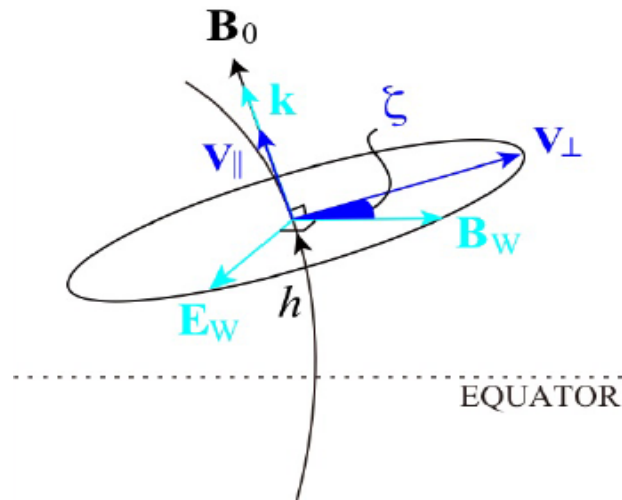


Generation of lower band and upper band whistler-mode chorus emissions and associated electron acceleration in the inner magnetosphere

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Nonlinear motion of resonant electrons is described by the pendulum equations with the inhomogeneity factor S controlled by the frequency sweep rate, the gradient of the magnetic field, and the wave amplitude.



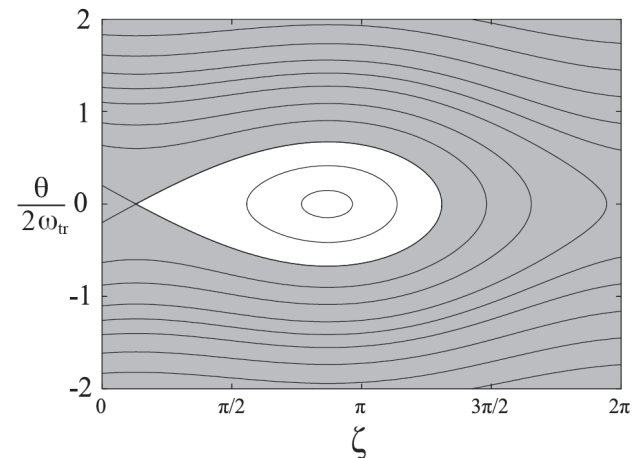
$$\theta = k(v_{\parallel} - V_R)$$

Inhomogeneity Factor

$$S = -\frac{1}{s_0 \omega \Omega_w} \left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h} \right)$$

$$\frac{d\zeta}{dt} = \theta$$

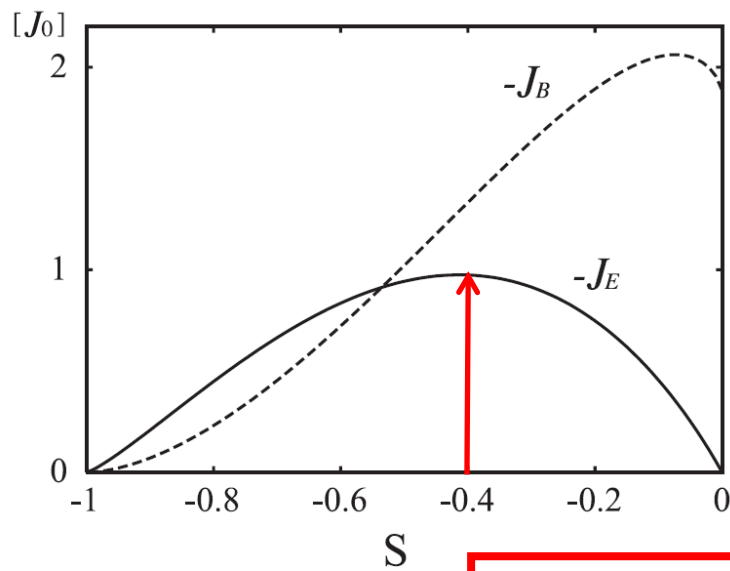
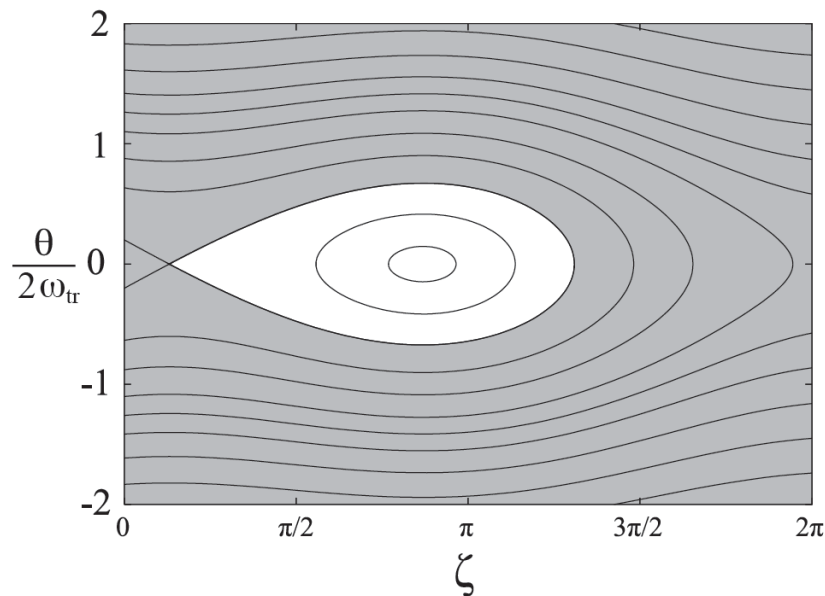
$$\frac{d\theta}{dt} = \omega_t^2 (\sin \zeta + S)$$



Nonlinear wave growth is caused by J_E due to formation of electromagnetic electron hole along with frequency sweep rate (rising-tone) near the equator. The rising tone frequency is due to J_B , resonant current parallel to the wave magnetic field.

$$\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E$$

$$c^2 k^2 - \omega^2 - \frac{\omega \omega_{pe}^2}{\Omega_e - \omega} = \mu_0 c^2 k \frac{J_B}{B_w}$$



Maximum $-J_E$

$$S_{EQ} = -0.4$$

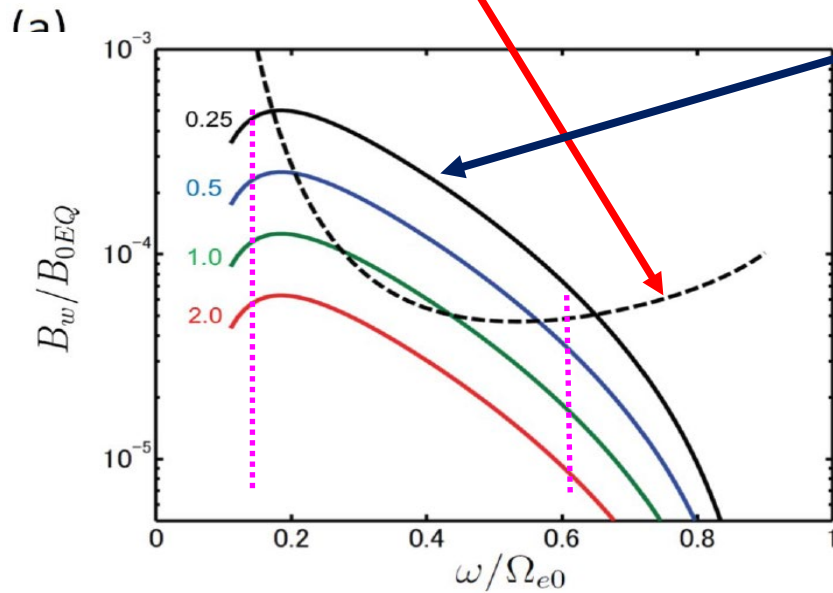
$$S = -\frac{1}{s_0 \omega \Omega_w} \left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h} \right)$$

$$\frac{\partial \omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1} \Omega_w$$

[Omura et al., JGR, 2008]

Nonlinear wave growth is possible between Optimum Wave Amplitude and Threshold Amplitude.

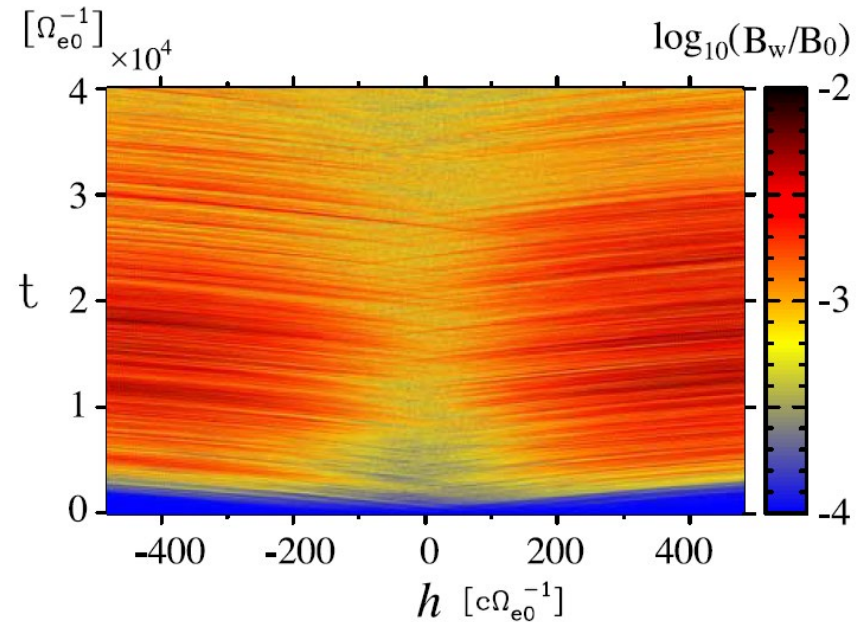
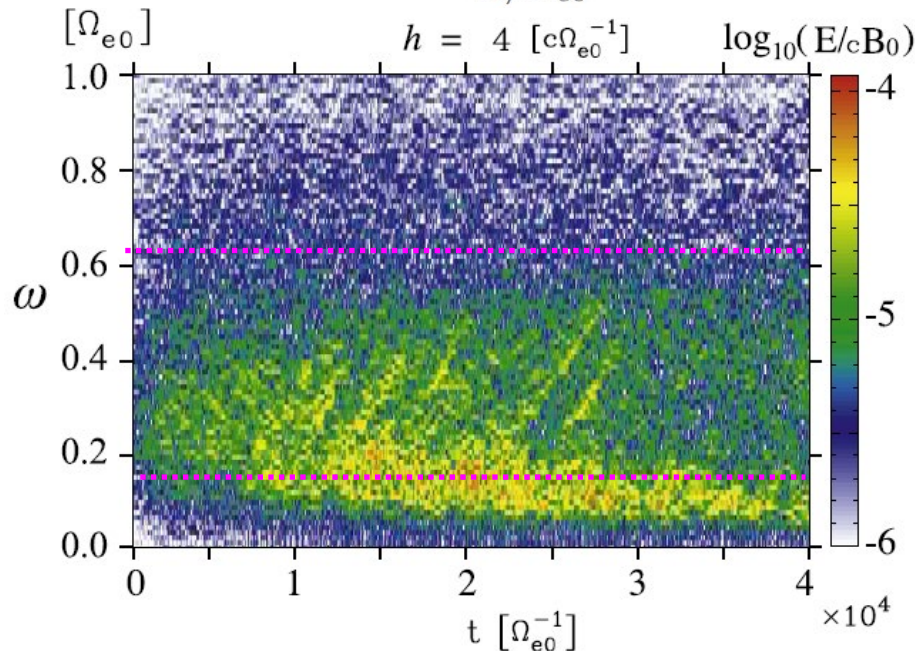
[Omura and Nunn, JGR, 2012]



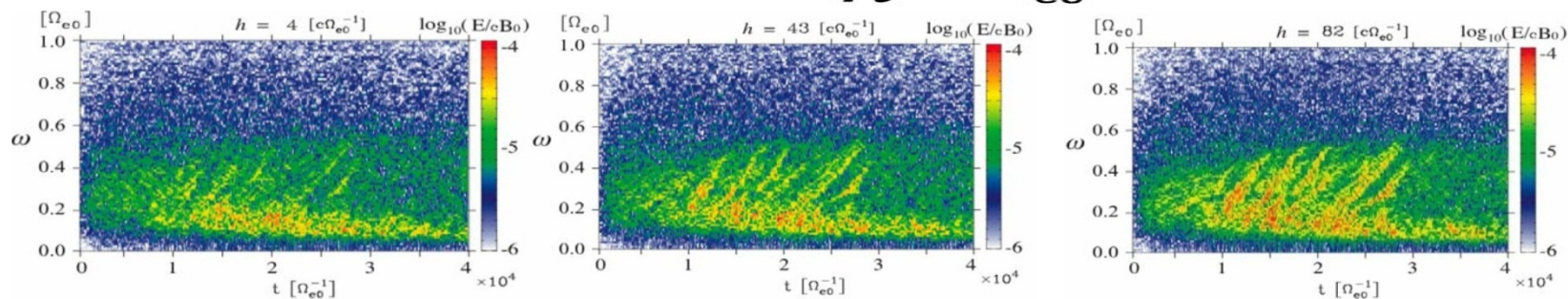
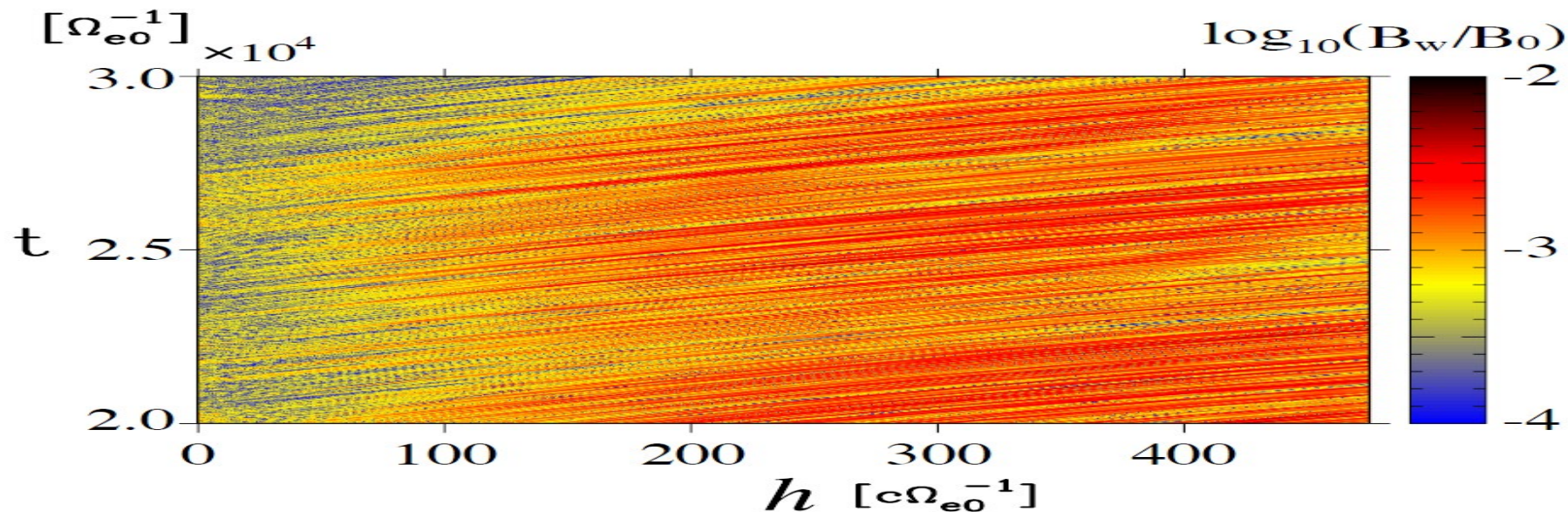
Electron Hybrid Simulation
[Katoh and Omura, JGR, 2007]

$$\tau = T_N/T_{tr}$$

$$\tau = 0.25 \sim 0.5$$



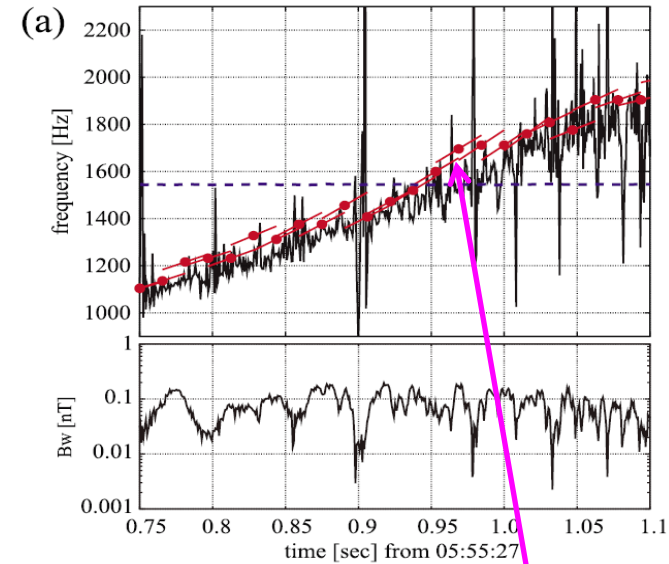
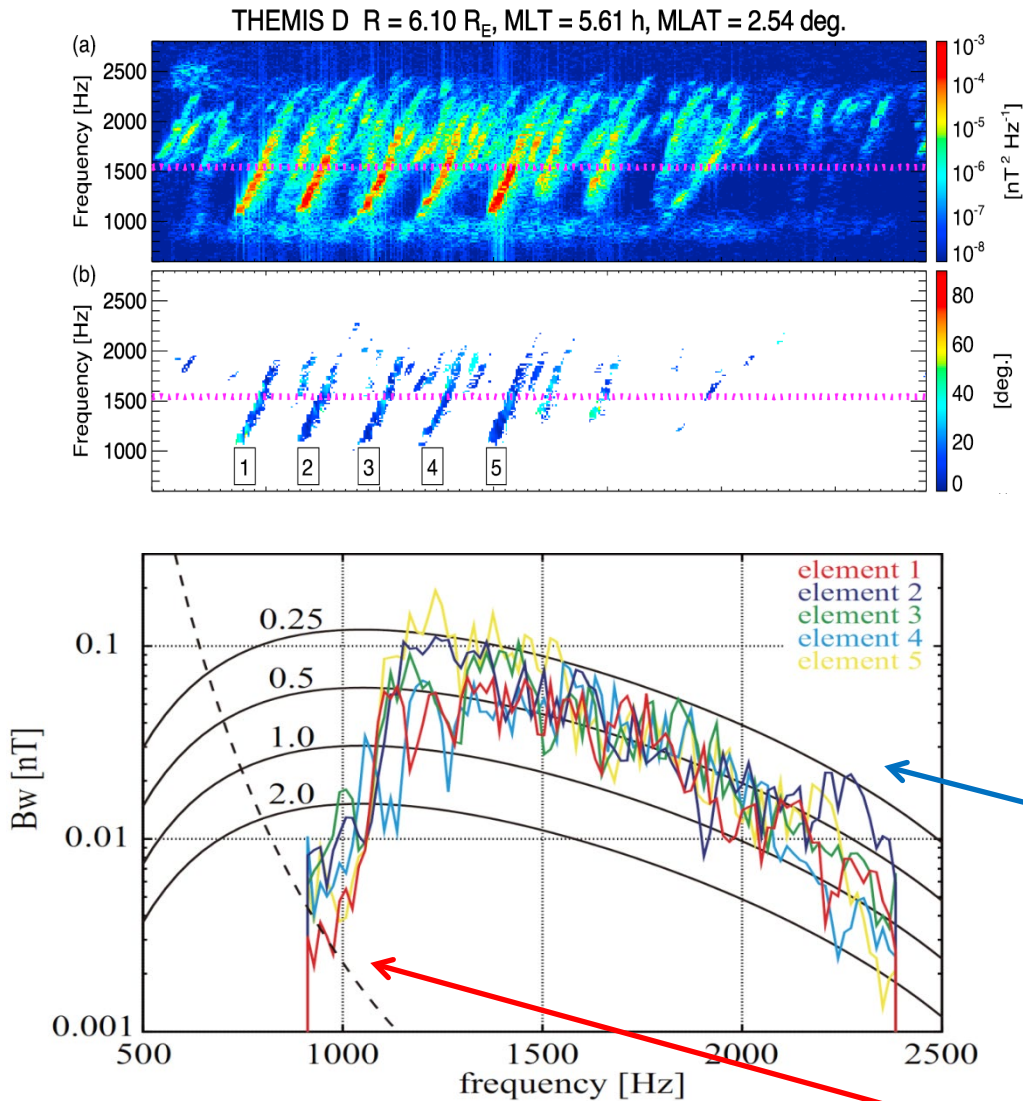
Nonlinear wave growth also takes place through propagation away from the equator due to the gradient of the magnetic field.



Equator \rightarrow Self-sustaining Mechanism $\rightarrow h$

$$S = -\frac{1}{s_0 \omega \Omega_w} \left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h} \right) \sim -0.4$$

Nonlinear wave growth is repeated around the optimum wave amplitude with gradually increasing frequencies, forming many subpackets in one chorus element.



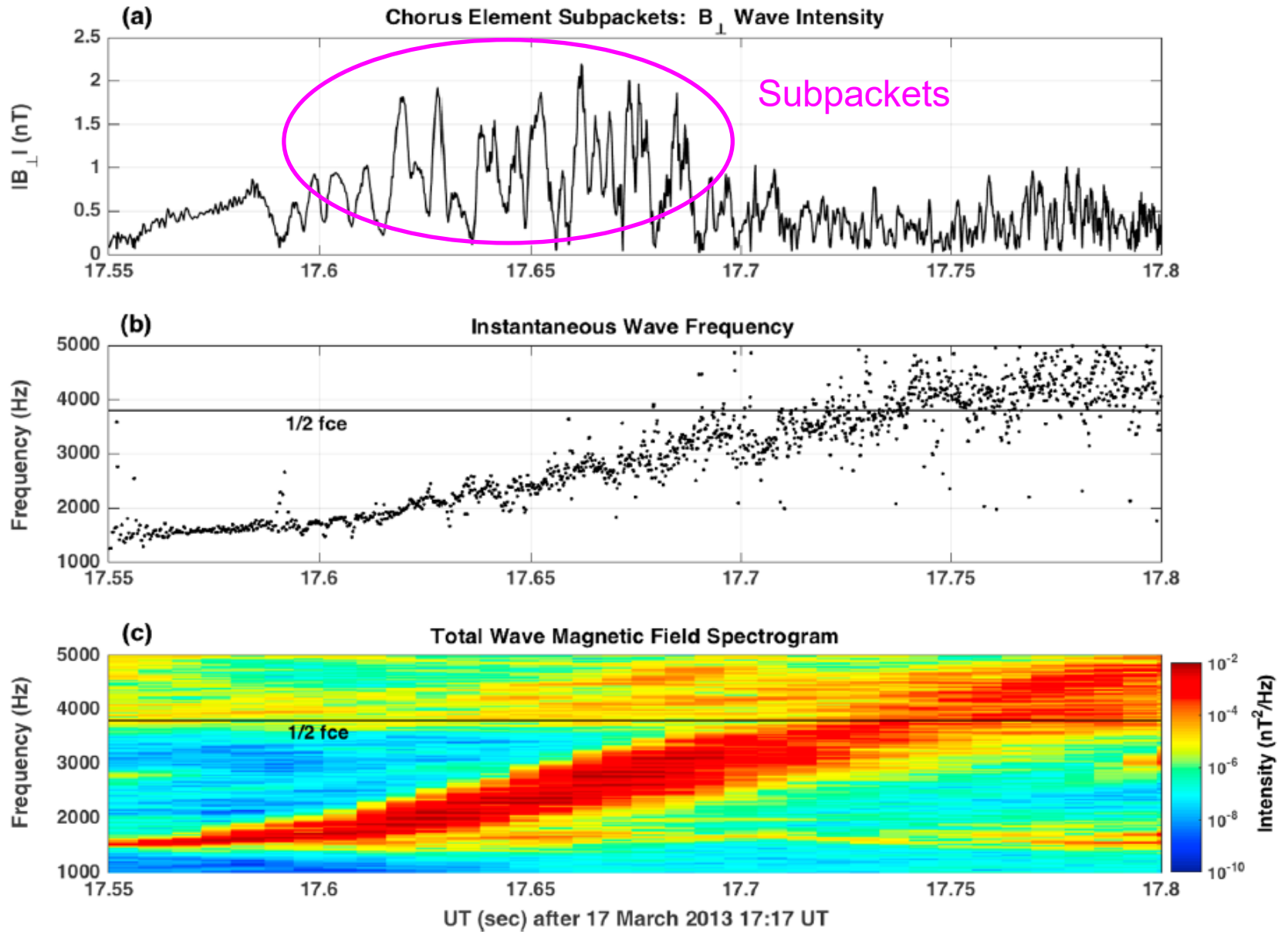
$$\frac{\partial \omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1} \Omega_w$$

Optimum Amplitude

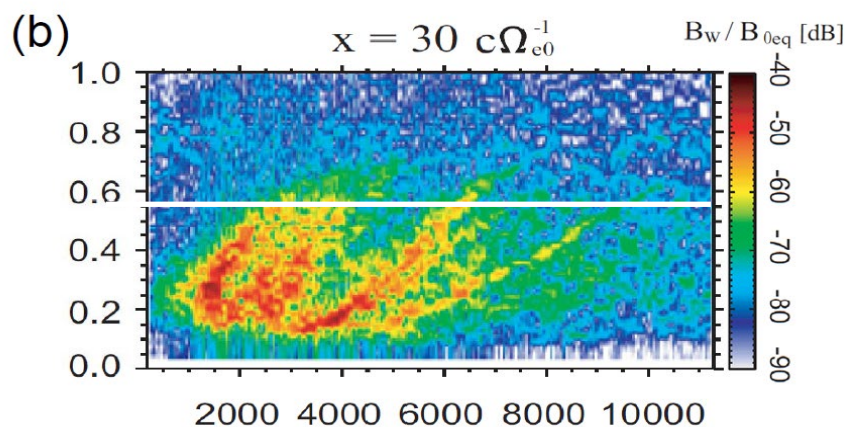
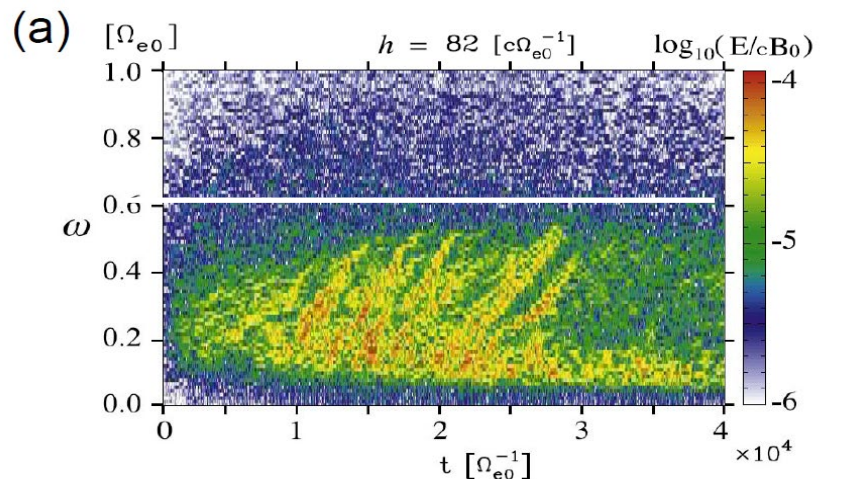
Threshold Amplitude

[Kurita et al., JGR, 2012]

Here is an example of subpacket structure in a chorus element.

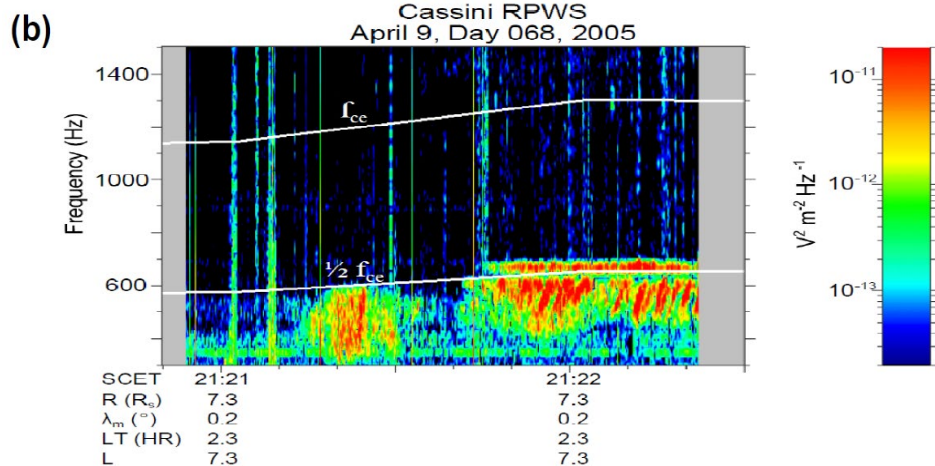
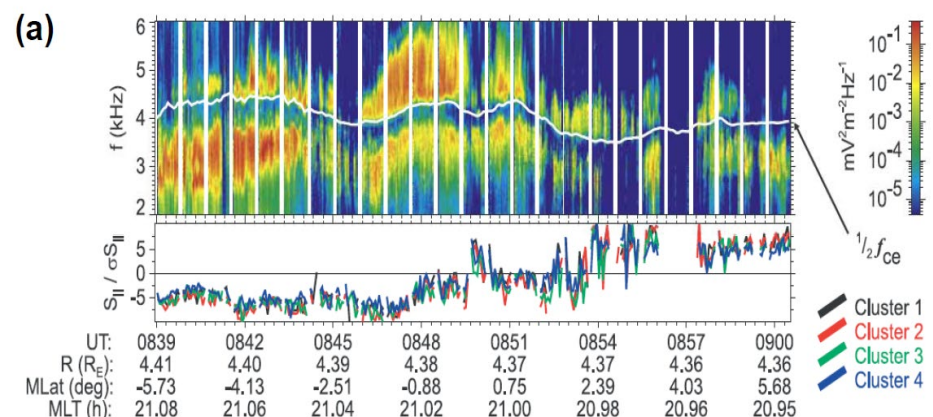


Simulations of chorus emissions in the parallel propagation show generation of chorus without a gap at half gyrofrequency, while observations show the gap. Oblique propagation is essential.



[Omura et al., JGR, 2008]

[Hikishima et al., JGR, 2009]



[Santolik, et al., JGR, 2003]

[Hosphodarsky et al., JGR, 2008]

In quasi-parallel propagation (oblique), the group velocity V_g and the phase velocity V_p become equal near half the gyrofrequency, while the parallel wave electric field appears.

$$E_{w\parallel} = \frac{\omega \sin \Psi}{\delta^2 \Omega_e - \omega} E_w$$

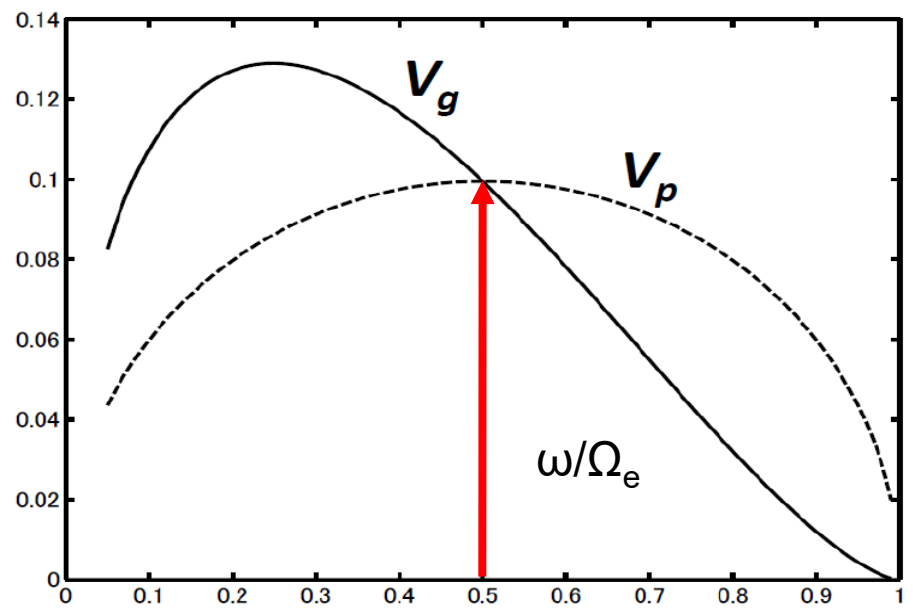
$$\sin^2 \Psi \ll 1$$

Ψ : Wavenormal Angle

With $\omega = 0.5 \Omega_e$

$$V_g = V_p$$

$$\tilde{v}_{\parallel} = v_{\parallel} - V_p$$

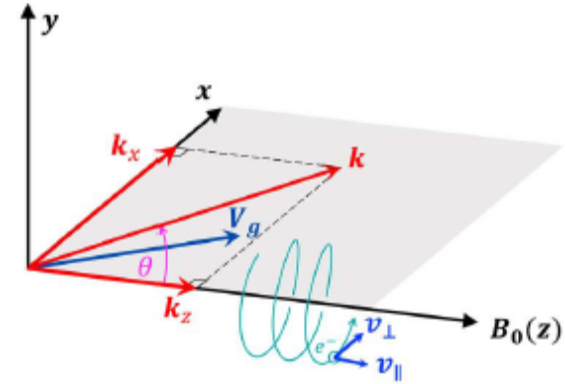


Exact formulation of the second-order resonance condition for Landau resonance is given by the following equations with S_0 .

$$\frac{d^2 \zeta_0}{dt^2} = \omega_{t1,0}^2 (\sin \zeta_0 + S_0) = 0$$

$$\omega_{t1,0}^2 = \frac{\omega_{t,0}^2}{\gamma} \left(1 - \frac{v_{\parallel}^2}{c^2} \right)$$

$$\omega_{t,0}^2 = \frac{ek_z}{m_0} [E_z^w J_0(\beta) + v_{\perp} B_R^w J_{-1}(\beta) - v_{\perp} B_L^w J_1(\beta)] \quad \beta = \frac{\gamma v_{\perp} k_x}{\Omega_e}$$

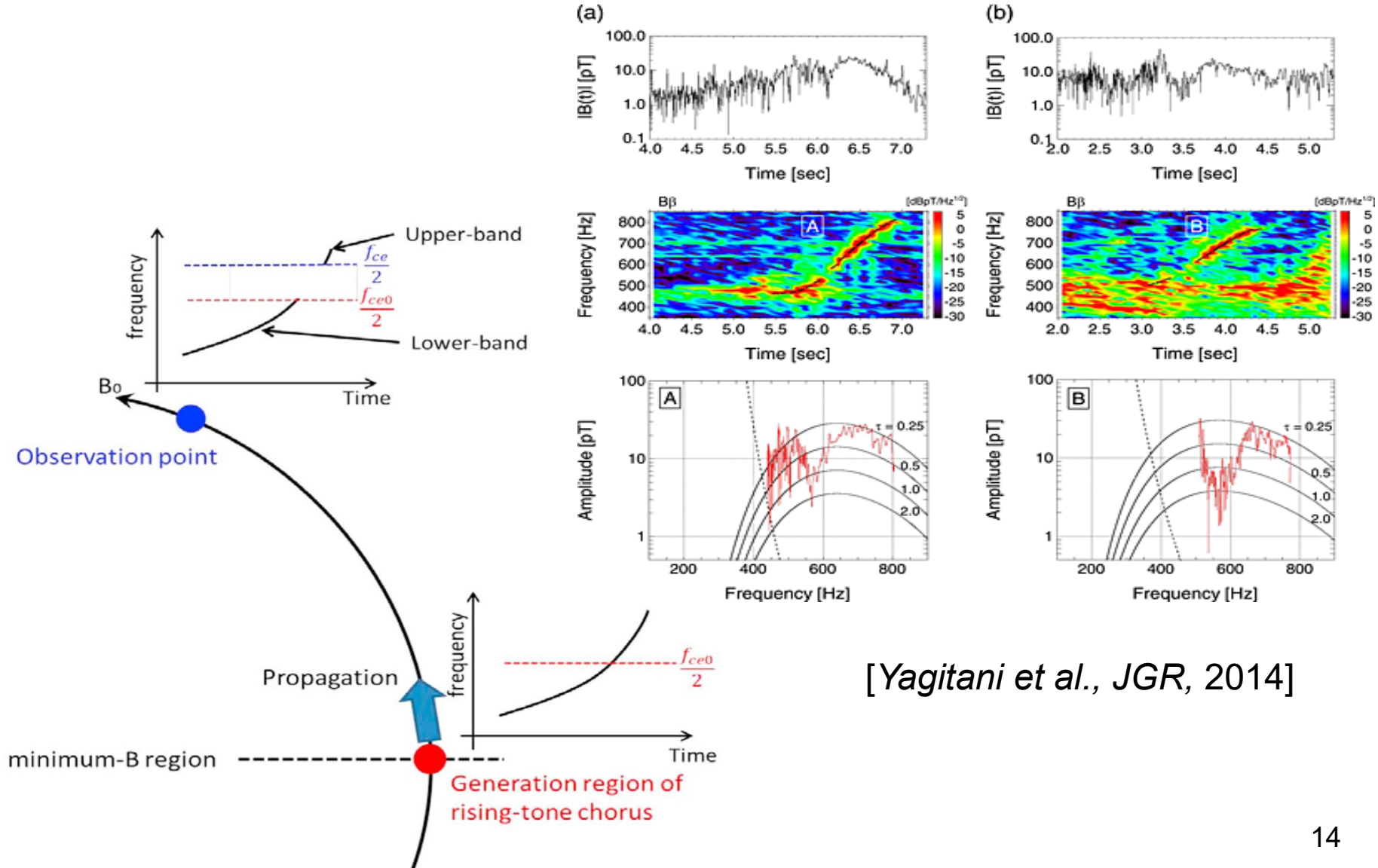


Assuming the quasi-parallel condition $\sin^2 \theta \ll 1$

$$S_0 = -\frac{1}{\omega_{t1,0}^2} \left\{ \left(1 - \frac{V_{p\parallel}}{V_{g\parallel}} \right)^2 \frac{\partial \omega}{\partial t} + \frac{c}{2} \left[\frac{\omega \chi V_{p\parallel}^2}{\xi(\Omega_e - \omega)c^2} + \frac{\omega v_{\perp}^2}{\Omega_e c V_{p\parallel}} \right] \frac{\partial \Omega_e}{\partial z} \right\}$$

$$\frac{dK_0}{dt} \sim \frac{m_0}{k_z} v_{\parallel} \omega_{t,0}^2 \sin \zeta_0$$

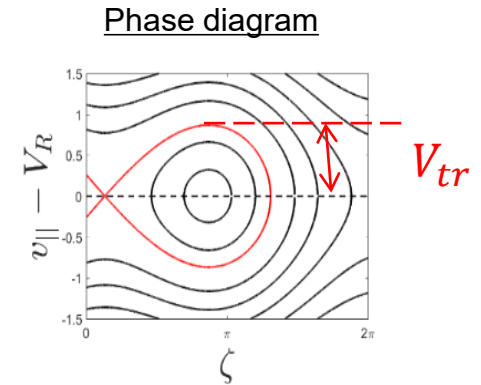
Chorus element undergoes nonlinear damping at half the local gyrofrequency. The gap increases as the distance from the equator increases.



[Yagitani et al., JGR, 2014]

Test particle simulations with different frequency wave packets demonstrate energy variation through both Landau ($n=0$) and cyclotron ($n=1$) resonances.

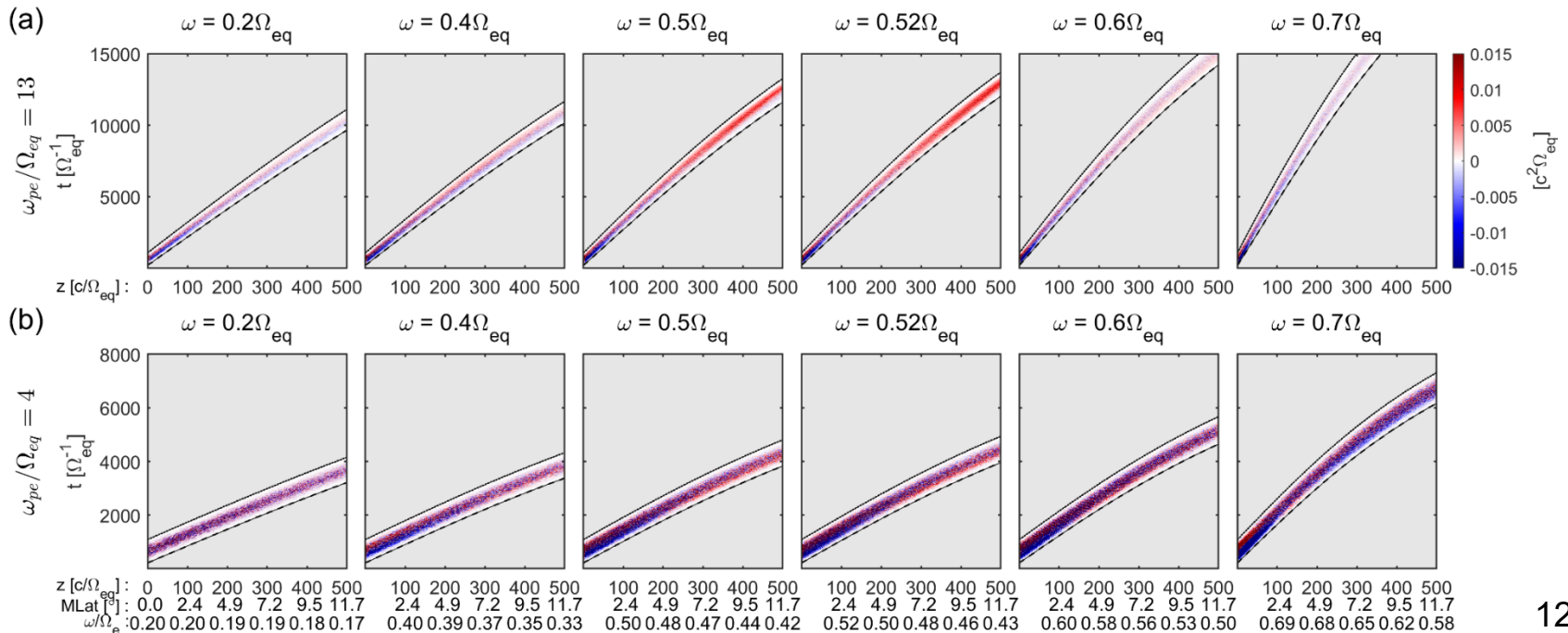
$v^{(n)}$: Selected electrons with $-2V_{tr}^{(n)} \leq v_{\parallel} - V_R^{(n)} \leq 2V_{tr}^{(n)}$
 (n : harmonic number)



Including resonant electrons and enough number of transient electrons.

$$P = q/m_e \left(\sum \mathbf{E} \cdot \mathbf{v}^{(0)} + \sum \mathbf{E} \cdot \mathbf{v}^{(1)} \right)$$

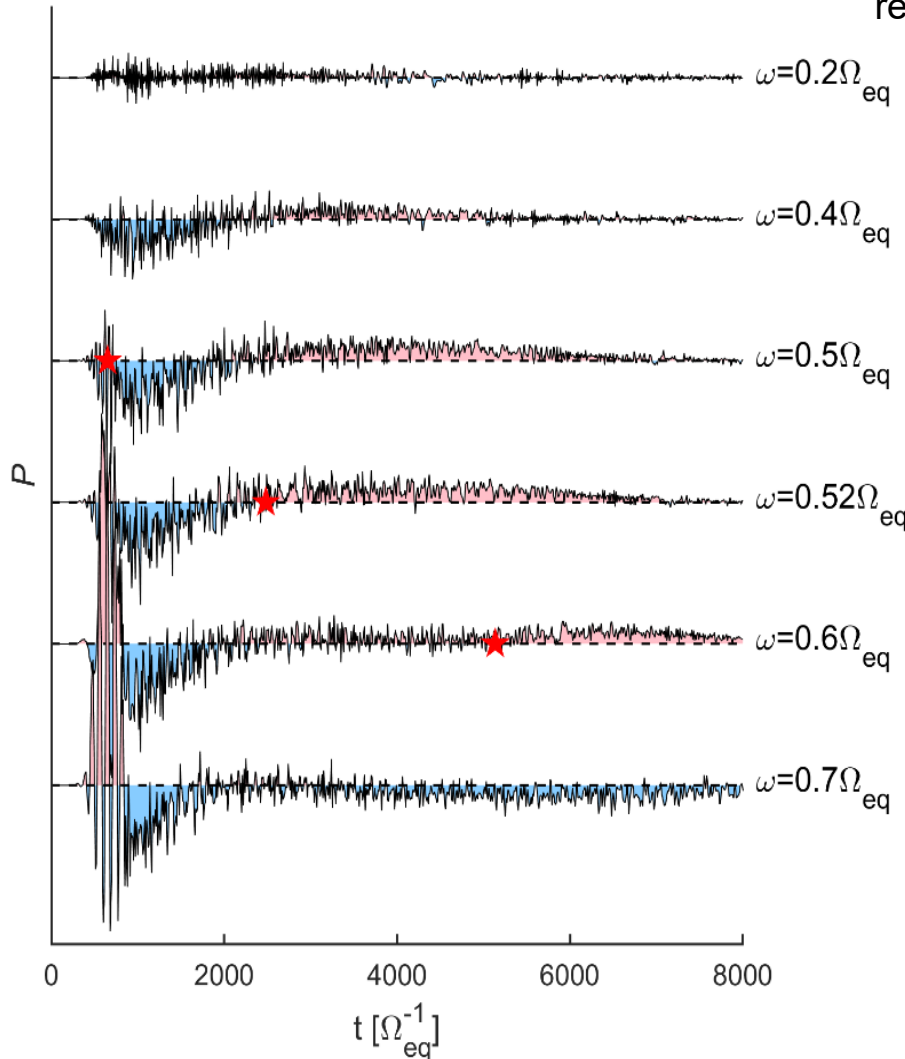
$$T_{\perp}/T_{\parallel} = 1.69$$



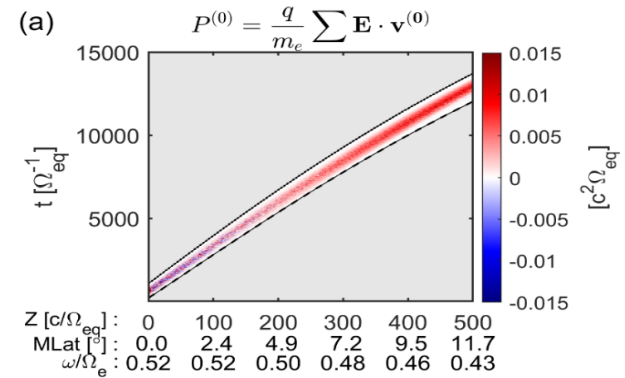
While the wave packets grow due to cyclotron resonance, wave damping due to Landau resonance becomes dominant as the wave packets propagate away from the equator near half the local gyrofrequency.

$$P = q/m_e \left(\sum \mathbf{E} \cdot \mathbf{v}^{(0)} + \sum \mathbf{E} \cdot \mathbf{v}^{(1)} \right) \quad \omega_{pe}/\Omega_{eq} = 4$$

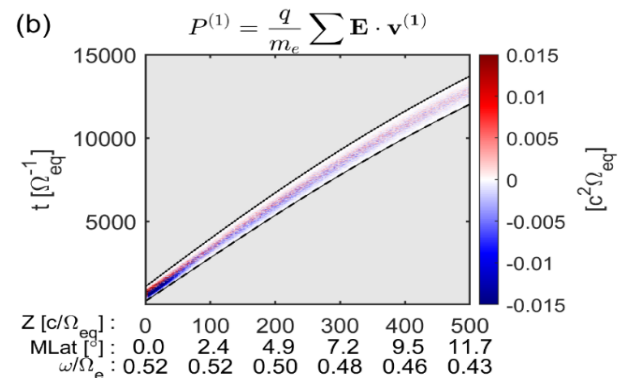
★: The timing when the maximum amplitude reaches the position with local $\omega/\Omega_e = 0.5$.



Landau resonance part

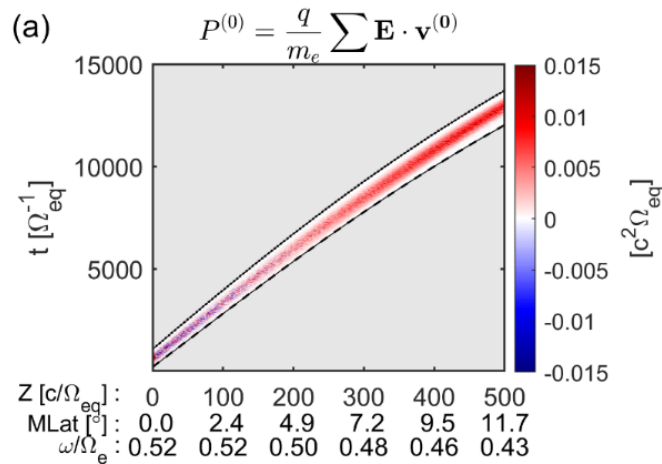


Cyclotron resonance part



Transverse components of $P^{(0)}$ contribute to nonlinear damping (Landau resonant electron acceleration) rather than longitudinal components.

$$\omega_{pe}/\Omega_{eq} = 13, \quad \omega/\Omega_{eq} = 0.52$$



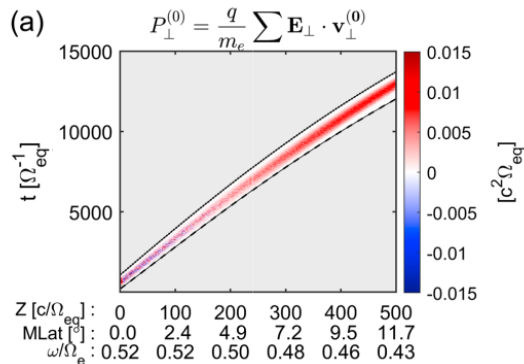
(background magnetic field)

$$\mathbf{B}_0 = \mathbf{e}_z B_0$$

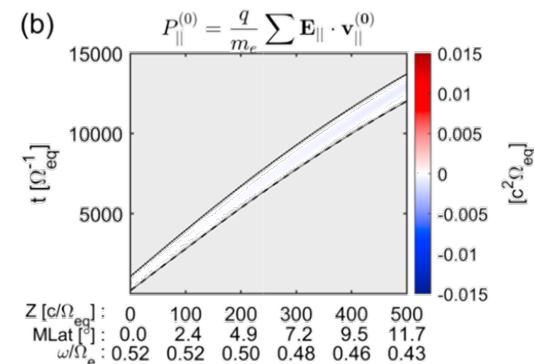
$$\mathbf{B}^w = \mathbf{e}_x B_x + \mathbf{e}_y B_y + \mathbf{e}_z B_z$$

$$\mathbf{E}^w = \mathbf{e}_x E_x + \mathbf{e}_y E_y + \mathbf{e}_z E_z$$

Transverse components



Longitudinal component



- The wave damping is formed by the perpendicular components of wave electric field and electron motions.

Summary

1. Rising tone chorus emissions are generated at the equator through **nonlinear wave growth process** due to formation of **electron holes**.
2. A series of subpackets are generated with **the optimum wave amplitude** and gradually increasing frequencies covering **half the electron gyrofrequency**, forming a rising-tone chorus element.
3. The chorus element propagates away from the equator undergoing **convective wave growth** with wave normal angle gradually deviating from the parallel direction.
4. There occurs **nonlinear wave damping** at half the gyrofrequency, separating the chorus element into the upper band and the lower band.
5. Test particle simulations confirm the **nonlinear wave damping** of chorus emissions at **half the gyrofrequency**
6. The **wave damping** is due to **Landau resonance** rather than cyclotron resonance for oblique waves at **half the gyrofrequency**.
7. Landau resonant electrons are accelerated in the nonlinear wave damping by **perpendicular wave electric field** rather than parallel component.

- Hsieh, Y.-K., & Omura, Y. (2018). Nonlinear damping of oblique whistler mode waves via Landau resonance. *Journal of Geophysical Research: Space Physics*, 123. <https://doi.org/10.1029/2018JA025848>
- Omura, Y., Hsieh, Y.-K., Foster, J. C., Erickson, P. J., Kletzing, C. A., & Baker, D. N. (2019), Cyclotron acceleration of relativistic electrons through Landau resonance with obliquely propagating whistler-mode chorus emissions. *Journal of Geophysical Research: Space Physics*, 124. <https://doi.org/10.1029/2018JA026374>