Guide

This presentation contains a short presentation of 9 slides (round buttons), followed by slides with more details (magnifying glass). You can either scroll through this document or click on the interactive buttons as explained below.

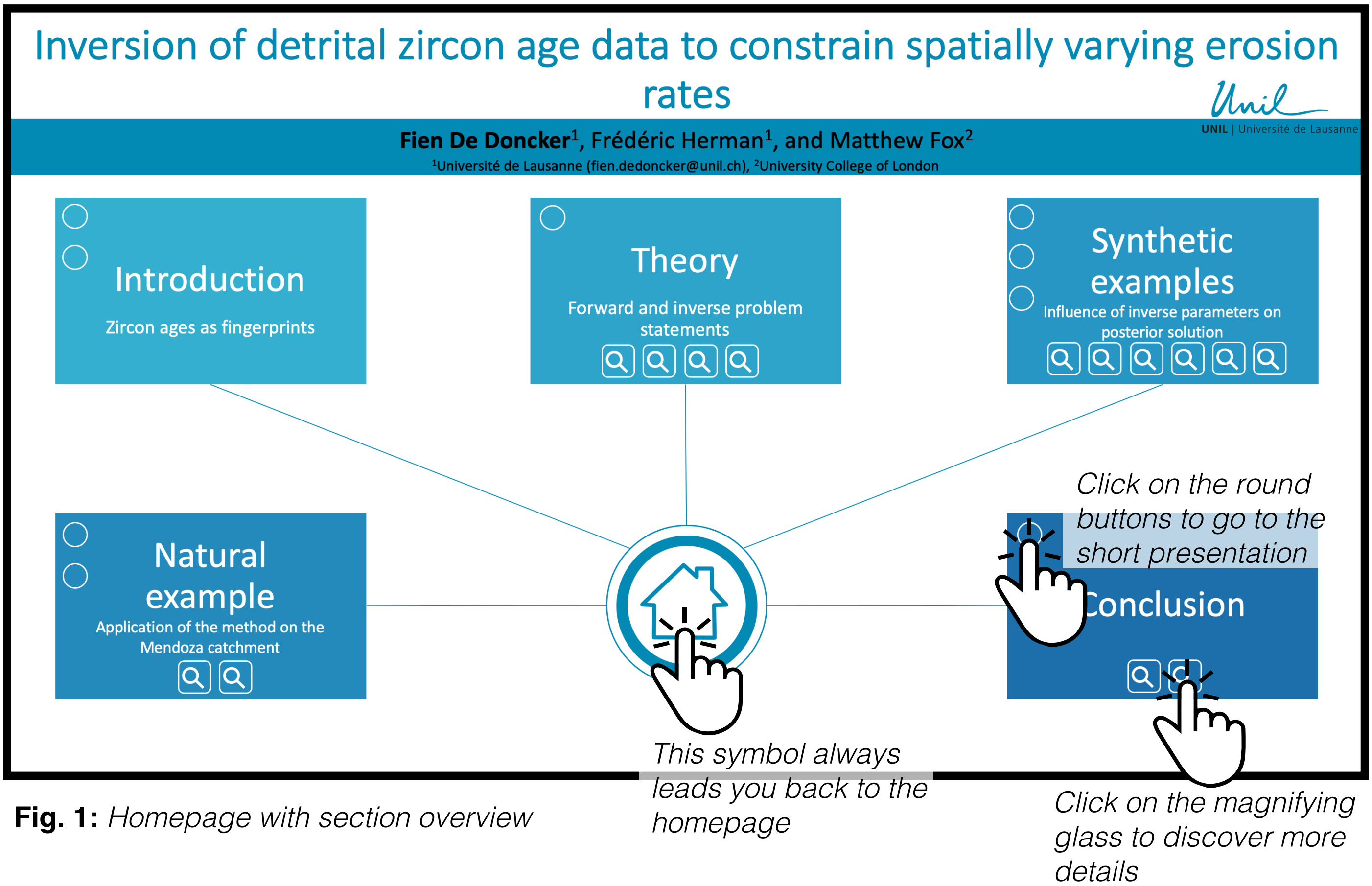


Fig. 1: *Homepage with section overview*

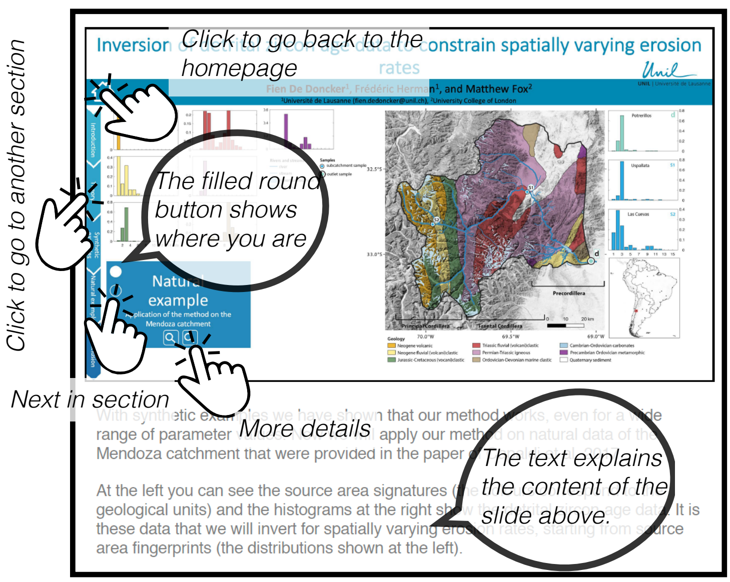


Fig. 2: *Example of a slide of the* ***short presentation*** *with the section bar at the left*

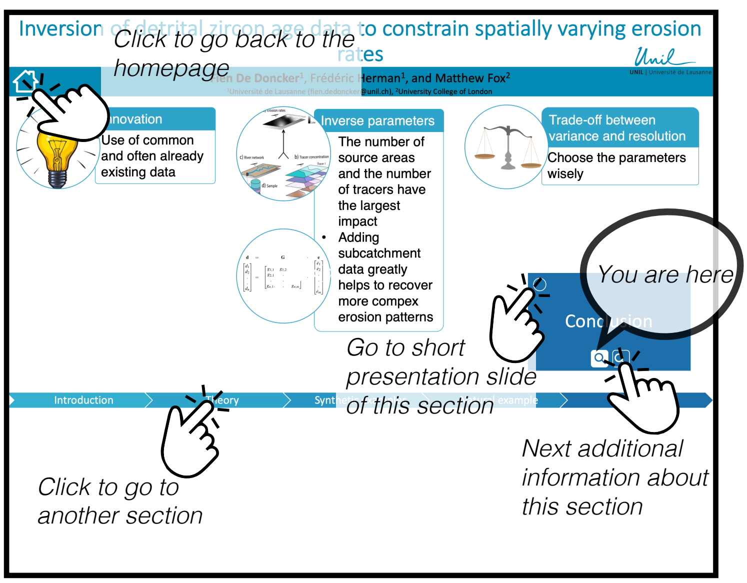
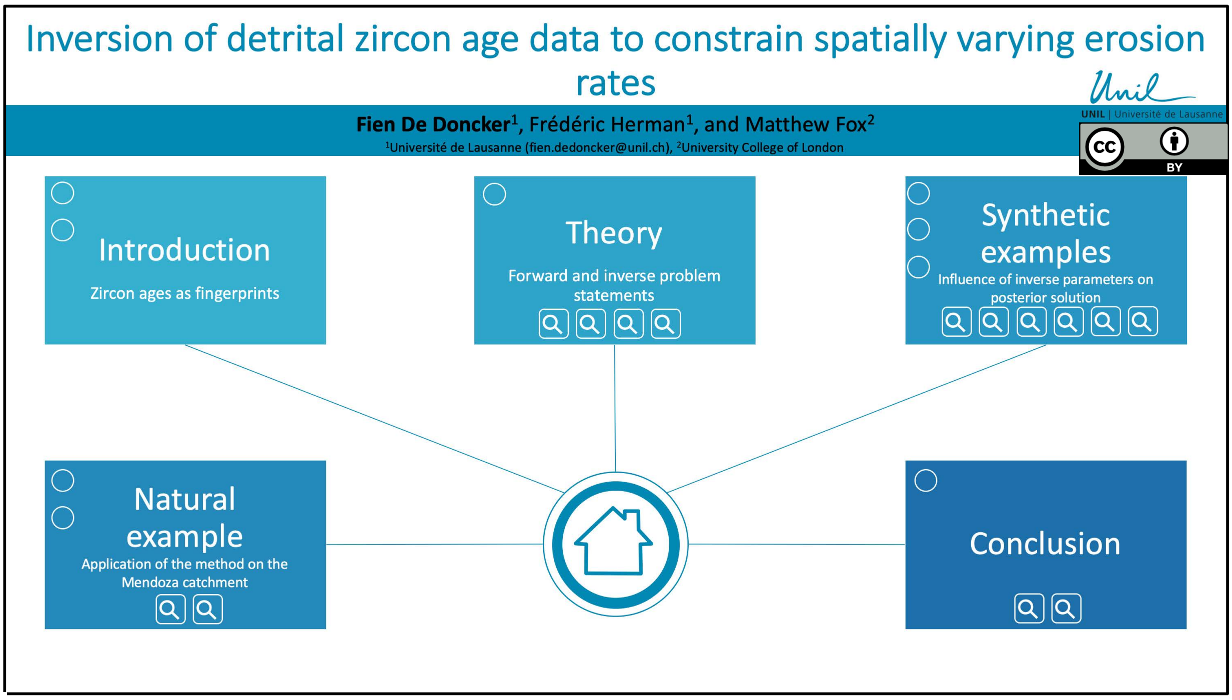
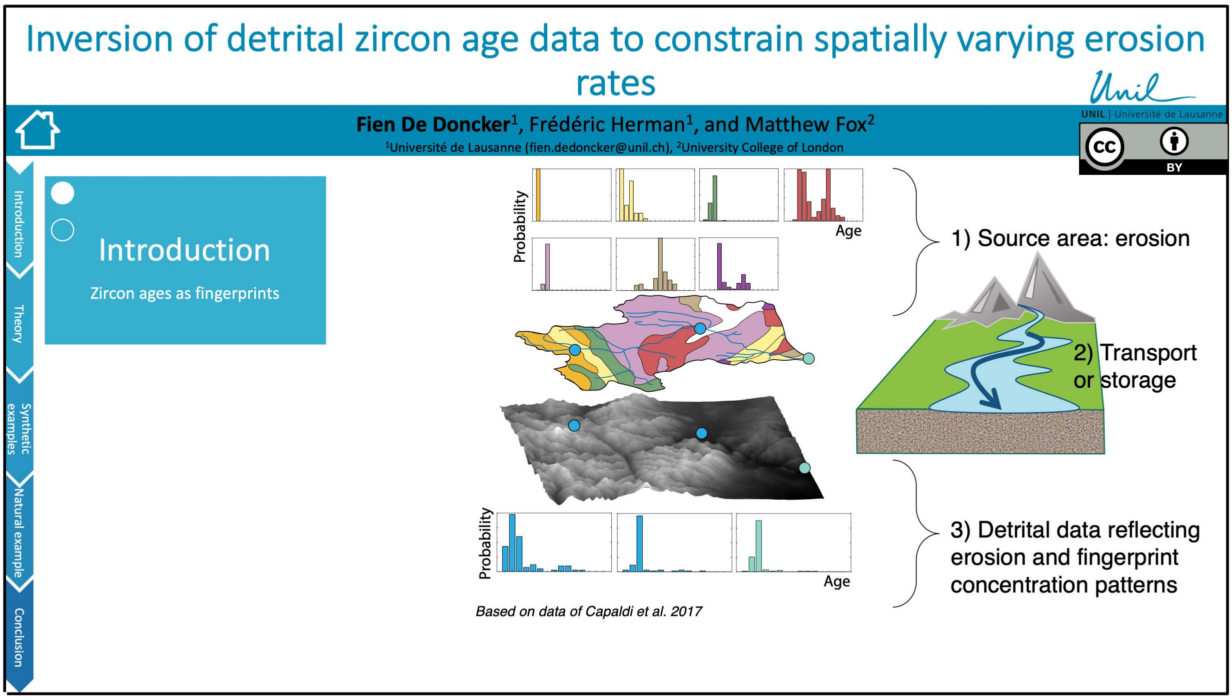


Fig.3: *Example of a slide of the* ***detailed information*** *with the section bar at the bottom*



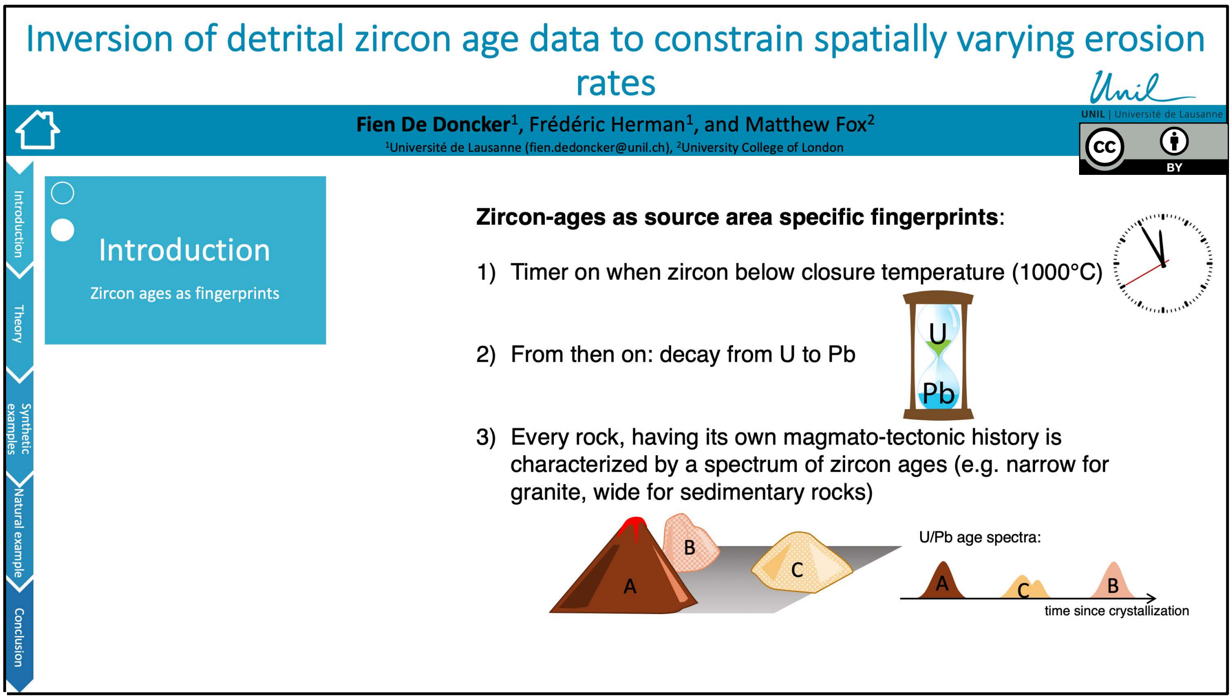
Erosion rates have an important impact on the (human) environment and it is therefore important to constrain them. However, that is easier said than done. We can calculate erosion rates from point measurements or river loads, but the spatial pattern of erosion rates – if any – that we will obtain with this will be very coarse. Another way to find erosion rates is with steady state assumptions, but these assumptions are often far from reality. In this presentation I will show how zircon ages can be used as fingerprints to derive the relative contribution of various source areas and how this information can be converted into spatially varying erosion rates. A special focus lays on the uncertainty assessment of the obtained erosion rates. At the end, I will present an application of our proposed method on zircon age data of the Southern Andes.



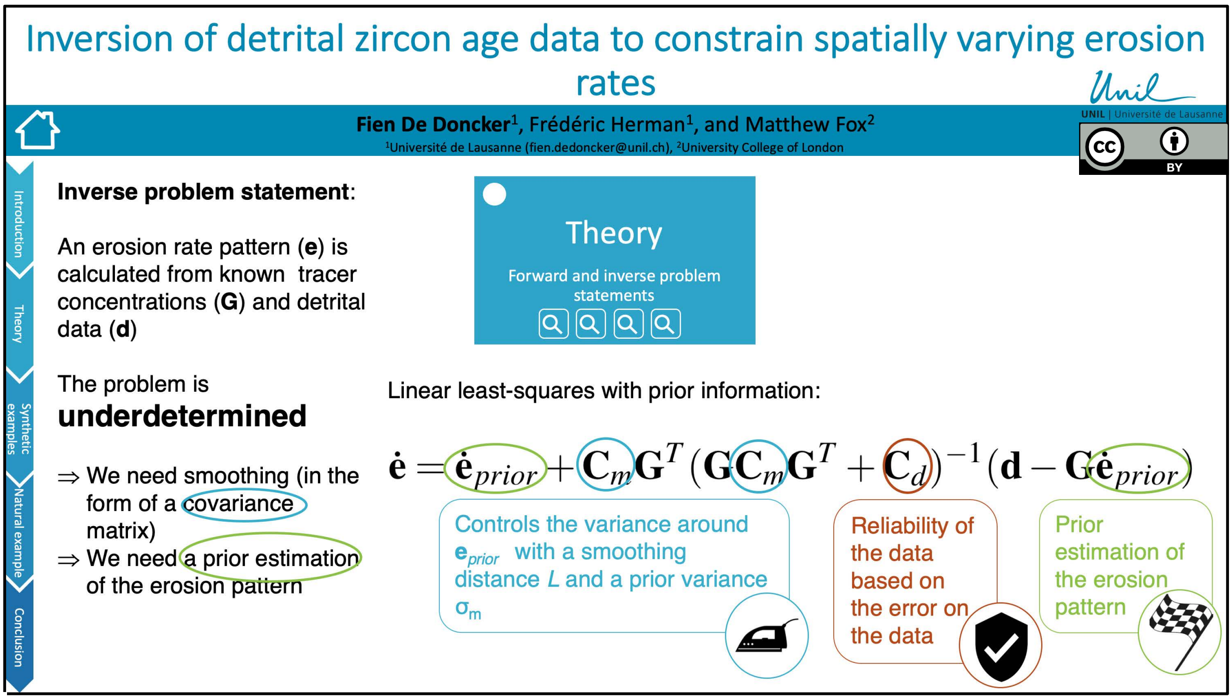
Provenance analysis is based on the following:
1. Each source area is characterised by a specific signature, also called fingerprint or tracer
2. Erosion happens in the source areas and tracers from different source areas are transported in the river.
3. Like that, modern sand samples represent a mixture of the different source fingerprints.

Our goal is to disentangle these and to convert this information into spatially varying erosion rates. Specifically, we use zircon age distributions as fingerprints. By dividing these age distributions into different age-bins with a specific proportion of the analysed zircons belonging to this age bin, these proportion-bins become passive tracers that are transported downstream.

At the left, an example is shown with data from Capaldi et al. 2017 showing different source signatures and detrital data representing a mix of these. The mix is controlled by the concentration of zircons in the different units – the so called zircon fertility – and the erosion rates that occur in the different units.



Zircon ages are specific for each source area – geological unit – since they record the magmato-tectonic history of the rocks. Once zircon crystals are cooled below 1000°C, a clock starts to tick. This clock is represented by the hourglass; at the beginning, the zircon minerals contain no Pb, but after crystallization, the U starts to decay to Pb. Like this, every geological unit, having its own specific history, is characterised by a unique zircon age spectrum.



Now that we understand how zircon-ages can be used as fingerprints in provenance analysis, let's have a look at how this information can be converted into spatially varying erosion rates.
Specifically, we want to invert the concentration of the different tracers found in modern sand samples into erosion rates. The concentration of the tracers in the detrital samples is called **d**, the detrital data.
Since the number of data is smaller than the number of unknowns (we want to find the erosion rate at every pixel of a map), we have to apply smoothing and need a prior estimation. This makes the model Bayesian (theory from Tarantola, 2005; Jackson, 1979).

⇒ The smoothing makes that adjacent pixels will have about the same erosion rates
⇒ The prior estimation serves as a starting point for the erosion rate estimation

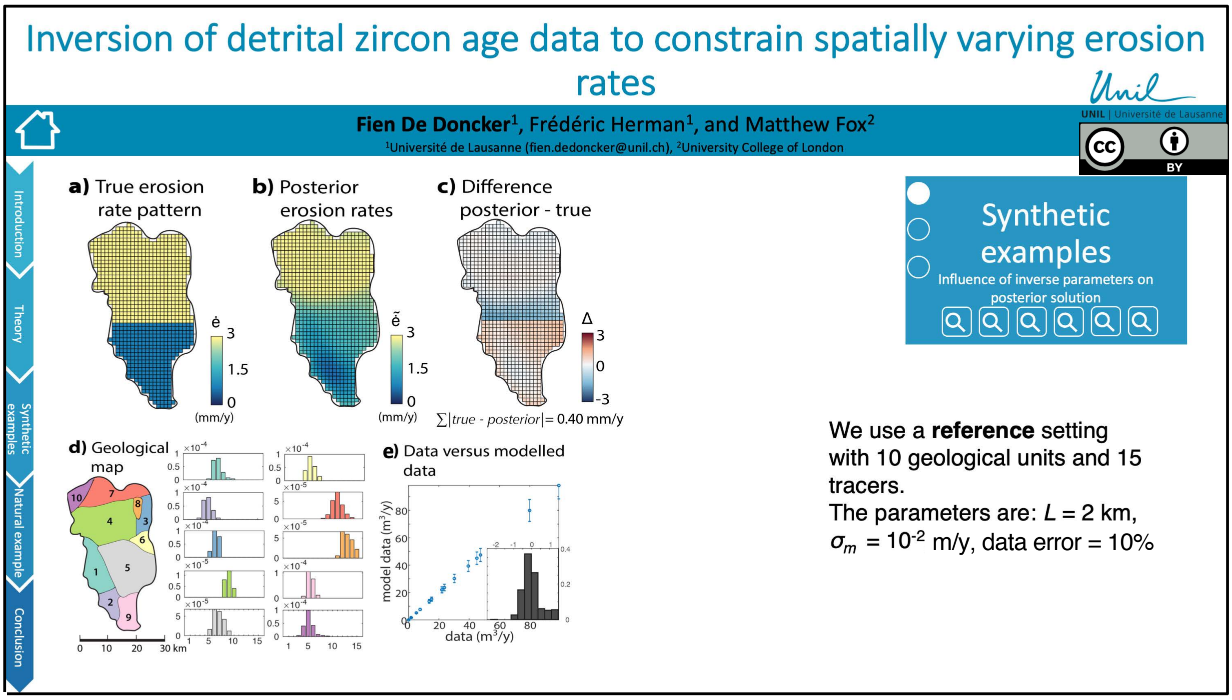
The inversion scheme that we use here is the linear least-squares method with prior information. It allows propagation of the errors in the posterior solution.

In general, to make our method work, we need:

- A tracer concentration map. More specifically, for every source area, we need its zircon-age distribution and we need to know the zircon fertility
- Detrital data: we need the zircon-age distribution of a modern sand sample
- A prior erosion guess; this can come from sediment load data or cosmogenic nucleide results

The parameters we can choose are:

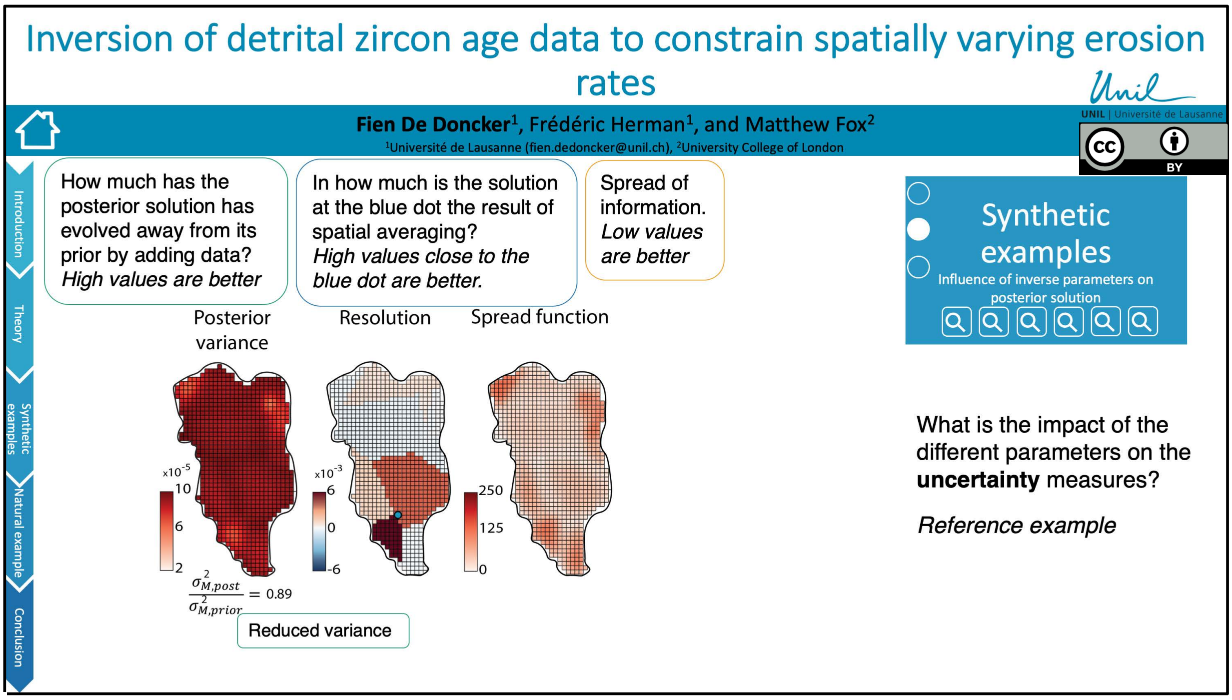
- The smoothing distance
- The prior variance that estimates the variance around the prior estimate



Now that we understand how the inverse method work, let's test how it behaves. In this part, we check how the posterior solution changes with different inverse parameters.

1. We generate synthetic detrital data from a 'true' erosion rate pattern that we multiply by our source area signatures.
2. We plug this synthetic detrital data into our inverse scheme to compute a posterior erosion map
3. We look at how close the posterior map is to the true erosion rate pattern

We start with a reference model with 10 geological units and other parameters that are listed above. The histograms show the zircon age distributions (or fingerprints) for the different source areas. As you see, the posterior map (b) is close to the true erosion rate pattern (a). In c, the difference between the posterior and true solution is given, red areas represent overestimations, blue pixels are regions where we underestimated the true model. When we plot the modeled data against the observed data, we see a good resemblance. The grey histogram shows the residuals.



To assess the meaning of the solution, we use 3 metrics:
1) The posterior variance
2) The resolution
3) The spread function

The posterior variance shows how much the solution has evolved away from the prior by adding data. Higher values are better. Below, the reduced variance is given, lower values mean that the added data helped to improve the solution.

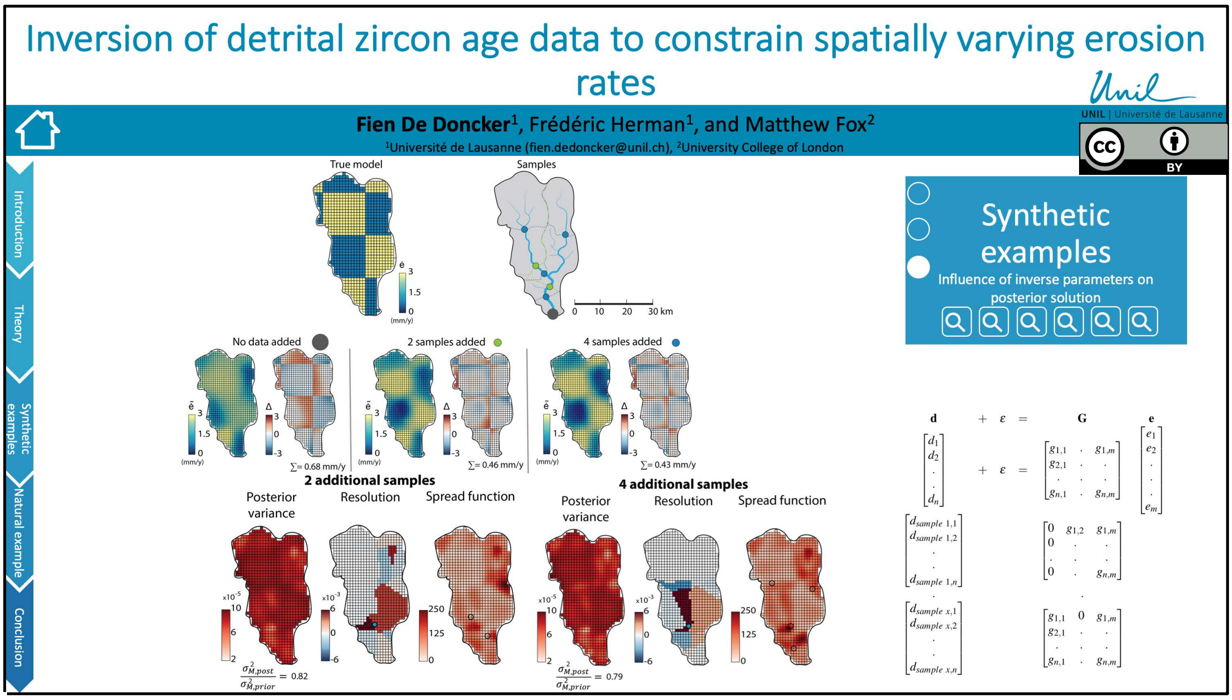
The resolution is some kind of filtering matrix that shows how much spatial averaging took place. Values close to one near the blue dot show that little spatial averaging was needed to constrain the solution at the blue dot.

The spread shows the same thing as the resolution, but in a different way. It shows for every pixel how much its information is used by other pixels to constrain their erosion rates. Low values indicate less spreading of information.

Here are the different metrics for the reference example.

There is a well-known ***trade-off between variance and resolution***:
A higher variance allows to detect local and small variations in erosion rates, but therefore, less smoothing or spatial averaging will be allowed so the errors will not be effectively averaged.

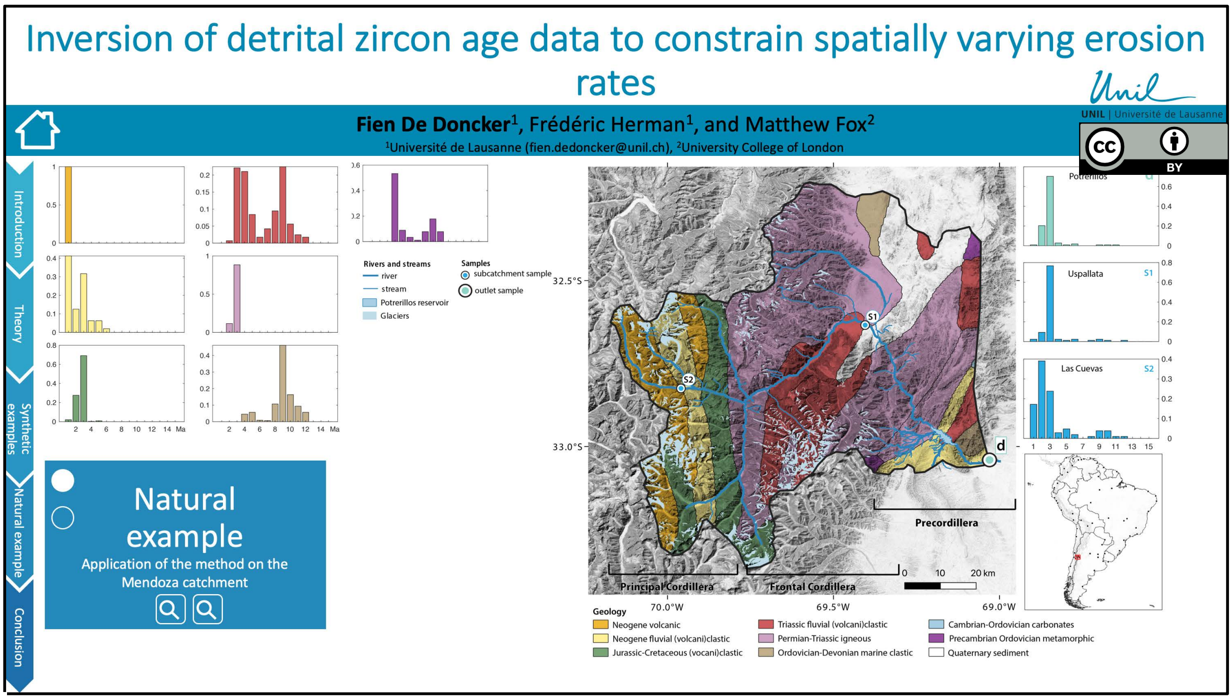
We will see this trade-off recurring in the other examples as well.



We can also add subcatchment data to recover more complex erosion rate patterns. As more data is added, the difference between the true and posterior solution decreases and so does the posterior variance, but the additional data increases the resolution.

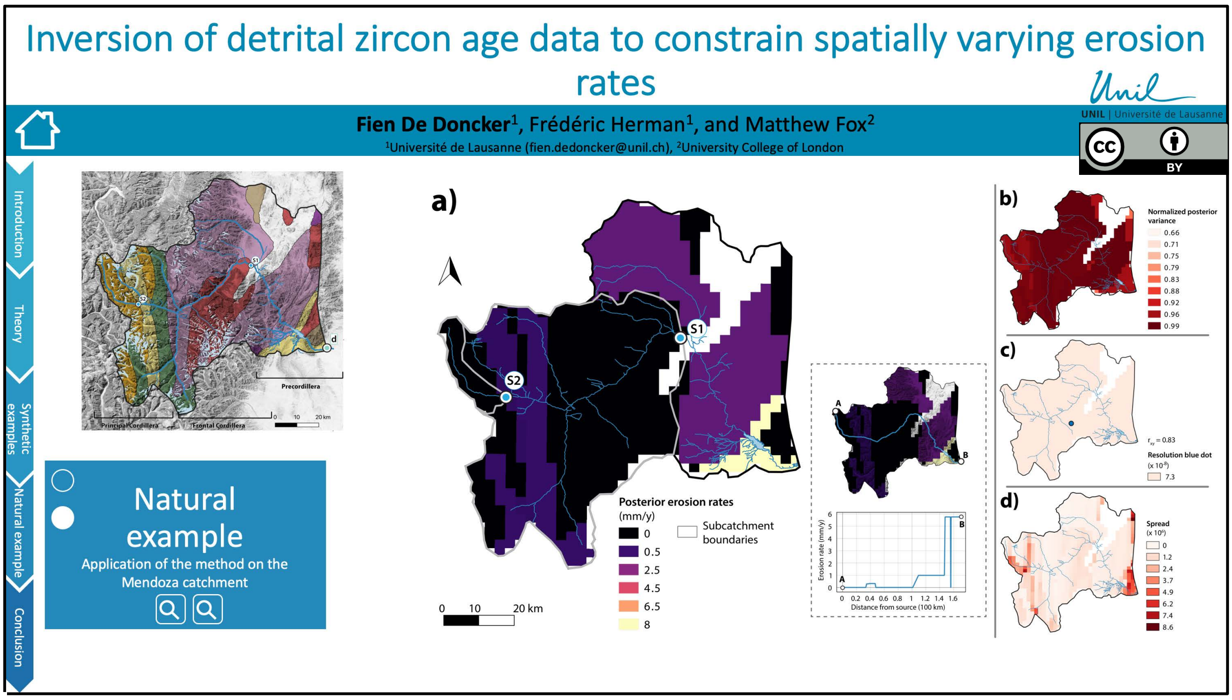
The data is simply added by appending the data on the data vector and by appending the ***G*** matrix with zeros for pixels outside of the subcatchment and the concentration values for pixels inside of the subcatchment for which data is added.

Let's have a look how this influences the meaning of our posterior solution. Adding data decreases the posterior variance at these points and increases the spread of information at these locations.



With synthetic examples we have shown that our method works, even for a wide range of parameter values. Now we will apply our method on natural data of the Mendoza catchment that were provided in the paper of Capaldi et al. 2017.

At the left you can see the source area signatures (the colours correspond to the geological units) and the histograms at the right show the detrital zircon-age data. It is these data that we will invert for spatially varying erosion rates, starting from source area fingerprints (the distributions shown at the left).



These are the results for the inversion with appended subcatchment data. The erosion rastes in the western part of the catchment are very low and high erosion rates are only visible in the east of the study area. Research around this lake has shown high hillslope instability and mass movements occur here. The intermediate erosion rates match the USLE-predicted actual hyrological erosion rates of around 1.5 mm/y.

We can see that the normalized (or reduced) posterior variance is high in the whole study area, except for the locations with small geological units. The resolution is rather low and the spread is high at the subcatchment boundaries. There is a 'blind spot' in the middle of the map, as there is little to no spatial variation in geology there. That is represented by high normalized variances, indicating that a lot of spatial averaging takes place here.

Inversion of detrital zircon age data to constrain spatially varying erosion rates

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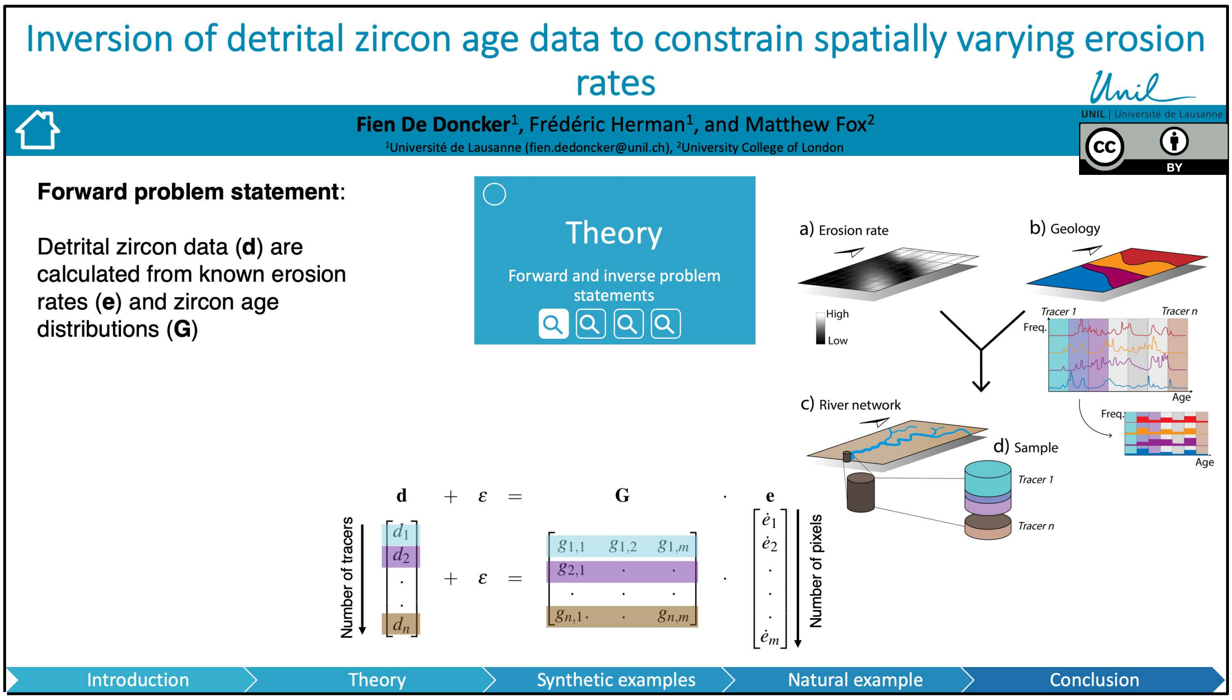
Introduction
Theory
Synthetic examples
Natural example
Conclusion

- Synthetic data showed that our method is robust
- The method is more suited for a well connected catchment
- The results of the natural example are in line with previous erosion rate studies in the Mendoza region
- We can extend the method to other tracers and other kinds of source areas

We illustrated an inversion method to calculate **spatially variable erosion rates** from **detrital zircon-age data**, starting from a prior erosion estimation, a yearly exported sediment load estimation and source area data. The method relies on the **recognition of zircon-age signatures of source areas in the zircon-age distribution of modern sands** sampled at the outlet of the investigated catchment.

Conclusion



To understand our inverse scheme, let's first have a look at what our data exactly mean.

At the right, the forward problem is illustrated: we want to calculate the concentration of zircons of different age intervals in the detrital sediments.
We consider discrete age bins as passive tracers. Hence, every source area is characterised by a unique tracer concentration pattern.
We calculate the concentration of every tracer that we will find in the sediments (**d**) by multiplying the known tracer concentration **G** times the erosion rate **e** at this location.
That means that **d** can be seen as a weighted average of the different tracer concentrations, with the weights being the erosion rates.

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Inverse problem statement:

An erosion rate patter (**e**) is calculated from known tracer concentrations (**G**) and detrital data (**d**)

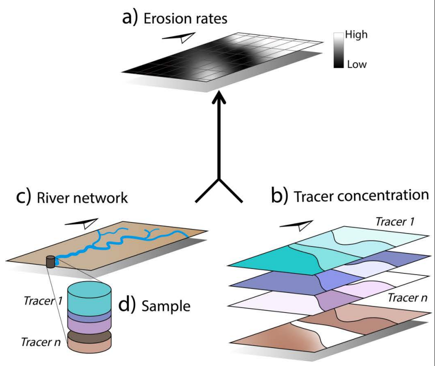
The problem is **underdetermined** as *m* (# pixels) is larger than *n* (# tracers)

⇒ We need smoothing (in the form of a covariance matrix)

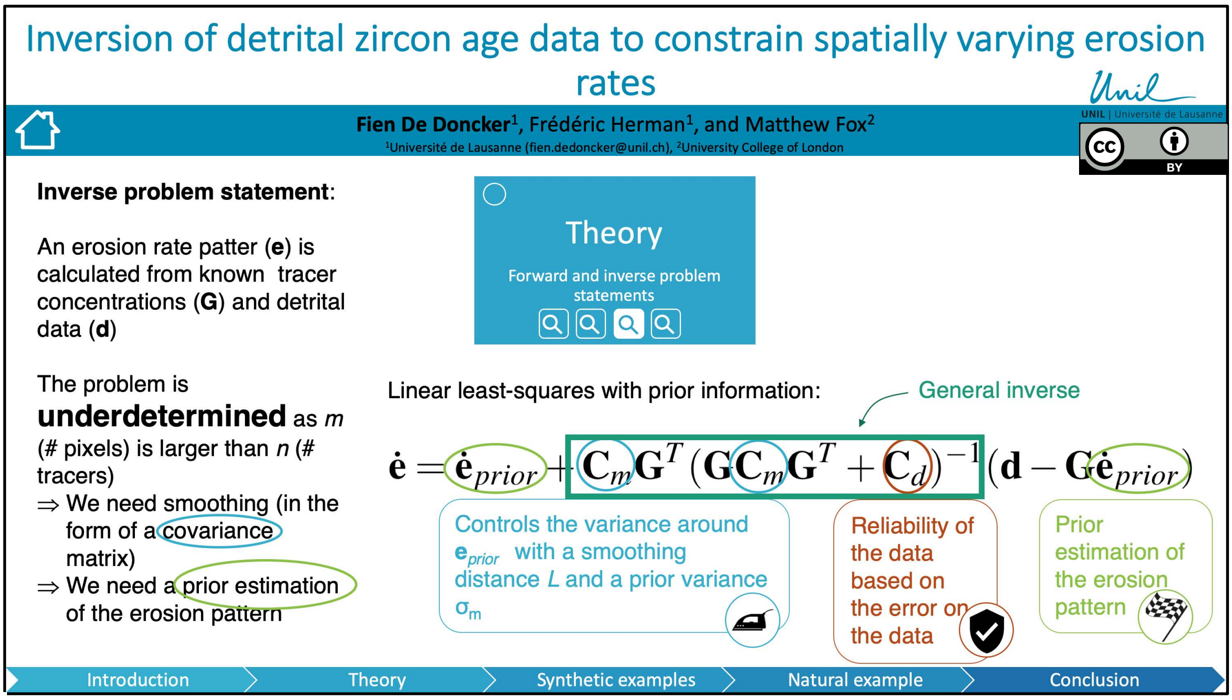
⇒ We need a prior estimation of the erosion pattern

Theory

Forward and inverse problem statements



Our goal remains to determine erosion rates and we want to obtain our goal by inverting our data into erosion rates.
We start from known tracer concentrations and detrital zircon age data to derive erosion rates. That means that we want to do the inverse of the forward problem, which is why this is called inverse modelling.
There is a problem however: our problem is underdetermined because the information we want to obtain (erosion rate at every pixel) is larger than the information we have (n number of tracers). To deal with the underdeterminedness of the problem, we need 1) smoothing and 2) a prior estimation of the erosion pattern.



This is how it looks like mathematically. Let's break this equation down.
Our posterior erosion rate is equal to a prior erosion rate estimation plus the difference between the posterior erosion rates (the general inverse times the detrital data) and the posterior erosion rates if prior data were to be used (**G***$\mathbf{e}_{prior}$ is in fact equal to $\boldsymbol{d}_{prior}$; and we do the the general inverse times $\boldsymbol{d}_{prior}$). That means that the data, the tracer concentration information, the model covariance, and the data covariance control in how much the posterior solution can deviate form the prior estimation.
The model covariance is governed by the prior variance σ_m (how much will the posterior solution vary around $\mathbf{e}_{prior}$?) and the smoothing distance L (how much does the information of other, more distant points play a role in determining the posterior solution at this location?)

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To assess the **meaning** of the the posterior solution we use
1) The posterior variance
2) The resolution
3) The spread of the resolution

Theory

Forward and inverse problem statements

$$\mathbf{C}_{M,post} = (\mathbf{G}^T \mathbf{C}_d \mathbf{G} + \mathbf{C}_m^{-1})^{-1}$$

How much has the posterior solution has evolved away from its prior by adding data?

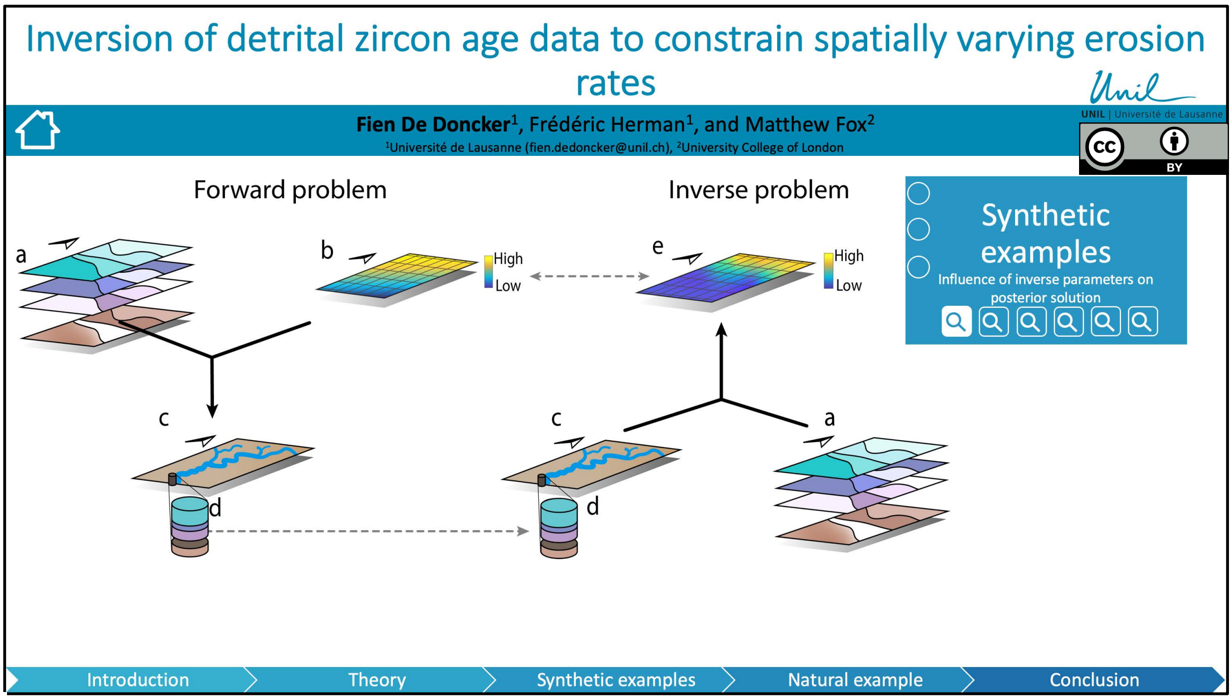
$$\dot{\mathbf{e}}_{post} = \mathbf{R}\dot{\mathbf{e}}_{true}$$

How much spatial filtering or averaging occurs?

$$spread(\mathbf{R}) = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{i,j}(R_{i,j} - \delta_{i,j})^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{i,j}R_{i,j}^2$$

How much is the available information spread?

Now that we know how we can convert the detrital data into an erosion map, we also want to know how we can assess the meaning of the posterior model. To do that, we use 3 metrics.
The first one is the posterior variance, which indicates in how much the posterior solution has evolved away from the prior by adding data. If we take the diagonal of this matrix, we obtain the posterior variance, which we can divide by the diagonal of the model covariance to obtain the normalized variance. Low values of the latter indicate a solution that evolved further away from its prior.
The second one is the resolution, which is a 'filtering matrix' between the true and posterior erosion rate. For a perfect solution where the posterior erosion rates are equal to the true erosion rates, the resolution ***R*** is equal to ***I***. We map a row of ***R*** corresponding to a specific location. For a perfect solution, this map would be 0 everywhere and 1 at the investigated location.
The third one is the spread function that shows in how much ***R*** deviates from identity. For a perfect solution, the spread function values would be 0 everywhere.



Now that we understand how the inverse method work, let’s test how it behaves. In this part, we check how the posterior solution changes with different inverse parameters. We try to recover a true erosion map (b in the figure) from data (d in the figure) created with our forward model that we plug into our inverse model; we then check the difference between the true model (b in the figure) and the posterior model (e in the figure).

Inversion of detrital zircon age data to constrain spatially varying erosion rates

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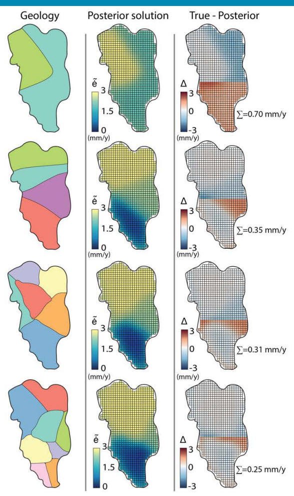
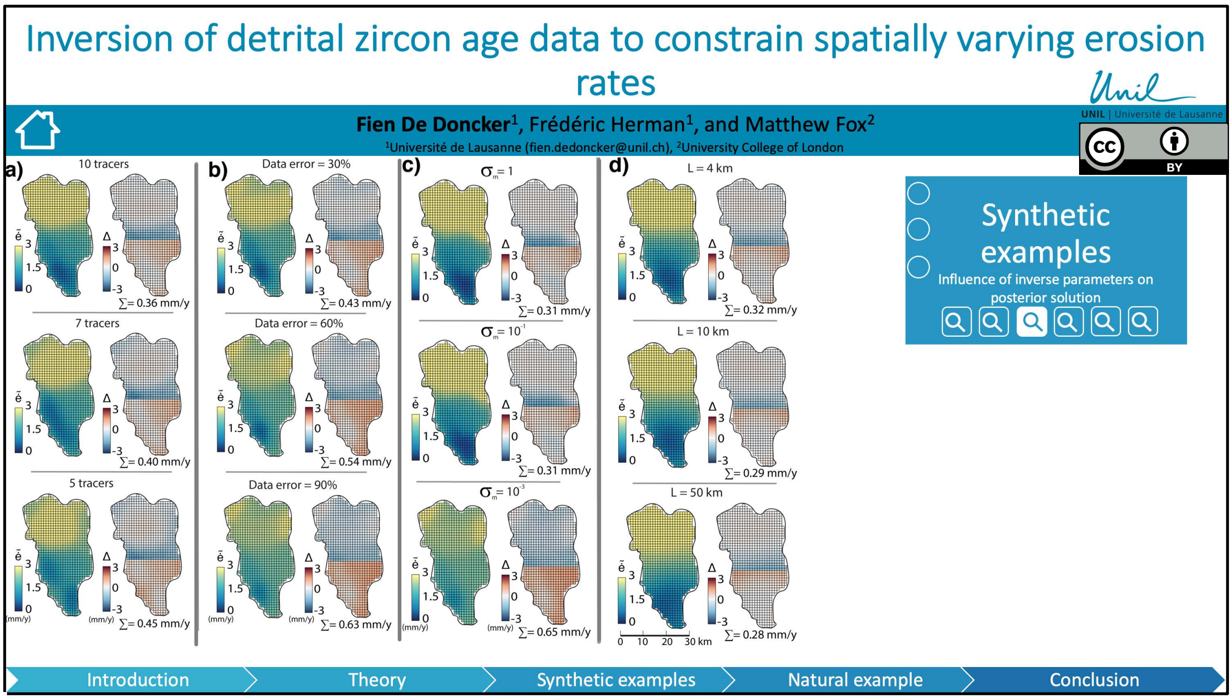


What happens if we change the geological map and decrease the **number of geological units**?
⇒ The posterior solution is less close to the true solution
⇒ The orientation and configuration of the units shape the posterior erosion pattern

Synthetic examples
Influence of inverse parameters on posterior solution

What would happen if we use a different geological setting? With less geological units, it's more difficult to recover the true erosion rate map as there are less source units that can be impacted by the erosion rate pattern. The orientation of the units is clearly visible in the example where only 2 units are used.



Now, we test the other parameters.

a) More tracers => more data => better solution

b) Higher data error => less trust in data => worse and smoother solution

c) Higher prior variance => higher variance around $\boldsymbol{e}_{prior}$ allowed => better solution

d) Higher smoothing distance => each pixel depends more on further-away information => better solution

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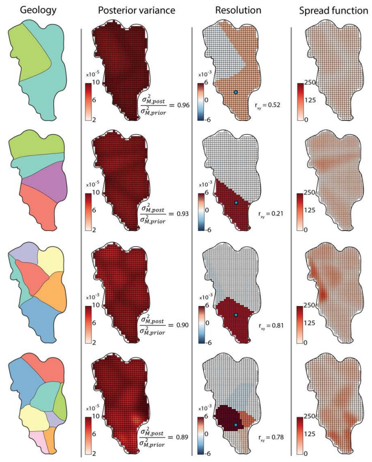


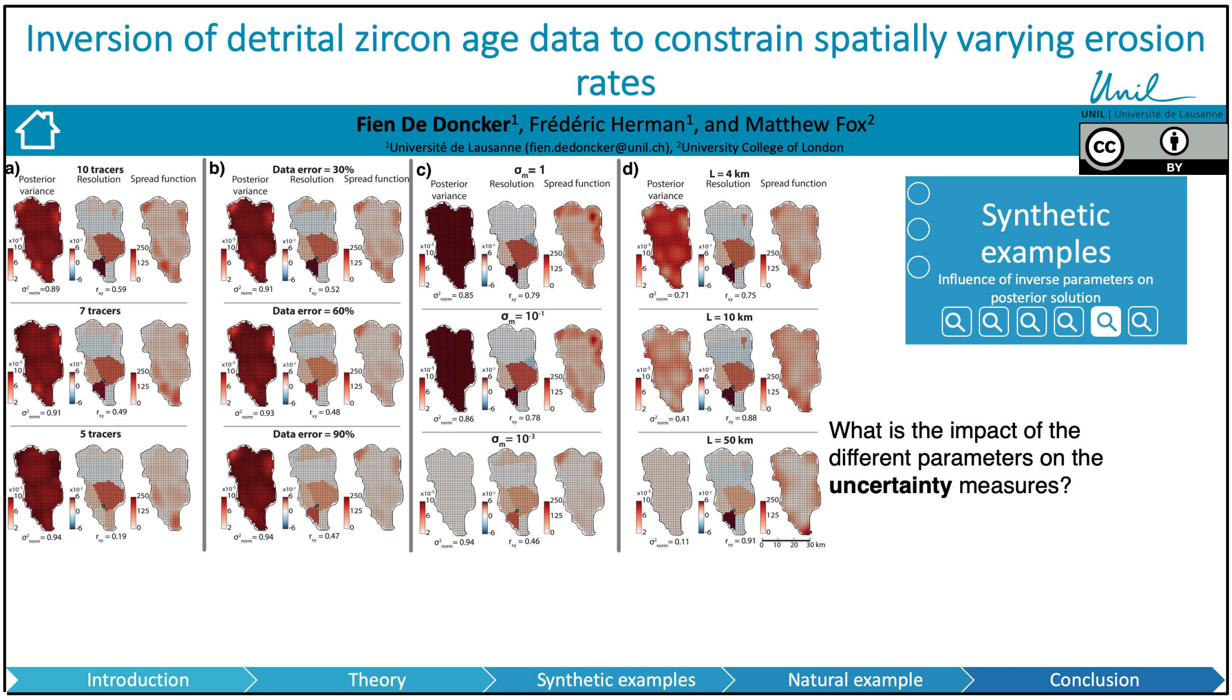
What happens if we change the geological map and decrease the **number of geological units**?
⇒ The posterior variance increases; low reduced variance
⇒ Low resolution (trade-off with variance)
⇒ Low spread function values due to low resolution values

Synthetic examples

Influence of inverse parameters on posterior solution

What is the impact of the different parameters on the **uncertainty** measures?

With less geological units, there is less data available so the posterior variance increases, while the resolution decreases.

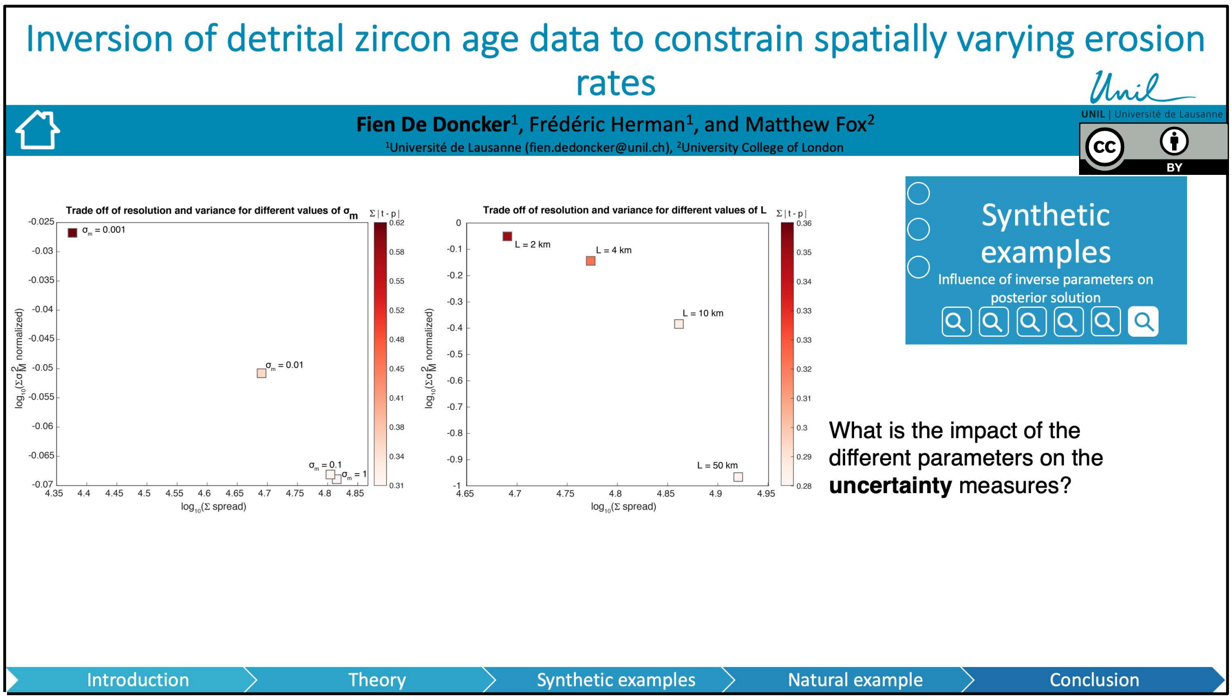


a) Less tracers
⇒ less data
⇒ lower resolution
⇒ higher variance

b) Higher data error
⇒ less trust in data
⇒ lower resolution
⇒ higher variance

c) Lower prior variance
⇒ lower variance around $\mathbf{e}_{prior}$ allowed
⇒ higher averaging: more units play a role

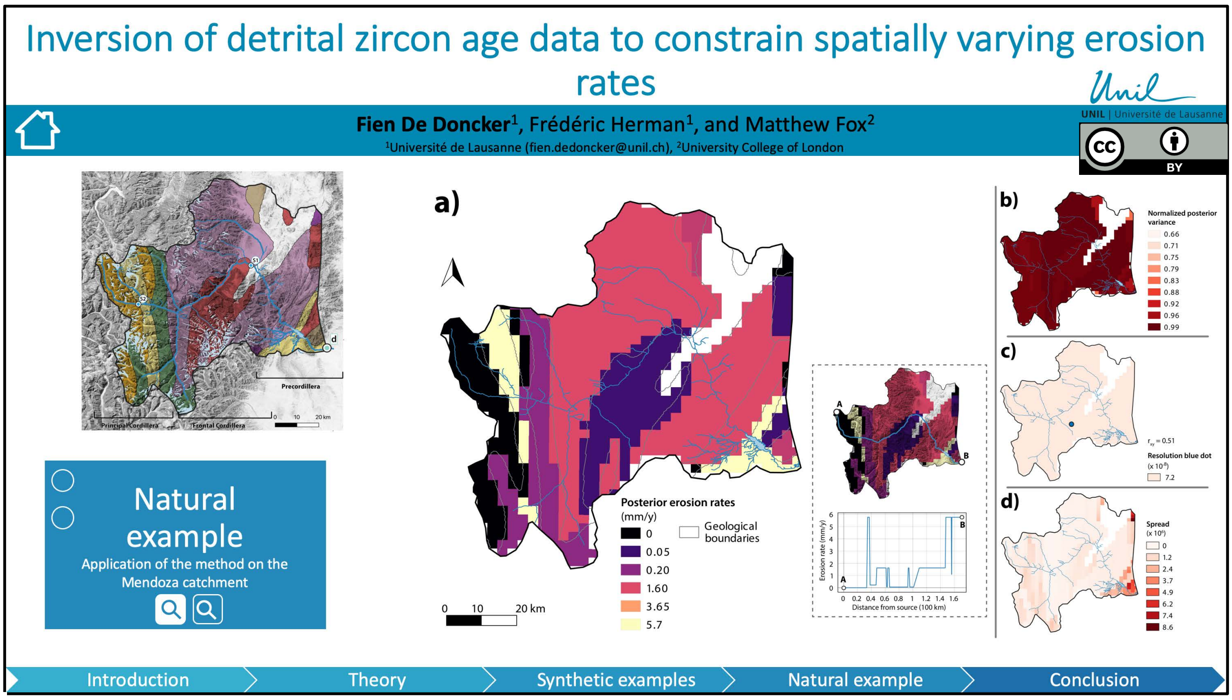
d) Higher smoothing distance
⇒ higher averaging: more units play a role
⇒ Lower variance



As discussed above, there is a well known trade-off between resolution and variance. Imagine that you want to count the number of leaves in a garden. For a high resolution, the variance in the number of leaves per m^2 will be large, but small imprecisions will be taken into account. If we want to decrease the variance by spatial averaging, we need to decrease the resolution. A resolution matrix that is closer to identity, indicating that the posterior solution is less a result of spatial averaging, corresponds to smaller spread function values.
Therefore, the plots above nicely illustrate this trade-off. The colours indicate the sum of the absolute difference between the true and posterior solution and on the y-axis the reduced variance is plotted and on the x-axis the sum of the spread function values. The posterior solution that is closest to the true solution is in both cases found for the smallest normalized variance, paired with a slightly higher spread, so a reduction in posterior variance leads to solutions closer to the true model, even though the resolution is slightly worse.

Higher prior variance
$\Rightarrow$ Further evolution from $\mathbf{e}_{prior}$ allowed

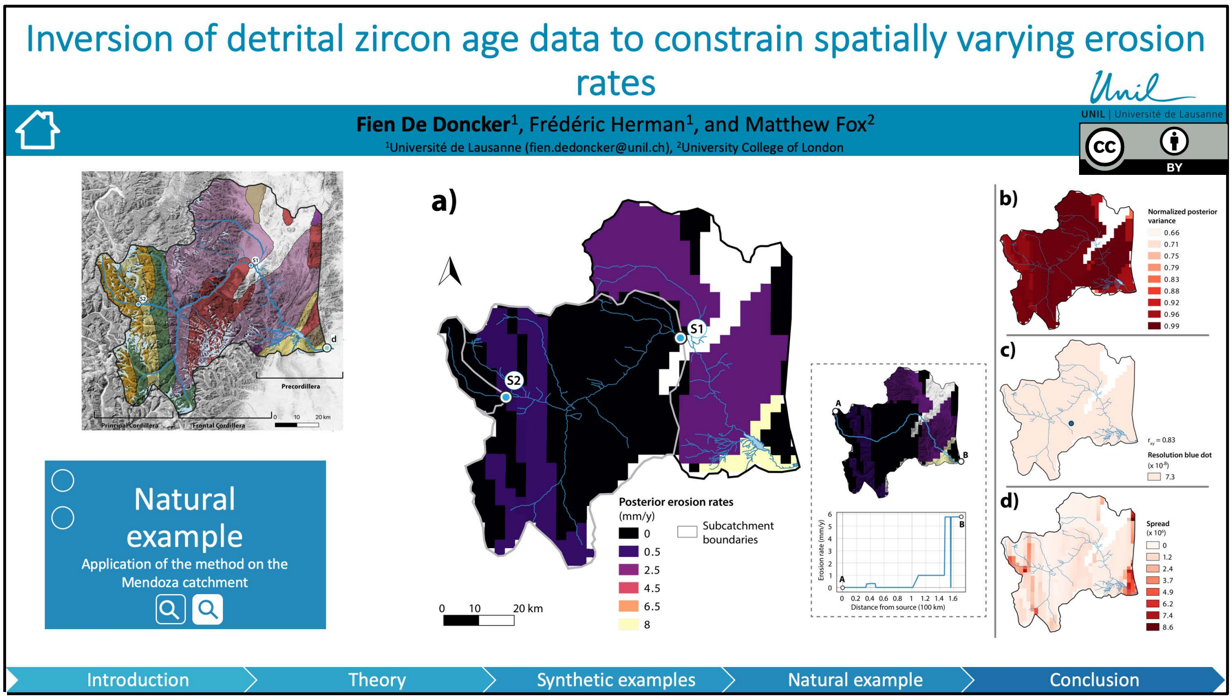
Higher smoothing distance
$\Rightarrow$ higher averaging: more units play a role
$\Rightarrow$ Lower variance



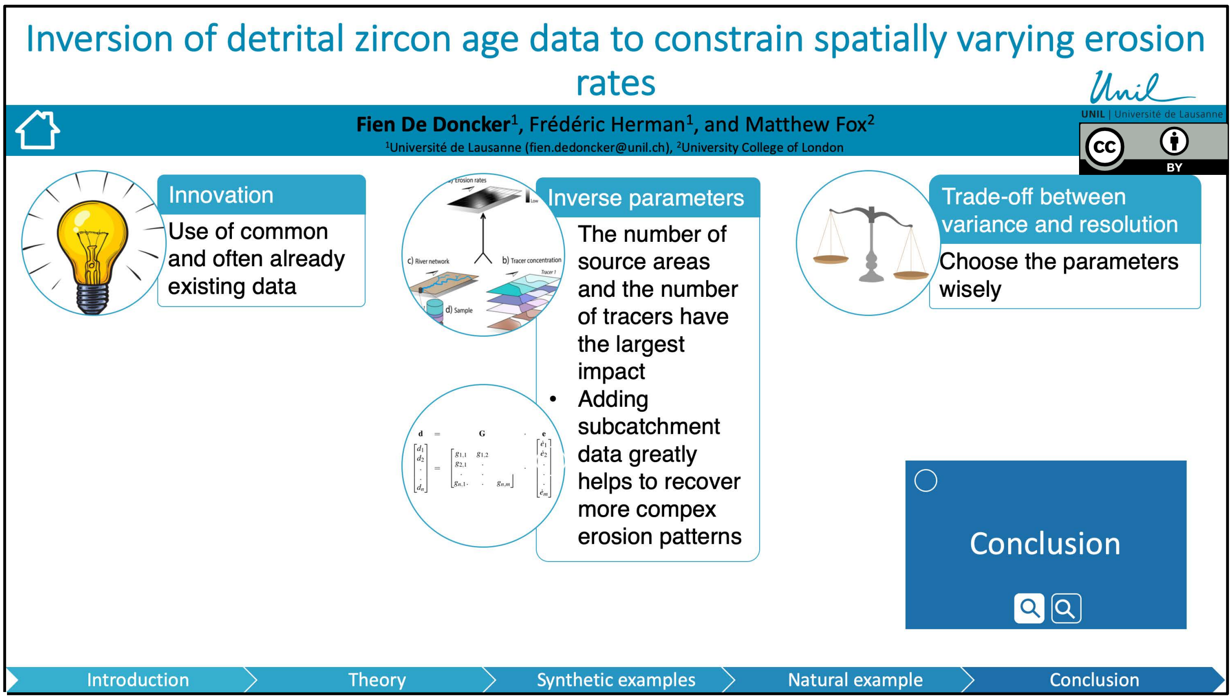
We chose this study area because forward experiments of Capaldi et al. 2017 resulted in synthetic data that match the actual detrital zircon age data very well.

The application of the inverse method results in posterior erosion rates with high erosion rates in the east and the west, intermediate erosion rates in the centre of the catchment and very low erosion rates in the very western end of the catchment. Note that since we have not a lot of geological variability in the centre of the catchment, the erosion rates here result from high spatial averaging, which is illustrated in the resolution mat (figure c). The normalized variance is rather high and sol are the spread values for the eastern part of the catchment, where the geological unit patches are smallest.

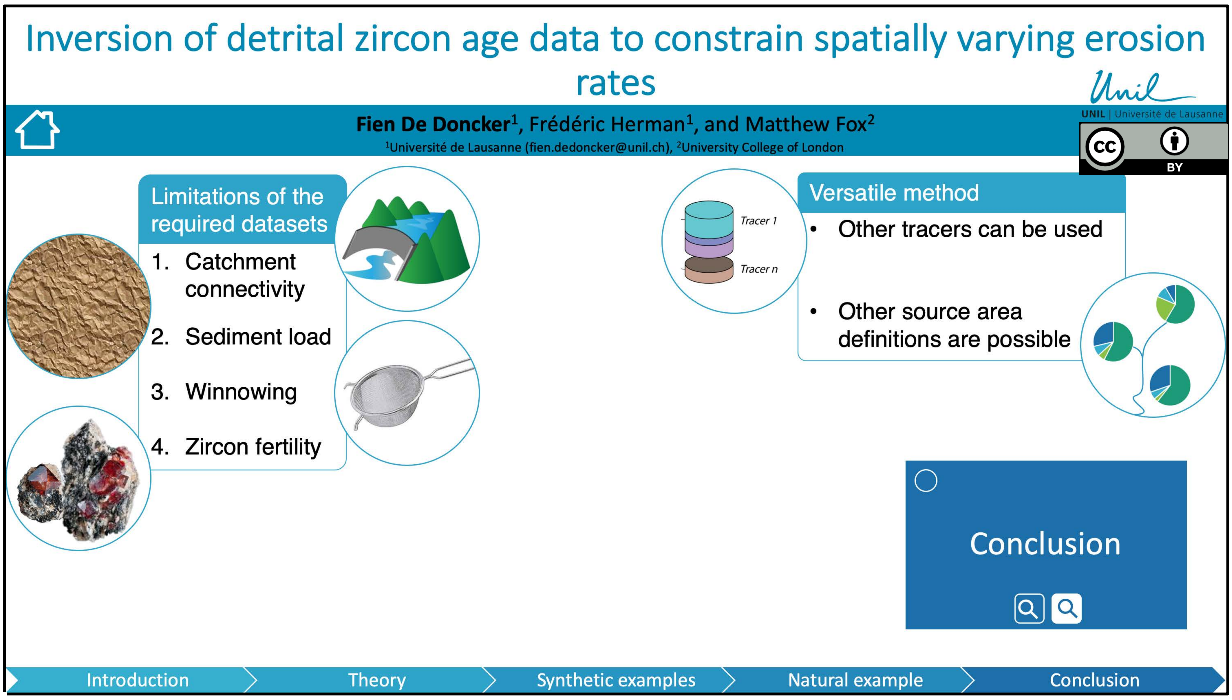
The suddenly high erosion rates in the western part of the catchment can be caused by two things. The first hypothesis is that these are caused by the high number of glaciers in this area which are known as strong erosional agents, but then, why wouldn't the other western catchments be characterized by high erosion rates as well? So we need to propose a second hypothesis. We see that around the lake at the eastern side of the catchment, the erosion rates are high, and that this region is characterised by the same lithology as the region in the west (the light yellow patches in the geological map on the left). That means that if the erosion rates around the lake are high - but not in the west of the region – that we will find a lot of fignerprints of this lithology in the detrital data, which will be interpreted by the model as high erosion rates everywhere where this lithology occurs. But how can we decouple these 2 regions? By adding subcatchment data (S1 and S2 in the geological map).



These are the results for the case where we appended subcatchment data. Now, the erosion rates have become very low in the western part of the catchment as well, which supports our second hypothesis from before. You can now indeed see that as the two regions have become decoupled, only the region around the lake is responsible for the high erosion rates, which means that the erosion rates here have increased up to 8 mm/y. Research around this lake has shown high hillslope instability and mass movements occur here. The intermediate erosion rates match the USLE-predicted actual hyrological erosion rates of around 1.5 mm/y.



- Zircon age analysis is common thanks to technological advances that allow analysis of large batches of zircon grains
- The trade-off between variance and resolution has been shown above and means that testing of parameters should always be done to pick the optimal parameter values.
- With the synthetic examples, we saw that with more geological units and more tracers (discrete age bins), our posterior solution was closest to the true solution.
- By adding subcatchment data, we can recover more complex true erosion patterns, sharp borders are better represented



The datasets that our method requires can have some limitations that might distort the detrital zircon age signal. 1.

1. If the catchment connectivity is low, some particles that were eroded cannot reach the detrital sampling location within a short timespan. That means that areas far away from the sampling location are possibly underrepresented, thereby distorting the resulting erosion rate map. However, adding subcatchment data can help to diminish these connectivity effects.
2. To convert the relative source contributions into erosion rates, we also need the sediment load, which is unfortunately quite hard to determine, certainly if we also want to quantify the bedload.
3. Hydraulic sorting leads to the preferential transport of grains with a certain grain size. If a relationship exists between age and grain size, this hydraulic sorting can lead to the distortion of the detrital signal. Verifying if this relationship exists is crucial and sampling grains of different grain sizes decreases winnowing effects.
4. Zircon mineral concentrations – so called fertility – can vary over 3 orders of magnitude within one catchment. We need to take this into account as the detrital tracer concentrations not only depend on the amount of erosion in each source area, but also on the zircon fertility. For example for a source area with very high erosion rates but low fertility, we will find its fingerprints back in low concentrations. If we don't account for fertility variations, our model will interpret this as very low erosion rates.

Our method is versatile: other tracers such as magnetism of minerals, colour of grains, stable isotopes, geochemical components, organic matter concentration and mineralogical properties can be used in stead of zircon distributions. The source areas also don't have to be defined as geological units, we can define them for example as subcatchments with fingerprints found in modern sand samples.