Have we misunderstood the Shields curve?

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Highlights

- Fluvial transport thresholds compiled in the Shields diagram are neither entrainment nor disentrainment thresholds
- Shields curve shows a "rebound threshold" associated with the kinetic energy balance of transported particles
- Conceptually simple rebound threshold model unifying viscous and turbulent aeolian and fluvial transport conditions
- Transport capacity requires exceeding the impact entrainment threshold, which is strictly larger than the rebound threshold

Outline

1. Introduction
2. Rebound threshold model
3. Experimental and numerical validation
4. Implications for capacity transport
5. Summary

Introduction

Environmental parameters:

- Particle density ρ_p [kg/m^3]
- Particle diameter d [m]
- Fluid density ρ_f [kg/m^3]
- Kinematic fluid viscosity ν_f [m^2/s]
- Fluid shear stress τ [N/m^2]
- Sediment transport rate Q [kg/(m.s)]
- Gravitational constant g [m/s^2]

Dimensionless numbers:

$$\begin{aligned} \text{Density ratio:} &\quad s \equiv \rho_p/\rho_f \\ \text{Galileo number:} &\quad \mathrm{Ga} \equiv d\sqrt{(s-1)gd}/\nu_f \\ \text{Shields number:} &\quad \Theta \equiv \tau/[(\rho_p-\rho_f)gd] \\ \text{Shear Reynolds number:} &\quad \mathrm{Re}_* \equiv \mathrm{Ga}\sqrt{\Theta} \\ \text{Dimensionless transport rate:} &\quad Q_* \equiv Q/\left[\rho_p d\sqrt{(s-1)gd}\right] \end{aligned}$$

Introduction

Two methods for measuring fluvial transport thresholds:

1. Visual: Θ_t is value of Θ at measured critical transport rate Q_*
2. Reference: Θ_t from extrapolating $Q_*(\Theta)$ to small or zero Q_*

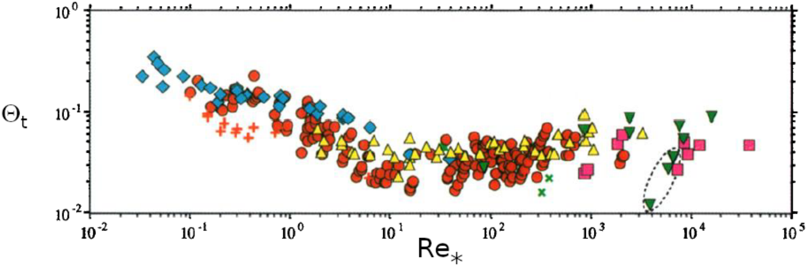


Figure: Shields diagram after Buffington & Mongomery[1]: reference thresholds (triangles) and visually measured thresholds (other symbols).

Reference (click to open):

(1) Buffington & Mongomery (WRR, 1997)

Introduction (visual method)

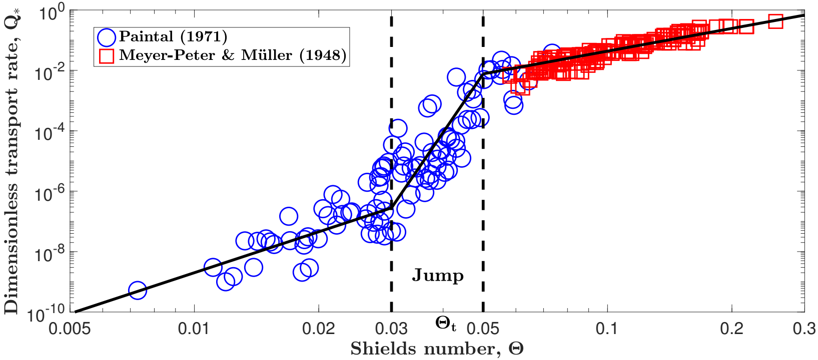


Figure: Measurements[1] of $Q_*(\Theta)$. Notice the jump of Q_* around $\Theta_t \approx 0.04$. Also, notice that $Q_* > 0$ even at $\Theta \approx 0.007 \ll \Theta_t$.

References (click to open):

(1) Paintal (JHR, 1971); Meyer-Peter & Müller (TU Delft, 1948) after Wong & Parker (JHE, 2006)

Introduction (visual method)

1. The visually measured threshold is usually interpreted as a measure for flow-driven entrainment of bed sediment[1].
2. However, flow-driven entrainment is mainly driven by extreme flow events, associated with very-large-scale motions[2].
3. Entrainment events occur even for Shields numbers nearly an order of magnitude below the Shields curve[3].
4. When turbulence is suppressed, almost all entrainment events are driven by impacts of transported particles onto the bed[4].

Conclusion:

The visual threshold does not describe flow-driven entrainment.

References (click to open):

(1) Dey & Ali (Sedimentology, 2019)
(2) Valyrakis et al. (WRR, 2011); Cameron et al. (JFM, 2020)
(3) Paintal (JHR, 1971)
(4) Heyman et al. (JGR, 2016); Pähtz & Durán (PRF, 2017); Lee & Jerolmack (ESD, 2018)

Introduction (reference method)

General transport rate relation of previous presentation:
$(\kappa, \mu_b, c_M) = (0.4, 0.63, 1.7)$

$$Q_* = \frac{2\sqrt{\Theta_t}}{\kappa\mu_b}(\Theta - \Theta_t)\left[1 + \frac{c_M}{\mu_b}(\Theta - \Theta_t)\right] \quad \text{if} \quad \frac{\Theta}{\Theta_t} \gtrsim 1.5\text{–}2 \quad (1)$$

1. According to Shields[1], extrapolating paired measurements of Θ and Q_* to $Q_* = 0$ yields the reference threshold.
2. Hence, Θ_t in Eq. (1) is the reference threshold.
3. Because of the validity of Eq. (1) across aeolian and fluvial conditions, Θ_t should have a universal physical meaning.
4. Assuming reference threshold = visual threshold, this universal physical meaning should be consistent with a jump of Q_* around Θ_t (see slide #6).

What is the physical meaning of Θ_t and the Shields curve?

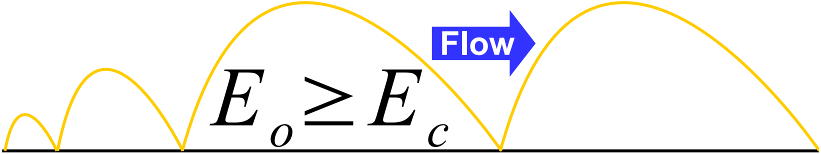
References (click to open):

(1) Shields (Caltech, 1936)
(2) Dey & Ali (Sedimentology, 2019)

Rebound threshold model

Thought experiment:

- Particle hop in nonfluctuating wall-bounded flow
- $E_{\uparrow(\downarrow)}$ = kinetic energy immediately after (before) a rebound
- $\theta_{\uparrow(\downarrow)}$ = rebound (impact) angle; $E_o \equiv E_\uparrow(t=0)$
- Mean rebound laws from experiments: $(E_\uparrow/E_\downarrow, \theta_\uparrow) = f(\theta_\downarrow)$



Findings:

- For sustained motion, a critical energy E_c must be exceeded:
- If $E_o \geq E_c(\Theta, \mathrm{Ga}, s)$, a periodic trajectory is approached.
- If $E_o < E_c(\Theta, \mathrm{Ga}, s)$, no motion is approached.
- For $\Theta < \Theta_t^{\mathrm{Rb}}(\mathrm{Ga}, s)$, only trivial solutions exist ($E_c = \infty$).

Hypothesis: Shields curve threshold $\Theta_t = \Theta_t^{\mathrm{Rb}}$.

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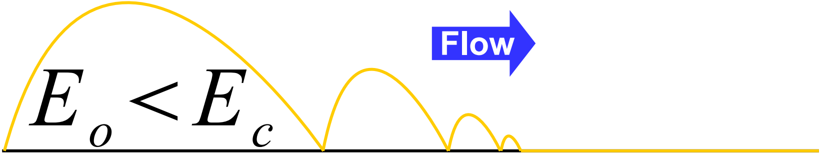

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Rebound threshold model

Hypothesis explains jump of Q_* around Θ_t:

- Particles entrained by turbulent events are, on average, suddenly able to stay much longer in motion.

Outline of mathematical model[1] for rebound threshold Θ_t^{Rb}:

1. For given values of Ga and s, find all periodic trajectory solutions $\Theta(\mathrm{Ga}, s, E_\uparrow)$ (various analytical solutions exist[1,2]).
2. Only consider particle trajectories with a rebound energy that exceeds the potential barrier energy E_b set by the pockets of the bed surface: $E_\uparrow \geq E_b$.
3. Consider that E_b is weakened by the near-surface flow. In particular, E_b vanishes when Θ exceeds Θ_t^{max}, the yield stress that imposes an upper limit on the Shields curve.
4. Obtain the rebound threshold from the trajectory for which Θ is minimal: $\Theta_t^{\mathrm{Rb}} = \min_{E_\uparrow} \Theta[\mathrm{Ga}, s, E_\uparrow \geq E_b(\Theta)]$.

References (click to open):

(1) Pähtz et al. (RoG, 2020); Pähtz et al. (submitted, 2020)

(2) Jenkins & Valance (POF, 2014); Berzi et al. (JFM, 2016); Berzi et al. (JGR: ES, 2017)

Experimental and numerical validation

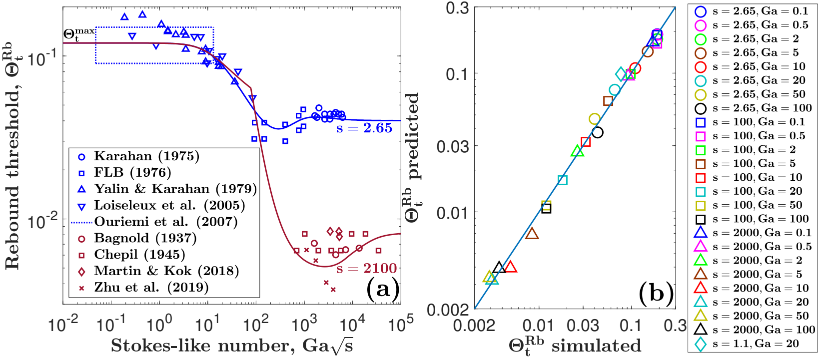

Figure: Rebound threshold model against (a) measurements[1−3] and (b) DEM-based sediment transport simulations. All model parameters in (a) have been obtained from experiments (not from fitting to threshold data).

References (click to open):

(1) Karahan (TU Istanbul, 1975); Fernandez Luque & van Beek (JHR, 1976); Yalin & Karahan (JHD, 1979);
(2) Loiseleux et al. (POF, 2005); Ouriemi et al. (POF, 2007)
(3) Bagnold (TGJ, 1937); Chepil (Soil Science, 1945); Martin & Kok (JGR: ES, 2018); Zhu et al. (JGR: ES, 2019)

Implications for capacity transport

Dimensionless average rebound threshold particle velocity $\overline{v_{x*t}}^{\mathrm{Rb}}$:

1. The minimization of Θ, used to obtain Θ_t^{Rb}, also yields $\overline{v_{x*t}}^{\mathrm{Rb}}$.
2. In particular, for transport in log-layer[1], $\overline{v_{x*t}}^{\mathrm{Rb}} \approx 2\kappa^{-1}\sqrt{\Theta_t^{\mathrm{Rb}}}$.

New understanding of capacity transport[2]:

1. Increasing transport load M leads to weakening of the flow via momentum transfer from the flow to transported particles.
2. Capacity transport is the weakest flow state (largest M) that allows for a sustained average rebound motion of transported particles (analogous to rebound threshold conceptualization).
3. This transport capacity definition leads to $M_* = \frac{1}{\mu_b}(\Theta - \Theta_t^{\mathrm{Rb}})$.
4. To keep M at capacity, a continuous supply of bed particles via impact entrainment is required (see next slide).

These expressions for $\overline{v_{x*t}}^{\mathrm{Rb}}$ and M_* were used to derive the general transport rate relationship presented in the previous presentation.

References (click to open):

(1) Pähtz & Durán (JGR: ES, 2018)

(2) Pähtz & Durán (PRF, 2018); Pähtz et al. (RoG, 2020)

Implications for capacity transport (impact entrainment)

1. Randomness in natural systems causes deposition.
2. Fluid entrainment, which occurs only in intermittent turbulent events, is unable to continuously balance this deposition[1].
3. Hence, entrainment by particle-bed impacts is required to continuously balance this deposition and sustain capacity[1].
4. However, we find that, in the limit $\Theta \to \Theta_t^{\mathrm{Rb}}$, the lift-off energy of particles entrained by impacts is necessarily smaller than the critical energy E_c required for sustained motion[2].
5. Hence, the impact entrainment threshold Θ_t^{ImE}, defined as the Shields number above which impact entrainment is able to balance deposition and sustain capacity, is larger than Θ_t^{Rb}.
6. A literature review[1] suggests $\Theta_t^{\mathrm{ImE}} \approx (1.5{-}2)\Theta_t^{\mathrm{Rb}}$.

References (click to open):

(1) Pähtz et al. (RoG, 2020)
(2) Pähtz et al. (submitted, 2020)

Summary

Summary of both presentations:

$$Q_* = \frac{2\sqrt{\Theta_t^{\mathrm{Rb}}}}{\kappa\mu_b}(\Theta - \Theta_t^{\mathrm{Rb}})\left[1 + \frac{c_M}{\mu_b}(\Theta - \Theta_t^{\mathrm{Rb}})\right] \quad \text{if} \quad \Theta \geq \Theta_t^{\mathrm{ImE}}$$

1. Θ_t^{Rb} can be predicted from rebound threshold model.
2. A model for Θ_t^{ImE} is currently missing, but it seems that

$$\Theta_t^{\mathrm{ImE}} \approx (1.5\text{–}2)\Theta_t^{\mathrm{Rb}}.$$

References regarding results in both presentations (click to open):

Pähtz & Durán (Physical Review Fluids **2**, 074303, 2017)
Pähtz & Durán (Journal of Geophysical Research: Earth Surface **123**, 1638–1666, 2018)
Pähtz & Durán (Physical Review Fluids **3**, 104302, 2018)
Pähtz et al. (Reviews of Geophysics **58**, e2019RG000679, 2020)
Pähtz & Durán (Physical Review Letters **124**, 168001, 2020)
Pähtz et al. (submitted, 2020)