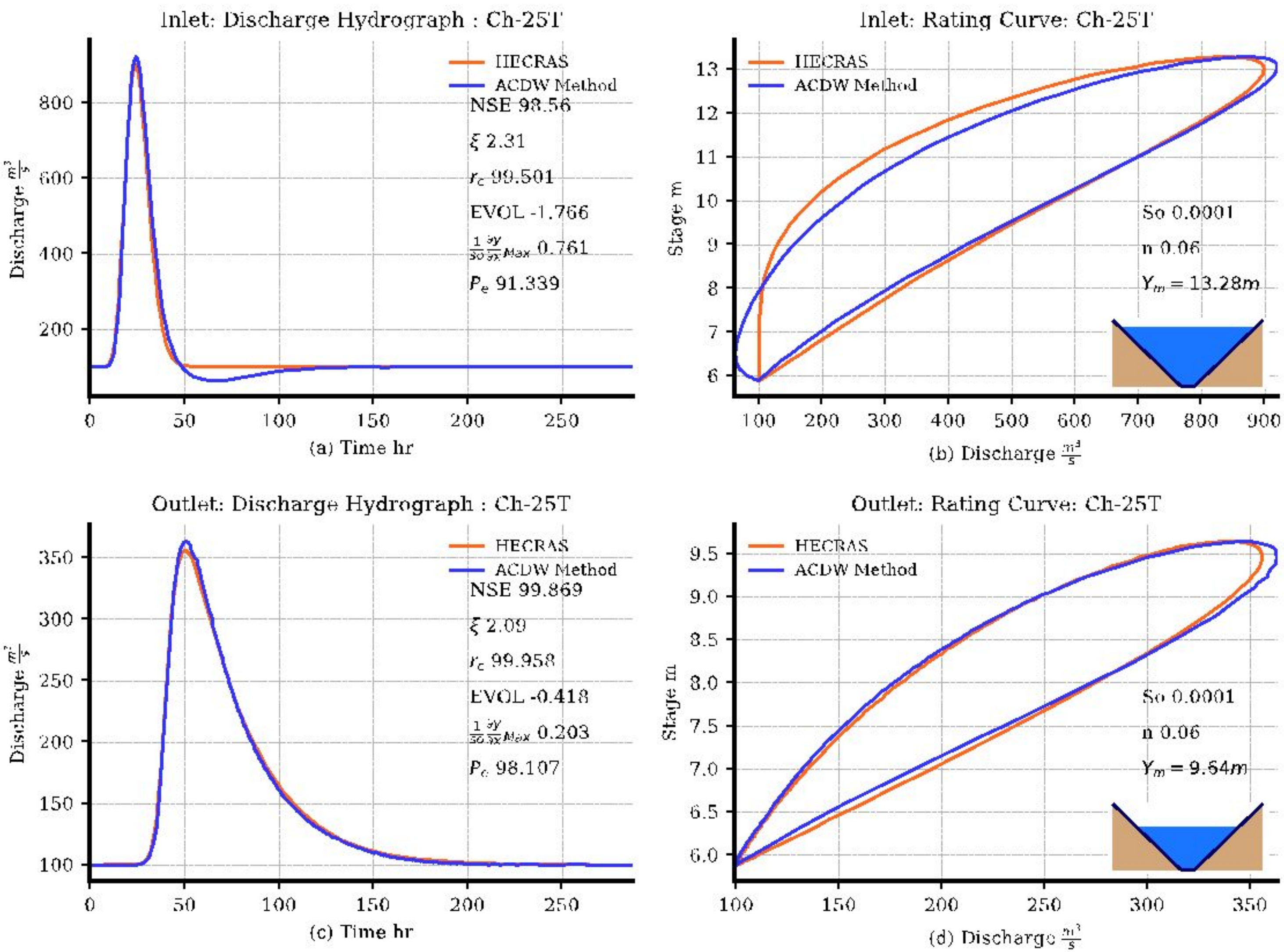


Result and Discussion

Approximation of ∂y/ ∂x according to ACDW

$$\frac{1}{s_o}\frac{\partial y}{\partial x} = \frac{1}{s_o T}\left(\frac{1}{c^2}\frac{\partial Q}{\partial t} - \frac{2}{c}\frac{\partial A}{\partial t}\right)$$

Benchmark ∂y/ ∂x

$$\frac{1}{s_0}\frac{\partial y}{\partial x} = 1 - \left(\frac{Q}{Q_0}\right)^2$$

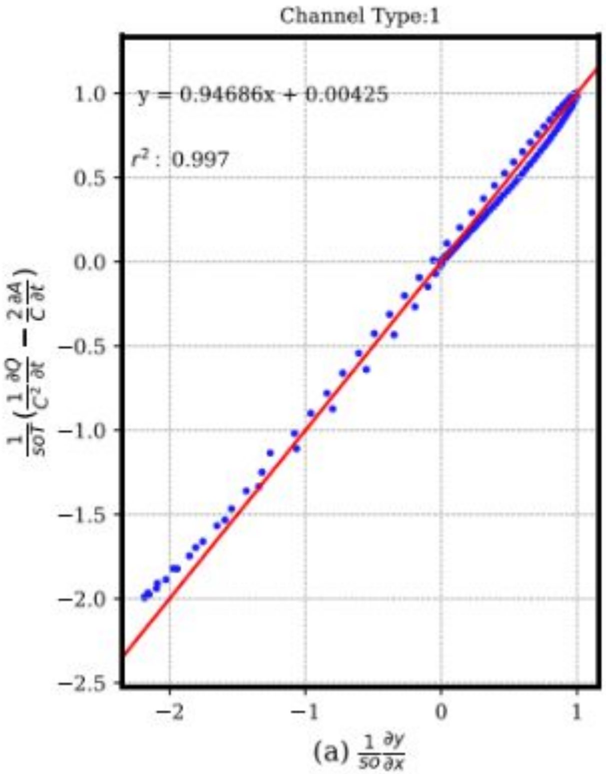
Results and Discussion

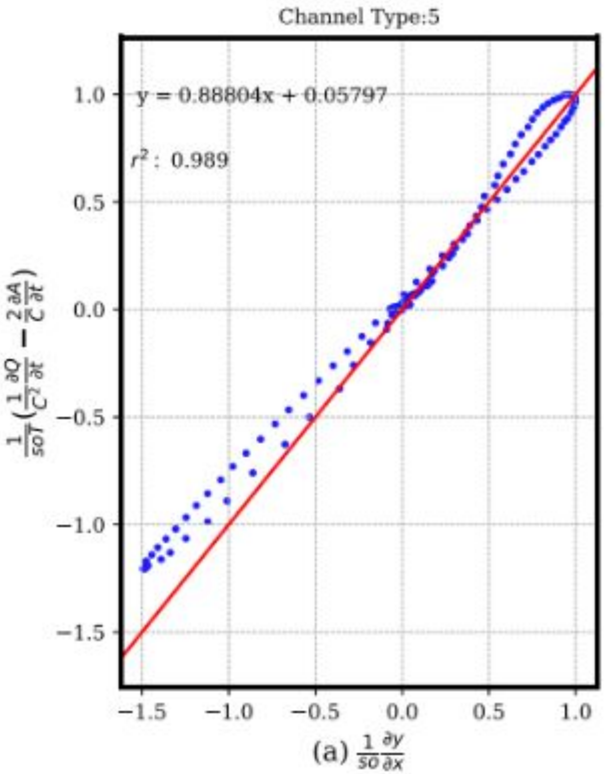
- Approximation of $\frac{\partial y}{\partial x}$

Channel Type	Bench Mark		ACDW r
	Minimum	Maximum	
1	-0.09629	0.04399	0.997
2	-0.12804	0.0613	0.997
3	-0.18636	0.09395	0.997
4	-0.30232	0.17438	0.995
5	-0.73486	0.49236	0.989
6	-0.12203	0.06439	0.972
7	-0.16847	0.09104	0.982
8	-0.25096	0.14138	0.993
9	-0.41214	0.26516	0.996
10	-0.42862	0.25982	0.996
11	-0.4378	0.26034	0.996
12	-0.4514	0.26232	0.995
13	-0.99744	0.66839	0.931
14	-0.16112	0.08669	0.974
15	-0.21743	0.12108	0.99
16	-0.31875	0.18761	0.995
17	-0.5322	0.33629	0.996
18	-1.18601	0.78786	0.893

Results and Discussion

Approximation of $\frac{\partial y}{\partial x}$





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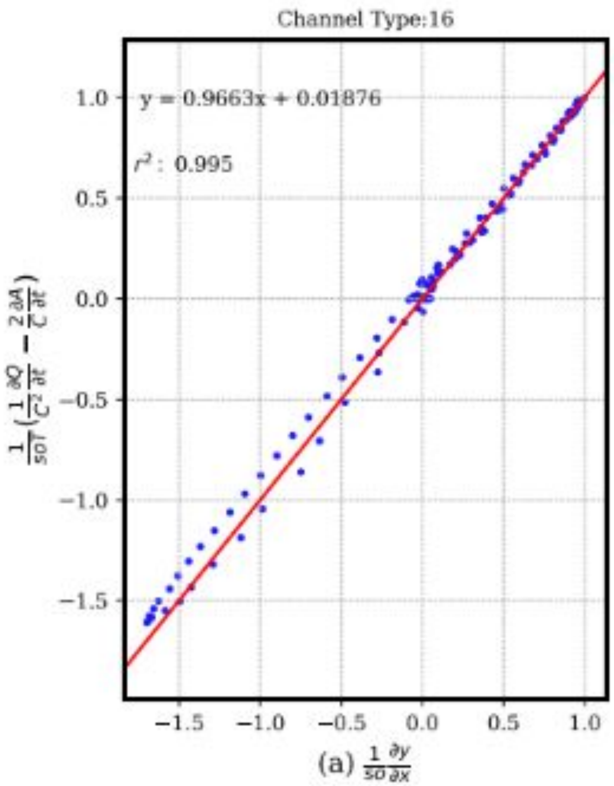
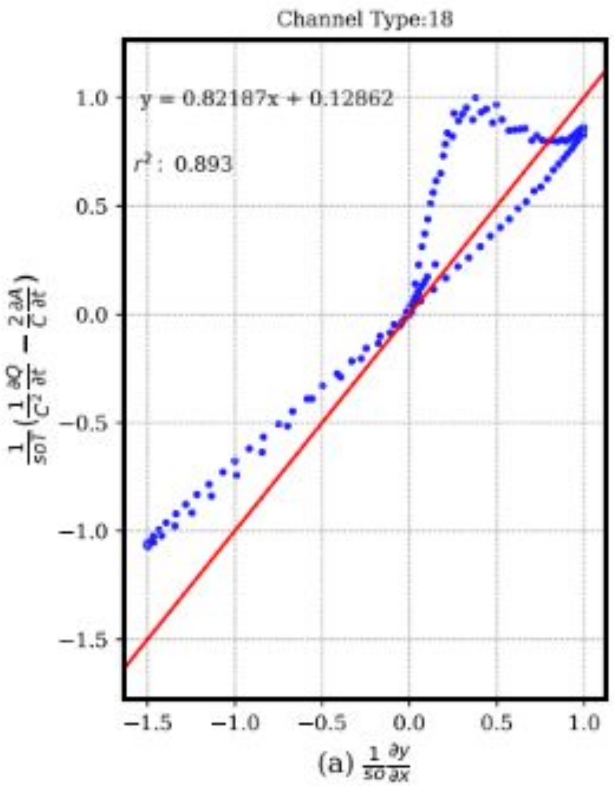
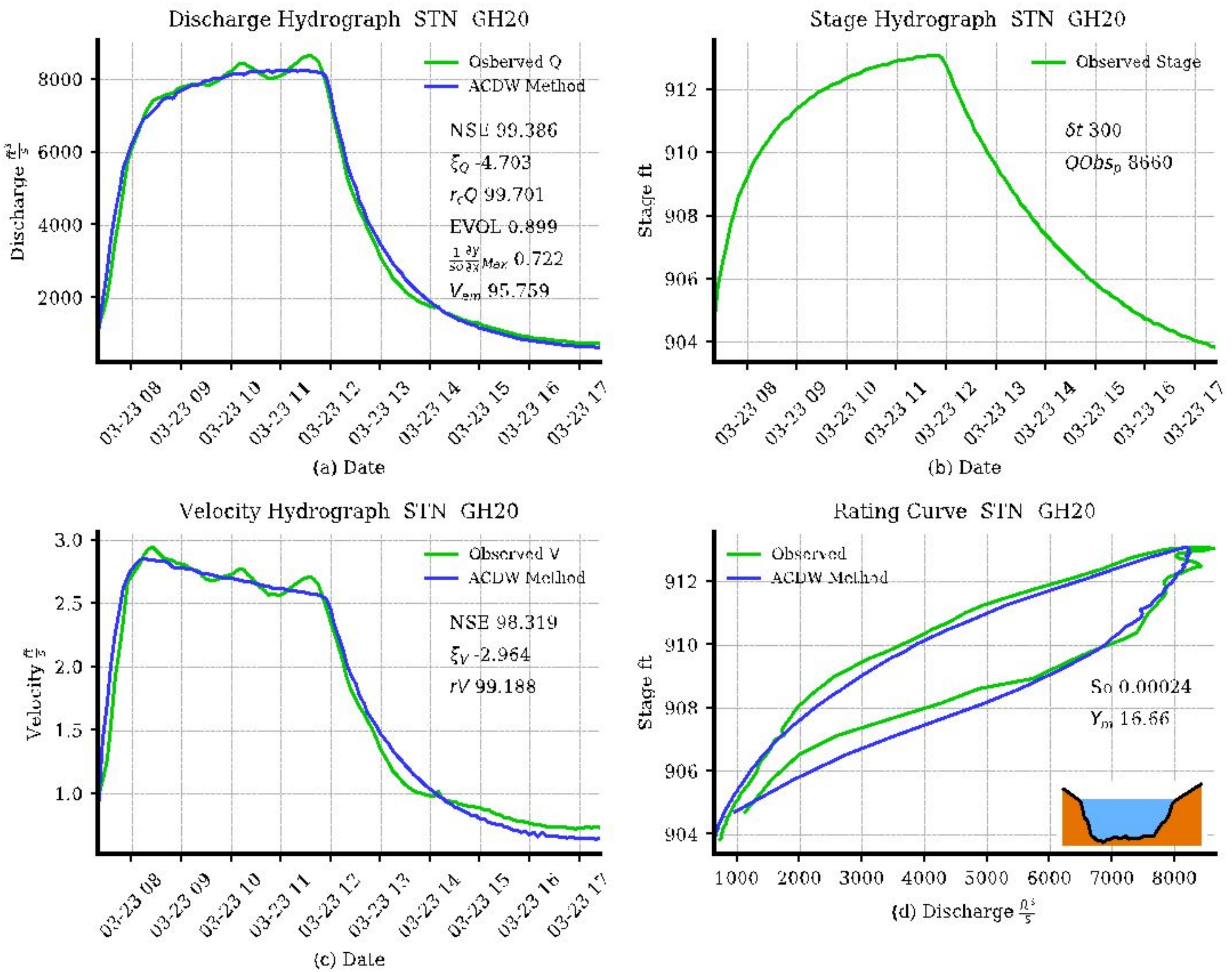
Approximation of $\frac{\partial y}{\partial x}$

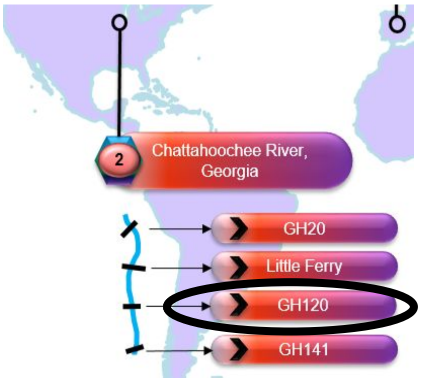




Results and Discussion

Application of ACDW Method on real data





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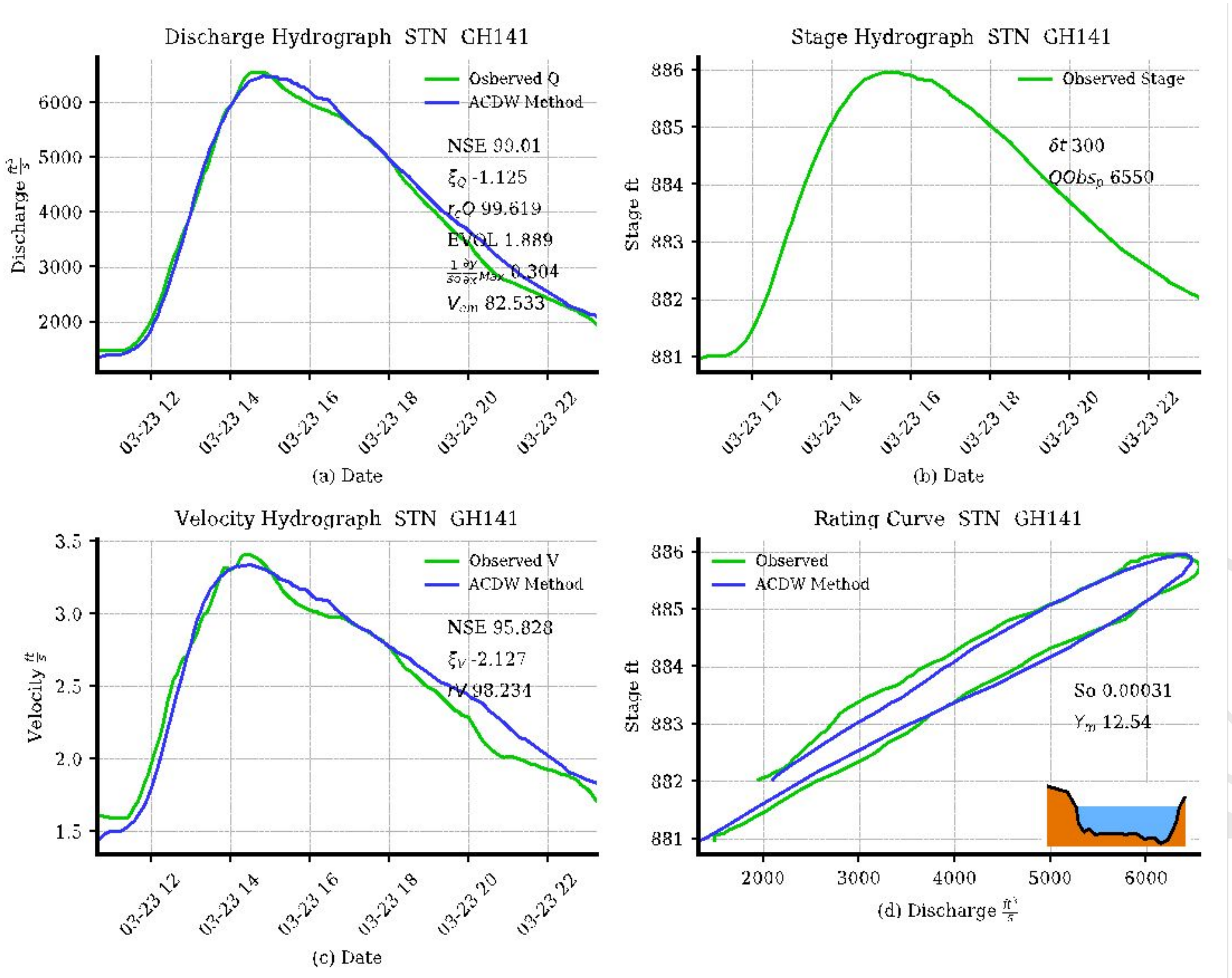
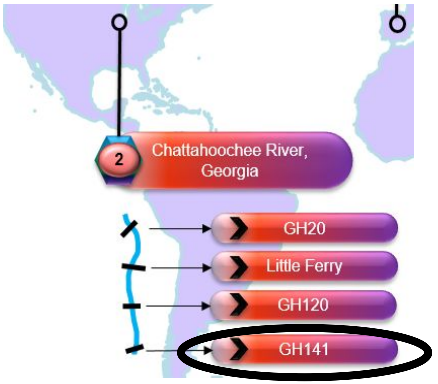
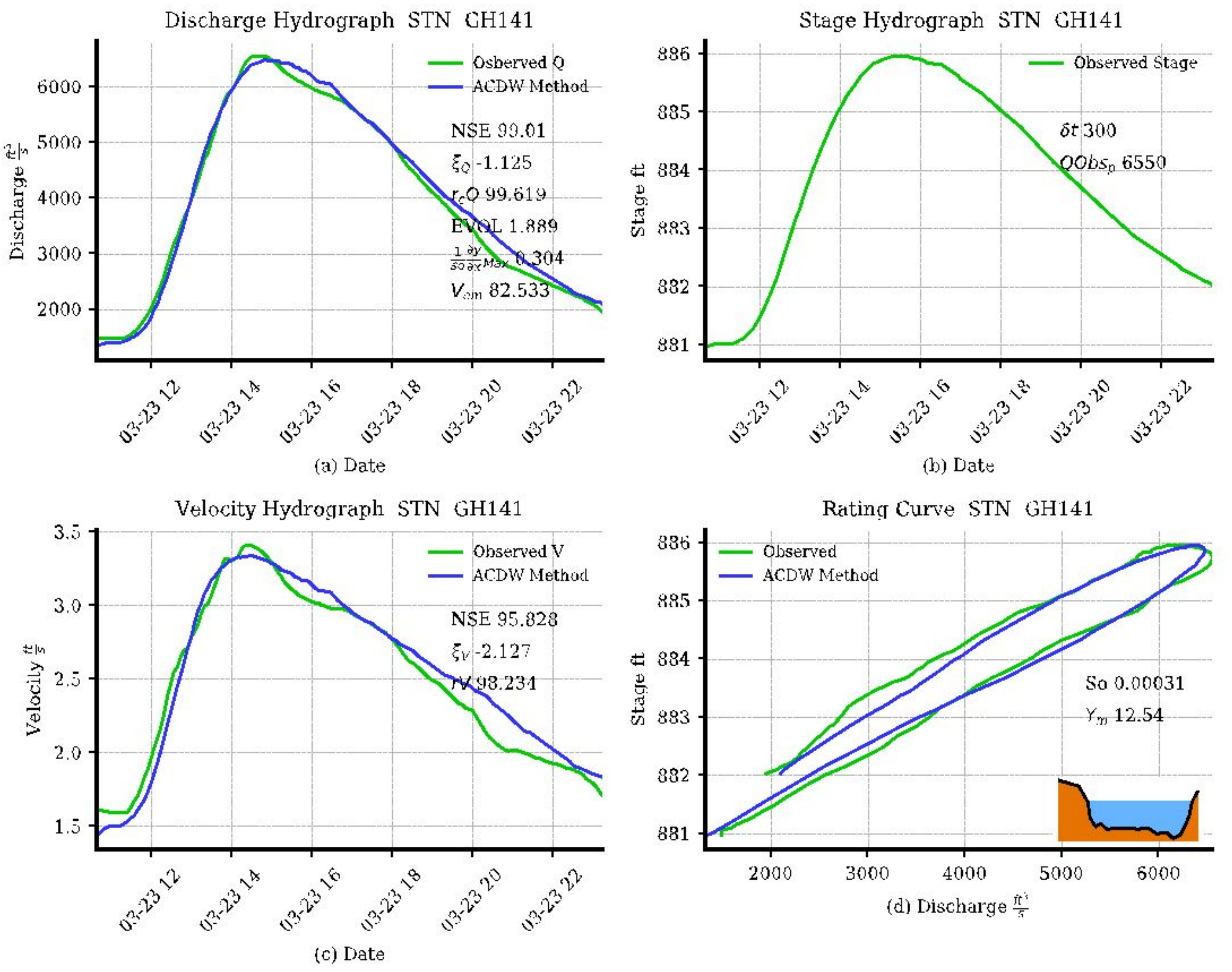
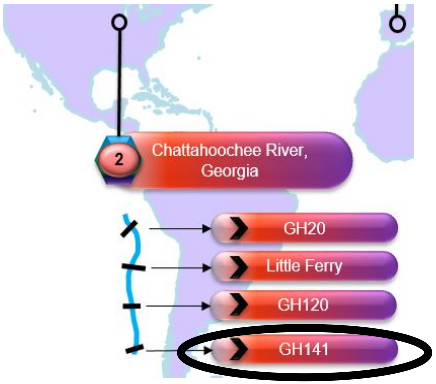




Results and Discussion

Application of ACDW Method on real data





Limitations

- It works flawlessly when the applicability criterion is satisfied

- The methodology to compute discharge using only stage data would be applicable to channel characterised by steep to moderate slope with cross-section of any shape (natural or artificial) where flow regime is characterized by any of supercritical, subcritical or critical flows. Conversely, the flow should be a virgin flow without any backwater effect at the gauging site.

Conclusion

- The new method (ACDW method) to estimate discharge using only stage data is developed based on hydrodynamic principles affected by hysteresis due to unsteady flow.

- Its applicability and suitability are extensively tested with hypothetical data and real data pertaining to Chattahoochee River. Data Source: Faye & Cherry (1980).

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SPYDER

julia

NumPy

SciPy

QGIS

THANK YOU

jupyter

Linux

NUMFOCUS
OPEN CODE = BETTER SCIENCE

matplotlib

pandas
$y_{it} = \beta' x_{it} + \mu_i + \epsilon_{it}$

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