Simulations of the Basal Forces Generated by Dam Breaks: Comparison Between Continuous and Discrete Models

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Different Model Strategies

Quantitative comparison between the models and with laboratory experiments?

- Shallow-Water continuum equations
- Navier-Stokes continuum equations
- Discrete Elements Methods (DEM)

SHALTOP
Bouchut et al. 2003,
Mangeney et al. 2007

BASILISK
Lagrée et al. 2011

SCoPI
Maury et al. 2005

Decrease computational cost!
Discrete Elements Method: Contact Dynamics

For each particle:

- **Signorini’s law**
  
  \[
  m_i \frac{d \mathbf{u}_i}{d t} = m_i \mathbf{g} + \sum_{j=i}^{N} \mathbf{f}_{i,j}
  \]

  Equivalent to the constrains:
  
  \[
  \lambda_{j,k} \geq 0 \\
  \mathbf{D}_{j,k} \geq 0 \\
  \lambda_{j,k} \mathbf{D}_{j,k} = 0
  \]

  Maury 2006, Lefevre 2009

- **Coulomb’s law**
  
  Equivalent to maximize the dissipated power:
  
  \[
  \max \quad -\tau_{j,k} \mathbf{v}_{j,k}
  \]

  such that \(|\tau_{j,k}| \leq \mu \lambda_{j,k}

  Tangential relative velocity
DEM with global computation of friction effects

Constrained optimisation problem  *Stewart 2000, Moreau 2003*

\[ \max_{u, \lambda, \tau} \mathcal{F}(u, \lambda) - \tau \cdot v_t(u) \]

**such that** Newton's laws are verified and:

- \( \tau \leq \mu \lambda \)  
  Coulomb's friction law
- \( \lambda \geq 0 \)  
  Repulsive normal forces

- Convex functional (numerically solved MOSEK)
- **Global computation of contact forces**
  stable implicit-scheme
- **No iteration on the contact network!**

Global dissipated power of tangential forces
Coulomb’s law:

\[ v_{i,j} \neq 0 \implies |\tau_{i,j}| = \mu \lambda_{i,j} \]
Continuum model rheology: \[ \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0 + I} I \]

\[ \mu_s = 0.48, \quad \Delta \mu = 0.25 \quad \text{and} \quad I_0 = 0.279 \]

Discrete Element Method

Grain-grain friction coefficient: \[ \mu = 0.9 \]

Martin et al., 2017

\[ t = 0.18 \text{ s} \]

\[ t = 0.30 \text{ s} \]

\[ t = 1.02 \text{ s} \]

3D discrete method is in good agreements with experiments data when \[ \mu = 0.3 \]

Previous studies used additional dissipation (e. g. rolling friction) Girolami et al. 2012

Viroulet et al., 2019
Strong fluctuations and spatial variability of the basal forces calculated from Discrete Element Methods!
Conclusion & Perspective

- **Global Model** for the Dry Friction Problem in Contact Dynamics codes
- **Numerical stability** and possibility of **large time steps** due to an implicit scheme
- Going further in **quantitative comparison** between the different models and experiments
- In the future: Understand and quantify the **physical origin of basal forces fluctuations**
\( N = 65,909 \) discs

Computation time: 47h 31mn

2 Intel Xeon E5-2650 2.00 GHz (2x8 cores)
Quantitative comparison with other models and experiments

Comparaison Contact Dynamics and Navier-Stokes simulations

\[ \mu = 0.5 \text{ and } e = 0.5 \]

\[ \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0 + I} I \]

\[ \mu_s = 0.32, \ \Delta \mu = 0.28 \text{ and } I_0 = 0.4 \]

Very good agreement BUT with parameters smaller than those measured experimentally

\[ \mu_s = 0.38, \ \Delta \mu = 0.26 \text{ and } I_0 = 0.279 \]

Jop et al., 2005

DEM goes further than experiments if no additionnal dissipation is accounted for!
Molecular Dynamics & Contact Dynamics

- **Molecular Dynamics**

  Contact forces are functions

  Hertz’s contact law

  \[ \mathbf{f}_{i,j} \cdot \mathbf{n}_{i,j} = \kappa \delta_{i,j}^{\frac{3}{2}} \]

- **Contact Dynamics**

  Contact forces verify contact laws

  Find contact forces such that beads do not overlap...

  Distances between spheres $i$ and $j$

  Constrained optimization problem

  Larger time steps

  \[ D_{i,j} \geq 0 \]

  \[ D_{j,k} \geq 0 \]

  \[ D_{i,k} \geq 0 \]
Molecular Dynamics & Contact Dynamics

- **Molecular Dynamics**

  \[
  m_i \frac{d \mathbf{u}_i}{dt} = m_i \mathbf{g} + \sum_{j \neq i}^N \mathbf{f}_{i,j}
  \]

  Contact forces are functions of positions and velocities.

  Hertz’s contact law:
  \[
  \mathbf{f}_{i,j} \cdot \mathbf{n}_{i,j} = \kappa \delta_{i,j}^{3/2}
  \]
  where \( \delta_{i,j} \) is the distance between spheres \( i \) and \( j \).

  Normal stiffness \( \kappa \)

- **Contact Dynamics**

  Constrained optimization problem:

  Contact forces verify contact laws:
  \[
  \text{Find contact forces such that beads do not overlap}...
  \]

  Larger time steps:

  Distances between spheres:
  \[
  D_{i,j} \geq 0 \quad D_{j,k} \geq 0
  \]
DEM with global computation of friction effects

Constrained optimisation problem  
**Stewart 2000, Moreau 2003**

Term from frictionless model (normal forces)

\[
\max_{u, \lambda, \tau} \mathcal{F}(u, \lambda) - \tau \cdot v_t(u)
\]

such that Newton’s laws are verified and:
\[
\begin{align*}
|\tau| & \leq \mu \lambda & \text{Coulomb’s friction law} \\
\lambda & \geq 0 & \text{Repulsive normal forces}
\end{align*}
\]

- Convex functional (numerically solved MOSEK)
- Global computation of contact forces
  stable implicit-scheme
- No iteration on the contact network!

- The choice of the functional implies
  the prevention of overlaps:
  \[
  D_{i,j}^{n+1} \geq 0
  \]
  is directly obtained as
  an optimality condition
Global friction effects in Contact Dynamics

Objective: Numerical scheme dealing with friction that can handle large time-step values

Add friction forces to the global frictionless model SCoPI

- Global computation of the contact forces at the same time: no iteration

Implicit scheme
Large time steps

1 000 000 spheres
S. Faure, A. Lefebvre-Lepot

40 000 spheres
S. Faure, A. Lefebvre-Lepot

Maury 2006, Lefevre 2009