The space-wise approach for cold-atom interferometry geodetic data analysis: the MOCASS study

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The MOCASS study

- MOCASS is a study carried out from 2016 to 2018 by POLIMI and other Italian research groups on behalf of the Italian Space Agency (ASI).

- The study regarded a new satellite mission proposal for continuously monitoring of the Earth gravity and its changes, based on a satellite-borne interferometer exploiting ultra-cold atom technology.

- Goal of the study: to investigate the performance of a cold atom interferometer flying on a low Earth orbiter and its impact on the modeling of different geophysical phenomena, considering:
  - the signal requirements from a geophysical point of view,
  - the estimation of the global signal characteristics as a result of the geodetic data analysis,
  - the technological characteristics of an atom interferometer that delivers the gravity and gradient field at the satellite altitude.
MOCASS as a GOCE follow-on mission

• In this study, the basic idea was that of a GOCE follow-on mission, with a unique spacecraft carrying an instrument capable of measuring functionals of the Earth gravitational potential.

• Instrument: ultra-cold atom interferometer

• Study of the signal requirements from a geophysical point of view

• Geodetic data analysis by the space-wise approach

• The study was funded by the Italian Space Agency (ASI Contract No. 2016–9-U-0)
MOCASS as a GOCE follow-on mission

• **In this study, the basic idea was that of a GOCE follow-on mission.**

• **Proposed payload: a spaceborne gradiometer based on laser-cooled atom interferometry, a sensor based on a concept proposed by (Carraz et al., 2014), exploiting the principle of Cold Atom Interferometry.**

• **These instruments operate as inertial sensors and have been used for fundamental physics experiments.**

Complete laser + vacuum system developed in Firenze (Space Optical Clocks Project, 2008), image provided by AtomSensors srl
The MOCASS payload

It has been proved that a sensitivity of $3.5 \text{ mE}/\sqrt{\text{Hz}}$ ($1 \text{ E} = 10^{-9} \text{ s}^{-2}$) over a wide spectral range can be reached.

ASD (Amplitude Spectral Density) of the gravity gradient noise for the cold atom interferometer, assuming fluctuations of $10 \mu\text{m/s}$ RMS for the residual velocity difference between atom clouds, a typical GOCE-like orbit and a compensation of the rotation rate at the level of $10^{-8} \text{ rad/s}$. Case: nadir-pointing mode, radial (z) sensitive axis.

(Migliaccio et al., Surveys in Geophysics 40:1029–1053, 2019)
The MOCASS payload

Transfer function (in the frequency and time domain) of the cold atom interferometer

Noise PSD parameters of the cold atom interferometer

<table>
<thead>
<tr>
<th>Interferometer time T</th>
<th>Cycle period $T_c$</th>
<th>Number of atoms</th>
<th>Distance d</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 s</td>
<td>1 s</td>
<td>$10^6$</td>
<td>0.5 m</td>
</tr>
</tbody>
</table>
MOCASS: numerical simulations scenarios

Nadir pointing mode

- High orbit (satellite altitude ~ 259 km)
- Low orbit (satellite altitude ~ 239 km)

Inertial mode

- Simulated data:
  - $T_{xx}, T_{yy}, T_{zz}$ $\gg$ Single-arm gradiometer
  - $T_{xx}$ and $T_{zz}, T_{yy}$ and $T_{zz}$ $\gg$ Double-arm gradiometer
For the geodetic data analysis, numerical simulations were performed at POLIMI applying the space-wise approach for different mission scenarios.

Space-wise simulation and data analysis strategy:

- Error estimates based on Monte-Carlo simulations.
The role of the SST solution (from the on-board GNSS receiver data) is to numerically stabilize the gridding solution, reducing the residual signal amplitude and correlation.

In the case of GOCE, it also plays a role in the low degree final estimation, which is not the case for MOCASS.
Wiener deconvolution filter

\[ W(f) = \frac{H(f) \times S_y(f)}{H(f)^2 \times S_y(f) + S_v(f)} \]

*Noise ASD of the MOCASS Cold Atom gradiometer compared with the corresponding ASD of the measurement error of the GOCE gradiometer*

*MOCASS Wiener deconvolution filter (black curve) vs. GOCE Wiener filter (red curve)*
MOCASS: local collocation gridding

Local collocation gridding

\[ \hat{\Delta}(\phi, \lambda, R)_{R=R} = C_{zy}(C_{yy} + C_{ee})^{-1} \tilde{Y}_0(\phi, \lambda, R) \]

\(T_{xx}, T_{yy}\), and \(T_{zz}\) observations were used to predict grids of \(T, T_{rr}\), and \(T_{\lambda\lambda}\) values.

A local collocation procedure was applied to the filtered data, using a suitable method for data selection, in order to enhance local information and reducing or possibly avoiding undersampling (Reguzzoni et al., 2014).

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### Error r.m.s. of predicted grids of $T_{rr}$, $T_{\lambda\lambda}$ and $T$ values

<table>
<thead>
<tr>
<th>Simulated observation data</th>
<th>Pointing mode</th>
<th>Orbit</th>
<th>Error r.m.s. of predicted $T_{rr}$ grid values [mE]</th>
<th>Error r.m.s. of predicted $T_{\lambda\lambda}$ grid values [m$^2$/s$^2$]</th>
<th>Error r.m.s. of predicted $T$ grid values [m$^2$/s$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{xx}$</td>
<td>Nadir</td>
<td>High</td>
<td>1.216</td>
<td>39.193</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td>0.915</td>
<td>30.203</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>Inertial</td>
<td>High</td>
<td>4.521</td>
<td>9.834</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td>1.499</td>
<td>5.816</td>
<td>0.057</td>
</tr>
<tr>
<td>$T_{yy}$</td>
<td>Nadir</td>
<td>High</td>
<td>1.765</td>
<td>7.928</td>
<td>0.051</td>
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<tr>
<td></td>
<td></td>
<td>Low</td>
<td>1.290</td>
<td>5.202</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>Inertial</td>
<td>High</td>
<td><strong>0.453</strong></td>
<td><strong>6.834</strong></td>
<td><strong>0.041</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td><strong>0.328</strong></td>
<td><strong>5.283</strong></td>
<td><strong>0.042</strong></td>
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<tr>
<td>$T_{zz}$</td>
<td>Nadir</td>
<td>High</td>
<td>0.358</td>
<td>6.069</td>
<td>0.042</td>
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<td></td>
<td></td>
<td>Low</td>
<td><strong>0.2174</strong></td>
<td><strong>4.879</strong></td>
<td><strong>0.041</strong></td>
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<td>Inertial</td>
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<td>2.906</td>
<td>21.396</td>
<td>0.070</td>
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<td>Low</td>
<td>2.770</td>
<td>25.717</td>
<td>0.067</td>
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<tr>
<td>$T_{xx}$ and $T_{zz}$</td>
<td>Nadir</td>
<td>High</td>
<td>0.525</td>
<td>22.742</td>
<td>0.064</td>
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<tr>
<td></td>
<td></td>
<td>Low</td>
<td>0.307</td>
<td>4.203</td>
<td>0.035</td>
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<td></td>
<td>Inertial</td>
<td>High</td>
<td>0.761</td>
<td>12.920</td>
<td>0.114</td>
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<tr>
<td></td>
<td></td>
<td>Low</td>
<td>0.403</td>
<td>5.194</td>
<td>0.038</td>
</tr>
<tr>
<td>$T_{yy}$ and $T_{zz}$</td>
<td>Nadir</td>
<td>High</td>
<td><strong>0.552</strong></td>
<td><strong>12.102</strong></td>
<td><strong>0.041</strong></td>
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<tr>
<td></td>
<td></td>
<td>Low</td>
<td><strong>0.338</strong></td>
<td><strong>3.771</strong></td>
<td><strong>0.035</strong></td>
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<td>Inertial</td>
<td>High</td>
<td><strong>0.546</strong></td>
<td><strong>8.249</strong></td>
<td><strong>0.055</strong></td>
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<tr>
<td></td>
<td></td>
<td>Low</td>
<td><strong>0.341</strong></td>
<td><strong>4.731</strong></td>
<td><strong>0.038</strong></td>
</tr>
</tbody>
</table>

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Grid r.m.s for $T_{rr}$ along parallels

With polar gaps

Excluding polar gaps
MOCASS: spherical harmonic analysis

\[ \hat{T}_{lm} = \frac{1}{4\pi a} \int \hat{z}(\theta, \lambda) Y_{lm}(\theta, \lambda) d\sigma \approx \frac{1}{4\pi a} \sum_{i} \sum_{j} \hat{z}(\theta_i, \lambda_j) Y_{lm}(\theta_i, \lambda_j) \Delta \sigma_{ij} \]

- Spherical harmonic coefficients were computed by numerical integration (Migliaccio et al., 2004), and then combined according to MC solution errors to obtain a final estimate of the global geopotential model.

- The MOCASS results were compared with GOCE and GRACE solutions based on the same amount of data (two-month solutions).
In the next slides:

- Error degree variances for reconstruction of a global model (2-month data).
- Cumulative error for Δg at ground level (2-month data).
- Cumulative error for $T_{rr}$ at satellite altitude (2-month data).
- Cumulative error for Δg at ground level and $T_{rr}$ at satellite altitude, for 1-year, 2-year and 5-year mission.
- Cumulative errors of Δg and $T_{rr}$: linear trend of monthly solutions.

Curves are plotted for the cases

- nadir / inertial pointing configuration;
- high / low orbit;
- one-arm / double-arm gradiometer.

A comparison with corresponding curves for GRACE and GOCE is shown. For all curves low-order coefficients affected by polar gap degradation are disregarded.
Estimated global gravity model: error degree variances

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Cumulative error for $\Delta g$ at ground level

Cum. gravity anomaly error $\Delta g$ at ground level, [mGal] (nadir-pointing mode)

Cum. gravity anomaly error $\Delta g$ at ground level, [mGal] (inertial mode)

Cum. gravity anomaly error $\Delta g$ at ground level, [mGal] (nadir-pointing mode)

Cum. gravity anomaly error $\Delta g$ at ground level, [mGal] (inertial mode)
Cumulative error for $T_{rr}$ at satellite altitude (2-month data)

Cumulative error $T_{rr}$ at satellite altitude, [mE] (nadir-pointing mode)

Cumulative error $T_{rr}$ at satellite altitude, [mE] (nadir-pointing mode)

Cumulative error $T_{rr}$ at satellite altitude, [mE] (inertial mode)

Cumulative error $T_{rr}$ at satellite altitude, [mE] (inertial mode)
Cumulative error for 1-year, 2-year and 5-year mission, for $\Delta g$ at ground level (left) and $T_{rr}$ at satellite altitude (right)

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Cumulative errors of $\Delta g$ (left) and $T_{rr}$ (right): linear trend of monthly solutions
Summarizing, the best solutions are obtained:

- with data along the $zz$ direction, for the nadir-pointing configuration;
- with data along the $yy$ direction, for the inertial-pointing configuration.

Note: in the case of double-arm gradiometer there is no improvement because the atom cloud is not duplicated but it is split into two directions.
Results

• **Regarding the recovery of the static gravity field, the MOCASS error estimates can improve the performances of GOCE over all the harmonic spectrum, especially at high degrees, with a commission error of about of 1.4 mGal at degree 300 (0.9 mGal at degree 250) for a 5-year mission.**

• **For the time-variable gravity field, the MOCASS error estimates are promising (the 2-month solution is better than the GRACE one already above degree 40, while in the case of GOCE this occurs above degree 90). However, the accuracy at very low degrees seems to be still insufficient for this type of applications.**
A new study called MOCAST+ has been started in February 2020 proposing an enhanced cold atom interferometer which can deliver gravity gradients and time measurements.

The study will investigate whether this could give the possibility of improving the estimation of gravity models even at low harmonic degrees, with inherent advantages in the modeling of mass transport and its global variations: this would represent fundamental information, e.g. in the study of variations in the hydrological cycle and relative mass exchange between atmosphere, oceans, cryosphere and solid Earth.