On the permanent tide and the Earth dynamical ellipticity (materials)

A. Escapa[1,2] *, T. Baenas*[3]*, & J.M. Ferrándiz*[1]

[1]*University of Alicante;* [2]*University of Leon;* [3] *Centro Universitario de la Defensa, Murcia*

Spain

Alberto.Escapa@ua.es

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These materials are related to the paper *Precession of the non-rigid Earth: Effect of the mass redistribution* by Baenas, Escapa, & Ferrándiz (A&A 626, A58, 2019) available at https://doi.org/10.1051/0004-6361/201935472.

We refer the reader to that work in order to find the precise definition of the notations and symbols used in the next slides.

Context

- Conventional Earth precession and nutation stem from the torque exerted by external bodies, mainly by Moon and Sun
- That torque can be obtained through a multipole expansion of the gravitational potential energy —Earth+Moon and Earth+Sun— the second degree term being the most significant
- That term is linked to the Earth structure via its instantaneous matrix of inertia

$$I'_{ij} = \int_{B'} \rho'\left(\mathbf{r}'\right)\left(r'^2\delta_{ij} - x'_i x'_j\right) d^3\mathbf{r}',$$

resulting the well-known MacCullagh's formula (1844)

$$\mathcal{V}\left(\mathbf{r}_p\right) = -\frac{G m_p}{2 r_p^5} \sum_{i,j=1\,(j\geq i)}^{3} I'_{ij}\left[r_p^2\delta_{ij} - \left(3 - 2\delta_{ij}\right) x_{i,p} x_{j,p}\right]$$

- This potential provides the total effect arising from the perturbers p and can be split into undeformed —direct— and deformed—indirect— parts

Context

- ❑ The undeformed part is related to a "fictitious" Earth with no deformation, characterized by $\mathbf{I} = \mathrm{diag}\,\{\bar{A}, \bar{A}, C\}$, leading to

$$\mathcal{V}\left(\mathbf{r}_p\right) = \frac{Gm_p}{2r_p^5}(C - \bar{A})\left(3x_{3,p} - r_p^2\right) = \frac{Gm_p}{r_p^3}(C - \bar{A})\mathcal{C}_{20,p}(\theta_p, \phi_p)$$

- ❑ The deformed part is due to the redistribution of the elastic Earth caused by the direct action of the perturbers

- ❑ That redistribution results in a time-dependent contribution to the matrix of inertia

$$\Delta\mathbf{I}_q(t) = k_2\left(\frac{m_q a_\oplus^5}{3r_p^3}\right)\begin{pmatrix} \mathcal{C}_{20,q} - \frac{1}{2}\mathcal{C}_{22,q} & -\frac{1}{2}\mathcal{S}_{22,q} & -\mathcal{C}_{21,q} \\ -\frac{1}{2}\mathcal{S}_{22,q} & \mathcal{C}_{20,q} + \frac{1}{2}\mathcal{C}_{22,q} & -\mathcal{S}_{21,q} \\ -\mathcal{C}_{21,q} & -\mathcal{S}_{21,q} & -2\mathcal{C}_{20,q} \end{pmatrix},$$

entailing the redistribution tidal potential expression

$$\mathcal{V}_t\left(\mathbf{r}_p\right) = \frac{Gm_p}{2r_p^5}\left[3\begin{pmatrix} x_{1,p} \\ x_{2,p} \\ x_{3,p} \end{pmatrix}^t \Delta\mathbf{I}_q(t)\begin{pmatrix} x_{1,p} \\ x_{2,p} \\ x_{3,p} \end{pmatrix} - r_p^2\,\mathrm{trace}\left\{\Delta\mathbf{I}_q(t)\right\}\right]$$

Context

- ☐ The redistribution tidal potential $\mathcal{V}_t(\mathbf{r}_p)$ contributes to the Earth precession and nutation (Baenas et al. 2019, 2020), summing up its effects to that of the direct potential $\mathcal{V}(\mathbf{r}_p)$
- ☐ The precessional part is specially relevant from the point of view of stablishing a fundamental parameter: the Earth dynamical ellipticity or flattening (IERS Conventions 2010) — precession constant (Bursa 1995, Groten 2004)

$$H = \frac{C - \bar{A}}{C}$$

- ☐ Its value is derived from that observed for the general precession in longitude p_A (e.g., Kinoshita & Souchay 1990), reduced to the its linear part p'_A —proportional to *H*—

$$p'_A = HF(\varepsilon) = p_A - p_S,$$

where p_S accounts for other small contributions to the general precession of diverse origins (e.g., Williams 1994)

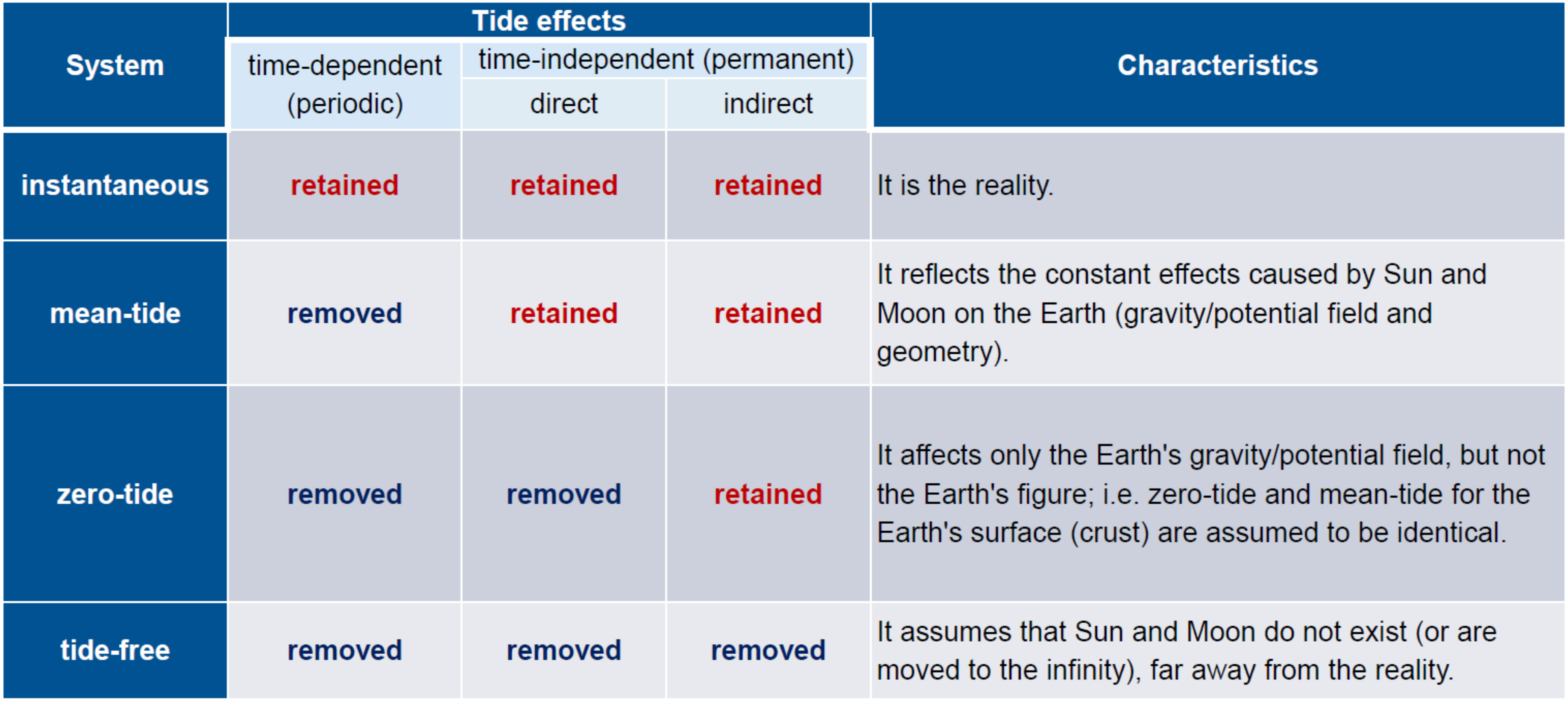
Context

- As we have pointed out, in the process of derivation of H from the precession it is involved the redistribution tidal potential $\mathcal{V}_t(\mathbf{r}_p)$
- That potential is a function of time of quasi-periodic nature driven by the orbital motion of the Moon and the Sun
- However, its time average is different from 0, due to the presence of a zero frequency in the Fourier expansion: it gives raise to the permanent tide (e.g., Bursa 1995)
- In this way, the determined value of H can be subject to different tidal systems as it is the case, for example, of the second-degree zonal geopotential —Stokes— parameter J_2 (e.g., Rapp et al. 1991, Bursa 1995, Groten 2004)
- Mean-tide, zero-tide, and free-tide systems can be considered

Context

Tidal systems

(see also Groten 2004, A2)



System	Tide effects			Characteristics
	time-dependent (periodic)	time-independent (permanent)		
		direct	indirect	
instantaneous	**retained**	**retained**	**retained**	It is the reality.
mean-tide	**removed**	**retained**	**retained**	It reflects the constant effects caused by Sun and Moon on the Earth (gravity/potential field and geometry).
zero-tide	**removed**	**removed**	**retained**	It affects only the Earth's gravity/potential field, but not the Earth's figure; i.e. zero-tide and mean-tide for the Earth's surface (crust) are assumed to be identical.
tide-free	**removed**	**removed**	**removed**	It assumes that Sun and Moon do not exist (or are moved to the infinity), far away from the reality.

Permanent tide: a station displacement? Or a permanent component of the station positions?, Laura Sánchez, Deutsches Geodätisches Forschungsinstitut (DGFI-TUM)- Technische Universität München, Unified Analysis Workshop, Paris, France, Oct 2 - 4, 2019

Context

- ☐ Resolution 16 adopted by the International Association of Geodesy (IAG) at its XVIII General Assembly (Hamburg, 1983) recommends that "*the indirect effect due to the permanent yielding of the Earth be not removed*" —zero-tide
- ☐ That was mainly motivated by the indistinguishability of that contribution from the constant part and the difficulties in modelling the Earth elastic response to long period forces —common Love number k_2 *versus* fluid one k_{2f}
- ☐ It is possible, however, to move from one tidal system to another applying the proper corrections —some of them conventional
- ☐ Regarding J_2, its zero-tide tidal distortion value δJ_2 (e.g., Bursa 1995, Groten 2004) goes back to a development by Zadro and Marussi (1973)

Context

- ☐ In the case of the dynamical ellipticity *H*, there is no explicit information about what tidal system is followed when providing its numerical value in the astronomical references
- ☐ This is the case in IAU2000/IAU2006 nutation/precession model (Mathews et al. 2002; Capitaine et al 2003) or in IERS Conventions 2010 —although there are some considerations about the permanent tide
- ☐ In the Current Best Estimates of the Parameters of Common Relevance to Astronomy, Geodesy, and Geodynamics (e.g., Bursa 1995, Groten 2000), it is supposed that the value of *H* is a zero-tide one but this is likely an assumption (e.g., Marchenko & Schwintzer 2003 or Marchenko & Lopushanskyi 2018)
- ☐ Since H and J_2 are combined to derive other Earth related parameters like, for example, $C/(MR^2)$, it is relevant to refer both to the same tidal system —besides to have a consistent set

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Some computations

- The effect of the redistribution tidal potential $\mathcal{V}_t(\mathbf{r}_p)$ on the Earth precession has been developed in Baenas et al. (2019)
- That process is also relevant in determining the tidal influence on *H*, entailing a consistent treatment of all the redistribution contributions that are worked out within the same framework
- We refer the reader to the complete derivations presented in Baenas et al. (2019) —see slide 2— giving here just an sketch of the main guidelines
- Within the scope of this communication, the central point is the Fourier expansion of $\mathcal{V}_t(\mathbf{r}_p)$ or, alternatively, $\boldsymbol{\Delta}\mathbf{I}_q(t)$ through

$$r_p^2\mathcal{C}_{20,p} = \frac{1}{2}\left(3x_{3,p}^2 - r_p^2\right),$$

$$\begin{aligned} r_p^2\mathcal{C}_{21,p} &= 3x_{1,p}x_{3,p}, \\ r_p^2\mathcal{S}_{21,p} &= 3x_{2,p}x_{3,p}, \end{aligned} \qquad \begin{aligned} r_p^2 C_{22,p} &= 3\left(x_{1,p}^2 - x_{2,p}^2\right), \\ r_p^2\mathcal{S}_{22,p} &= 6x_{1,p}x_{2,p}, \end{aligned}$$

Some computations

- ☐ The coordinates of the perturbers are given with respect to a reference system attached to the Earth, whereas their evolution is known in the ecliptic of date by some ephemeris (e.g., ELP, VSOP, etc.)
- ☐ Within the Hamiltonian framework the transformation involves the Andoyer variables —describing the rotation of the Earth— and some orbital functions and variables that characterize the Moon and the Sun motion. For example, we have

$$\left(\frac{a}{r}\right)^3 \mathcal{C}_{20}\left(\eta,\alpha\right) \simeq 3\sum_i B_i(I)\cos\Theta_i,$$

$$\left(\frac{a}{r}\right)^3 \mathcal{C}_{21}\left(\eta,\alpha\right) \simeq 3\sum_{i,\tau=\pm 1} C_i(I,\tau)\sin\left(\mu+\nu-\tau\Theta_i\right)$$

- ☐ Just the zonal second-degree $\mathcal{C}_{20}$ has a non-zero average, since $\mu+\nu\simeq\omega_{\oplus}t+\omega_0$ and there is a zero orbital frequency $\Theta_0=0$ in the Fourier decomposition set *i*

Some computations

- Hence, it is possible to compute $< \mathbf{\Delta I}_q(t) >$

$$< \Delta \mathbf{I}_q(t) >= k_2 \left(\frac{m_q a_\oplus^5}{a_q^3}\right) B_{0,q}(I) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- The orbital function $B_{0,q}(I)$, $I = -\varepsilon$ (Earth obliquity), was introduced in Kinoshita (1977)

$$B_i(I) \quad = \quad -\frac{1}{6}\left(3\cos^2 I - 1\right) A_i^{(0)} - \frac{1}{2}\sin 2I A_i^{(1)} - \frac{1}{4}\sin^2 I A_i^{(2)}$$

- The Love number appearing in the former equation should be the fluid one —$k_{2f} = 0.93$— (e.g., Lambeck 1980 or Bursa & Peck 1993), although in some conventional reduction it is is used $k_2 = 0.3$ (e.g., Rapp et al. 1991)
- From those expressions, we get the contribution of the zero tidal distortion of *H* —Darwin's theorem is involved —

$$\delta H \simeq \frac{3}{2}\frac{\Delta C}{C} = -k_2\left(\frac{3a_\oplus^5}{C}\right)\sum_{q=m,s}\left(\frac{m_q}{a_q^3}\right) B_{0,q} \simeq 8.8716\times 10^{-8} k_2$$

Some computations

- ☐ The former procedure can also be applied to derive δJ_2 with (Bursa, Groten, & Sima 2008)

$$\delta J_2 \simeq \frac{3}{2}\frac{\Delta C}{m_\oplus a_0^2},$$

  providing a value of 3.1279 × $10^{-8}k_2$, quite close from that reported in Groten (2004), 3.07531 × $10^{-8}k_2$

- ☐ The small differences (a relative error of 1.5%) can be due to the different orbital ephemeris used —the employed here being more precise— although it is necessary to revise the values of the Earth parameters entering into the computations
- ☐ From the point of view of obtaining H from the general precession in longitude, the precession due to δH must not be considered in the reduction process, since observationally we cannot distingue the indirect effect in H
- ☐ However, the precession rate due to the non permanent tides must be considered when deriving the p'_A value

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Summary

- ☐ As other Earth related parameters, the dynamical ellipticity H is affected by the redistribution of mass due to the action of the Moon and the Sun on the non-rigid Earth
- ☐ By using the same theoretical framework as for Earth precession/nutation modelling, we have derived the zero tidal distortion of H with $\delta H = 8.8716 \times 10^{-8} k_2$, providing an updated analytical expression with respect to that of Zadro and Marussi (1973) that can also be used to compute δJ_2
- ☐ When determining H from the general precession in longitude, it is necessary to recall that the precession due to δH must no be accounted for in the reduction process
- ☐ However, the redistribution contributions due to the non permanent tides must be considered and are not negligible (Baenas et al. 2019)

Summary

- ☐ When providing the value of H one should specify the used tidal system —mean, zero, or free— as it is the case, for example, of J_2 (Resolution 16, XVIII IAG GA, 1983)
- ☐ There is a lack of information about this point in current astronomical literature and standards (e.g., IAU 2000/2006, IERS Conventions 2010), which should be fixed in order to provide a consistent set of Earth parameters
- ☐ We are examining this question in detail, and the results will appear in a forthcoming paper