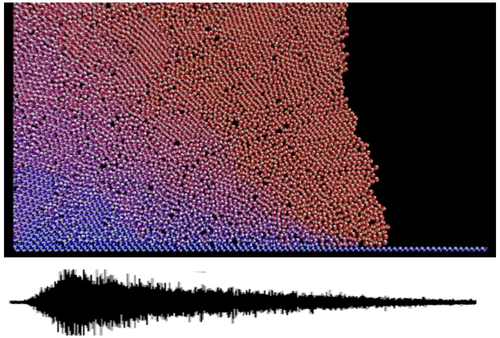
EGU Visio Conference | 4 - 8 May 2020

Simulations of the Basal Forces Generated by Dam Breaks: Comparison Between Continuous and Discrete Models

Hugo Martin[1,2,3], Bertrand Maury[4,5], Aline Lefebvre-Lepot[6], Yvon Maday[2], Sylvain Viroulet[7] &

Anne Mangeney[1,2,3]

Slidequakes



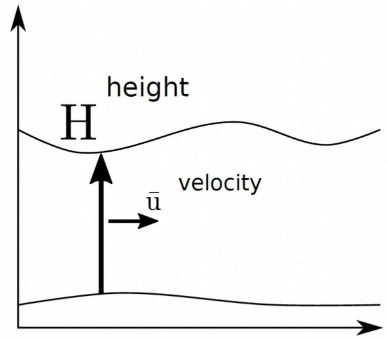
[1]*Université de Paris* (IPGP)
[2]*Sorbonne Université* (LJLL)
[3]*Inria Paris* (Ange)
[4]*Université Paris-Sud* (LMO)
[5]*École Normale Supérieure* (DMA)
[6]*École Polytechnique* (CMAP)
[7]*Université Toulouse* (IMFT)

Different Model Strategies

Quantitative comparison between the models and with laboratory experiments ?

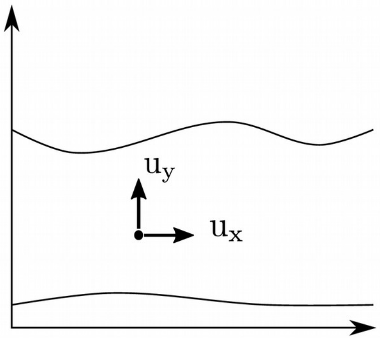
Landslide in Tuialamu. Photo/ Land Transport Authority Samoa

- Shallow-Water continuum equations



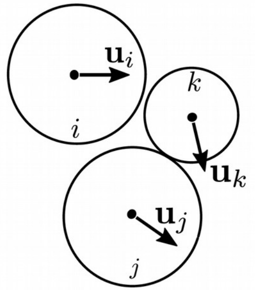
SHALTOP
Bouchut et al. 2003, Mangeney et al. 2007

- Navier-Stokes continuum equations



BASILISK
Lagrée et al. 2011

- Discrete Elements Methods (DEM)



SCoPI
Maury et al. 2005

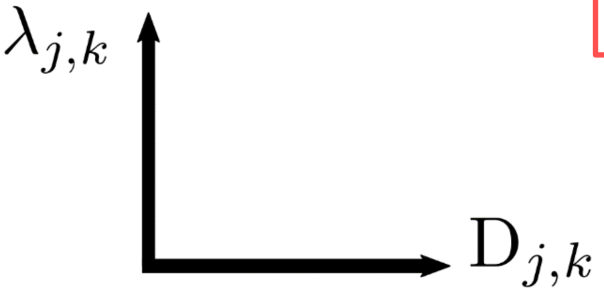
Decrease computational cost !

Discrete Elements Method: Contact Dynamics

For each particle:

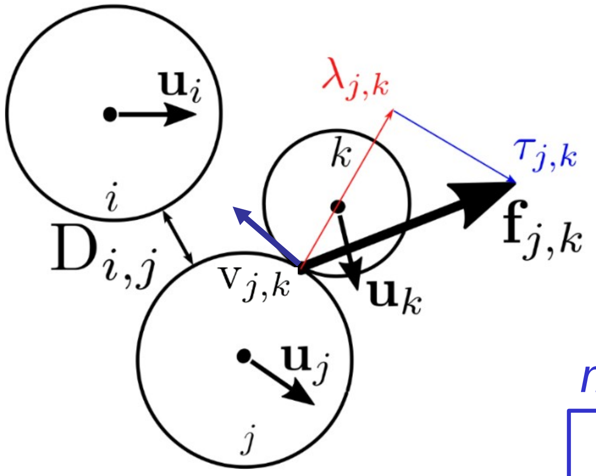
$$m_i \frac{\mathrm{d}\ \mathbf{u}_i}{\mathrm{d}\ t} = m_i\ \mathbf{g} + \sum_{j=i}^{N}\ \mathbf{f}_{i,j}$$

- Signorini's law



Equivalent to the constrains :

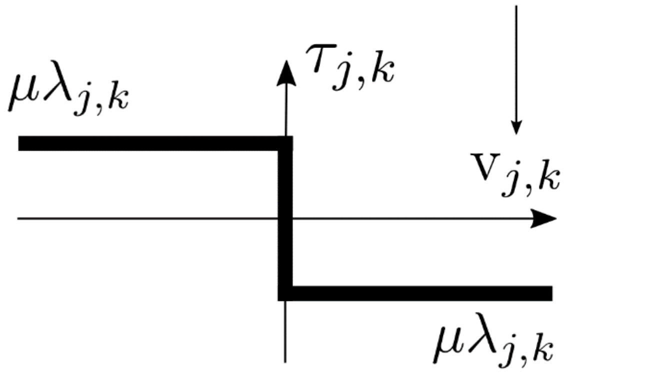
$$\lambda_{j,k} \geq 0$$
$$\mathrm{D}_{j,k} \geq 0$$
$$\lambda_{j,k}\mathrm{D}_{j,k} = 0$$



Maury 2006, Lefevre 2009

- Coulomb's law

Tangential relative velocity



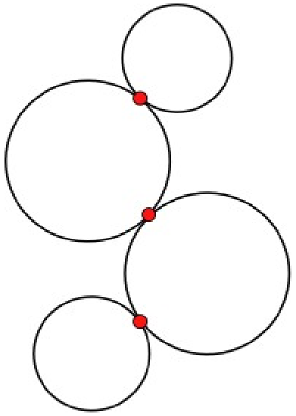
Equivalent to maximize the dissipated power :

$$\max_{\tau_{j,k}}\ -\tau_{j,k}\ \mathrm{v}_{j,k}$$

such that $|\ \tau_{j,k}\ | \leq \mu\lambda_{j,k}$

DEM with global computation of friction effects

Constrained optimisation problem *Stewart 2000, Moreau 2003*



Term from frictionless model (normal forces)

$$\max_{\mathbf{u},\ \boldsymbol{\lambda},\ \boldsymbol{\tau}} \mathcal{F}(\ \mathbf{u},\ \boldsymbol{\lambda}\ ) - \boldsymbol{\tau} \cdot \mathbf{v}_{\mathrm{t}}(\ \mathbf{u}\ )$$

Global dissipated power of tangential forces

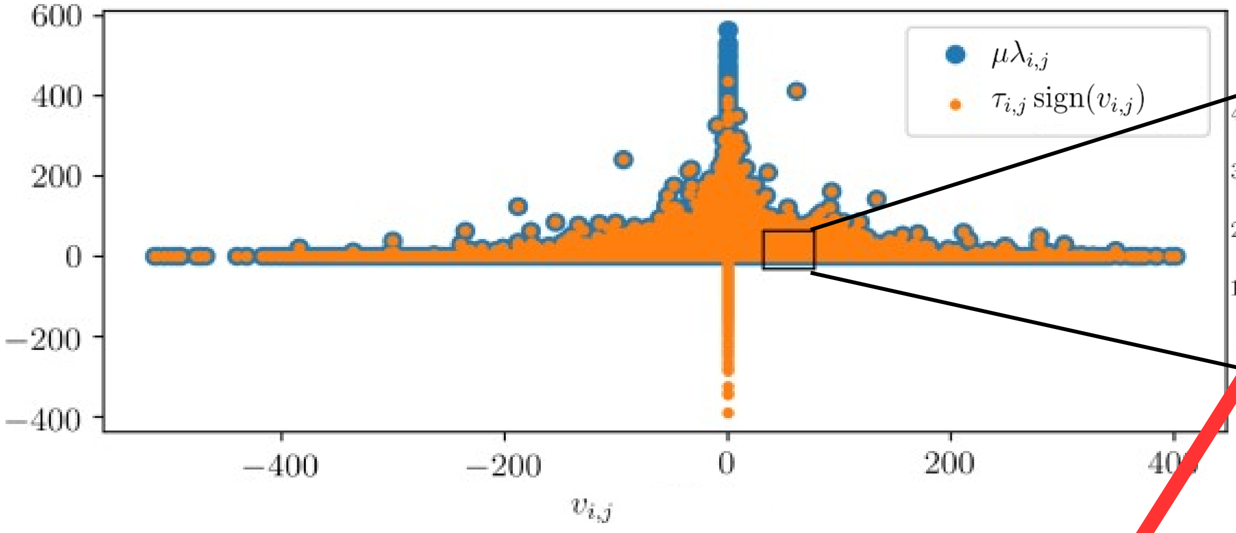
such that Newton's laws are verified and :

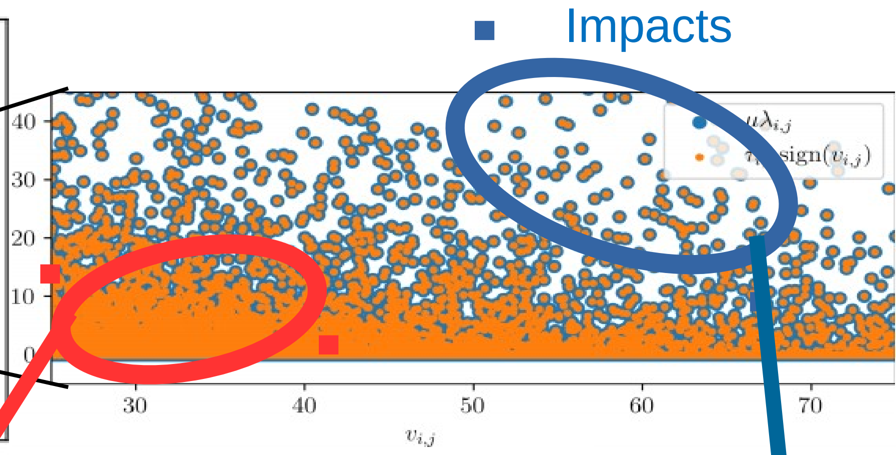
$$|\ \boldsymbol{\tau}\ | \leq \mu \boldsymbol{\lambda}$$ Coulomb's friction law

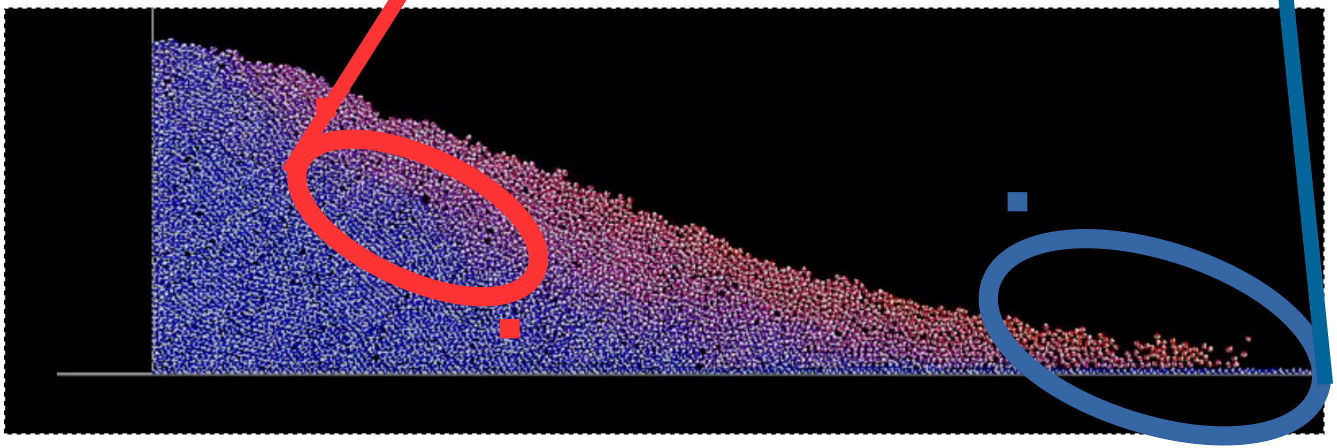
$$\boldsymbol{\lambda} \geq 0$$ Repulsive normal forces

- Convex functional (numerically solved MOSEK)
- Global computation of contact forces
  stable implicit-scheme
- No iteration on the contact network !

Local Coulomb's law







Coulomb's law :

$$v_{i,j} \neq 0 \quad \Rightarrow \quad |\tau_{i,j}| = \mu\lambda_{i,j}$$

2D Dam break over horizontal rigid bed

Continuum model rheology: $\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0 + I} I$

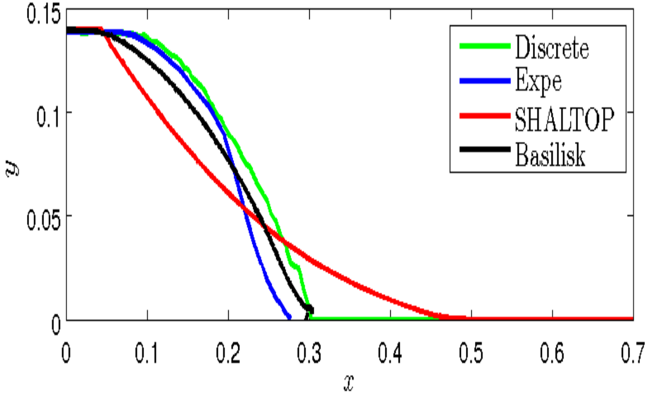
$\mu_s = 0.48,\ \Delta\mu = 0.25$ and $I_0 = 0.279$

Martin et al., 2017

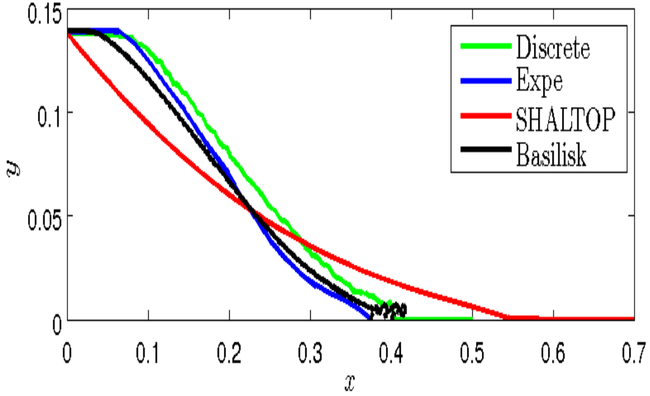
Discrete Element Method

Grain-grain friction coefficient: $\mu = 0.9$

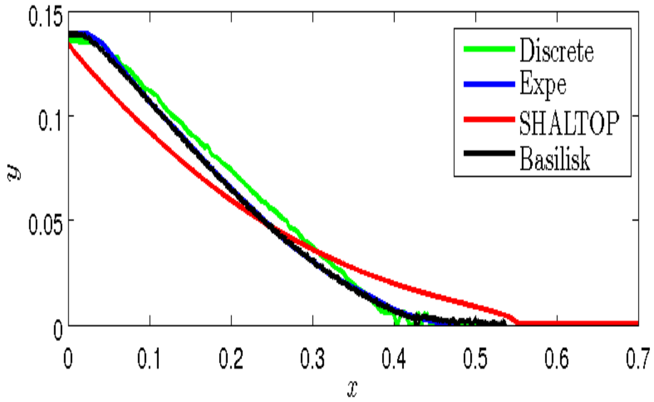
t = 0.18 s



t = 0.30 s



t = 1.02 s

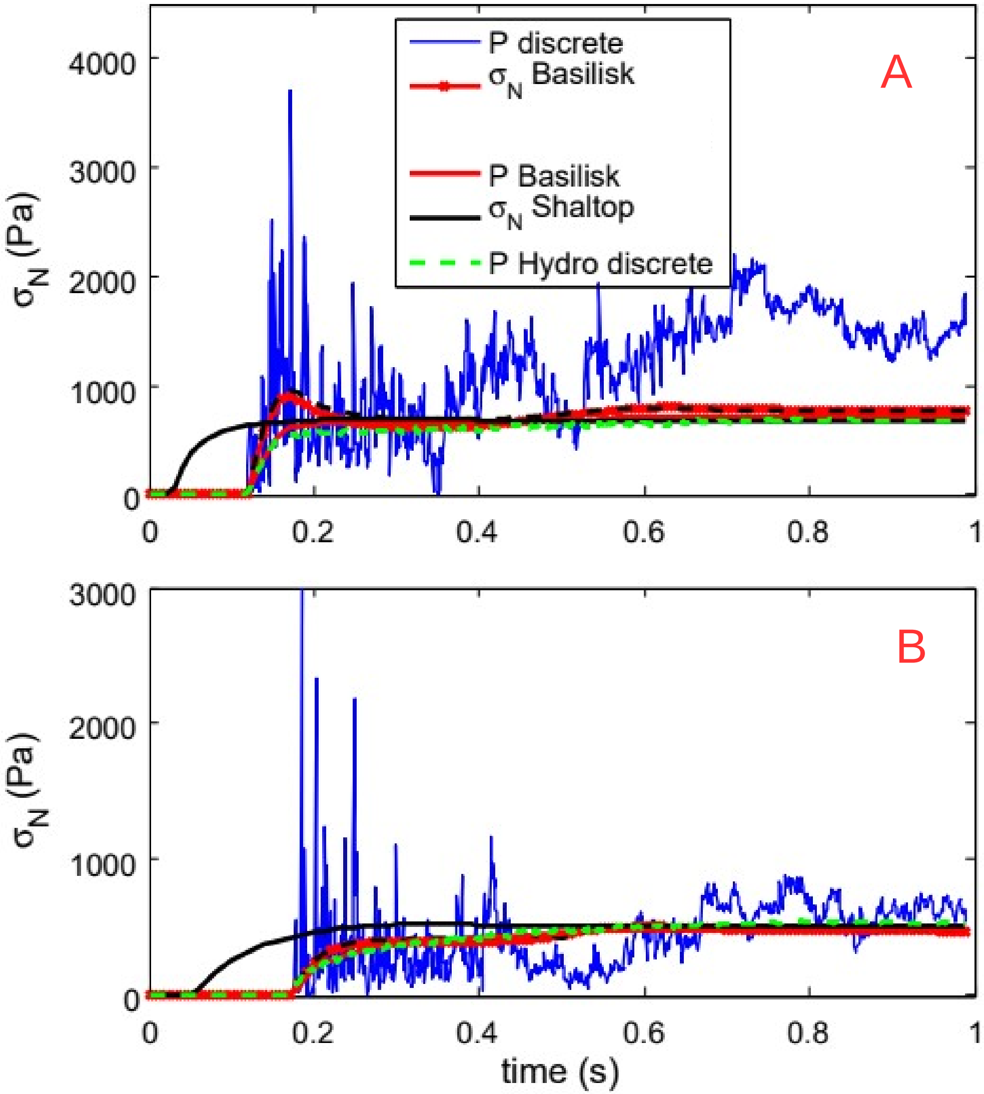


3D discrete method is in good agreements with experiments data when $\mu = 0.3$

Previous studies used additional dissipation (e. g. rolling friction) *Girolami et al. 2012*

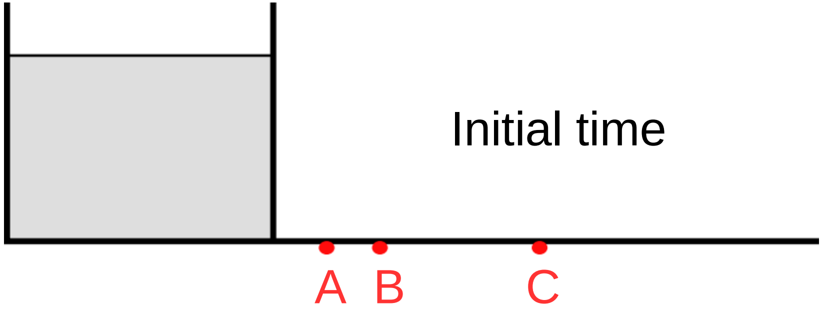
Viroulet et al., 2019

Basal forces



N = 65 909 discs
Computation time : 47h 31mn

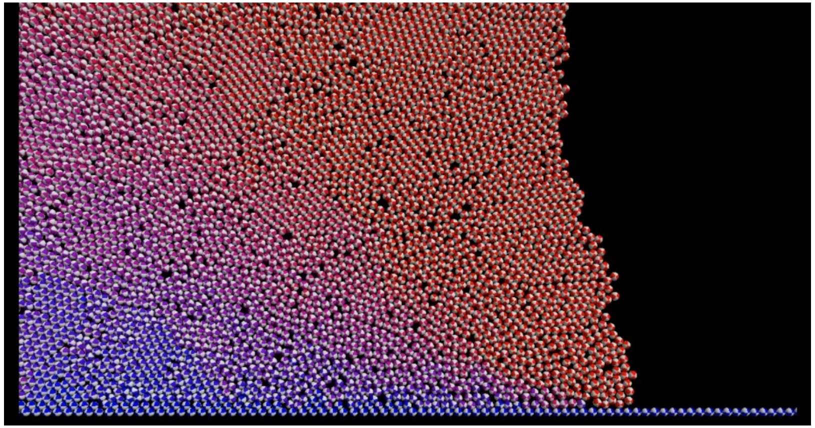
2 Intel Xeon E5-2650 2.00 GHz (2x8 cores)



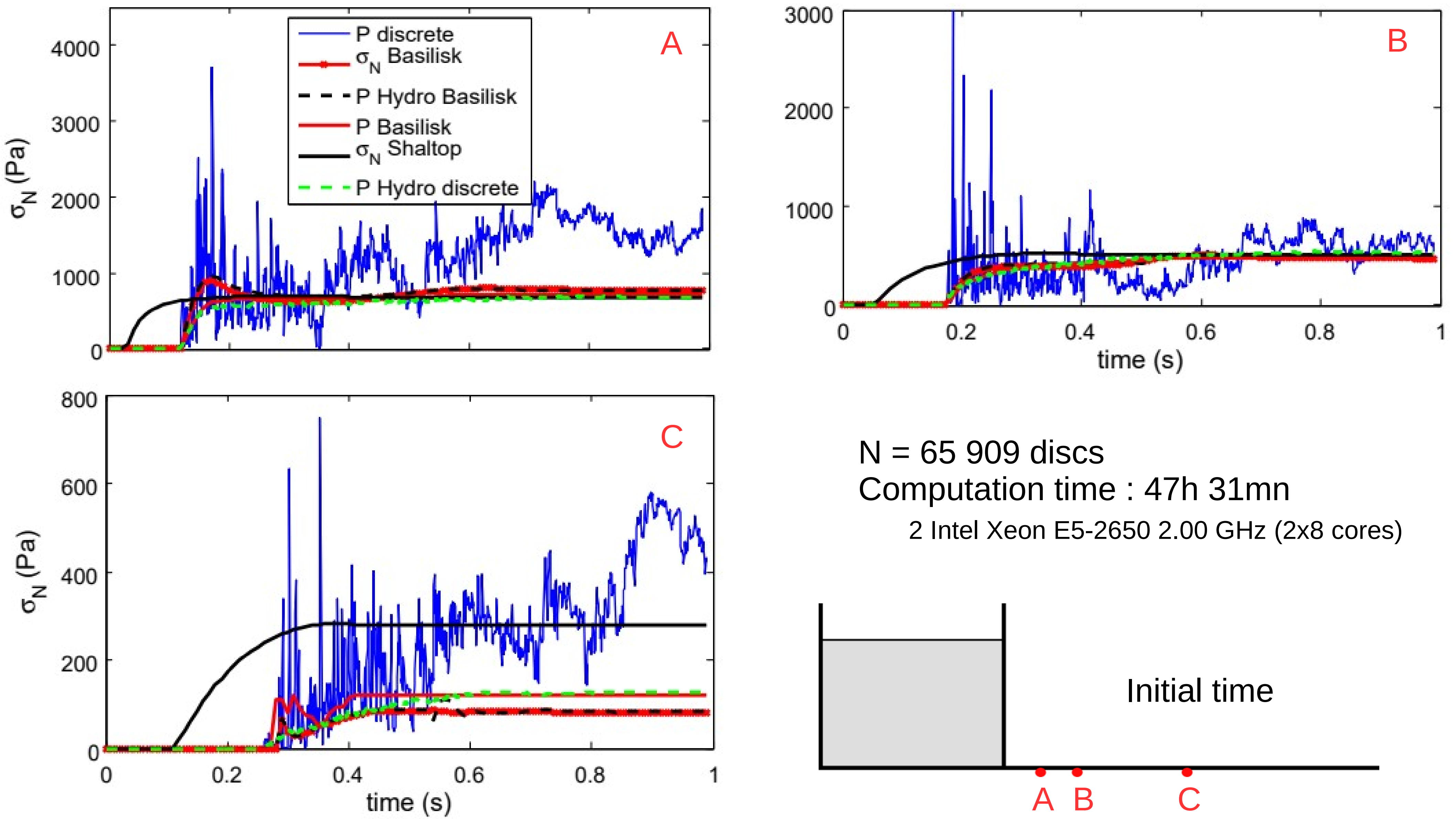
Strong fluctuations and spatial variability of the basal forces calculated from Discrete Element Methods !

Conclusion & Perspective

- **Global Model** for the Dry Friction Problem in Contact Dynamics codes
- **Numerical stability** and possibility of **large time steps** due to an implicit scheme
- Going further in **quantitative comparison** between the different models and experiments
- In the future : Understand and quantify the **physical origin of basal forces fluctuations**



erc ***Slidequakes***



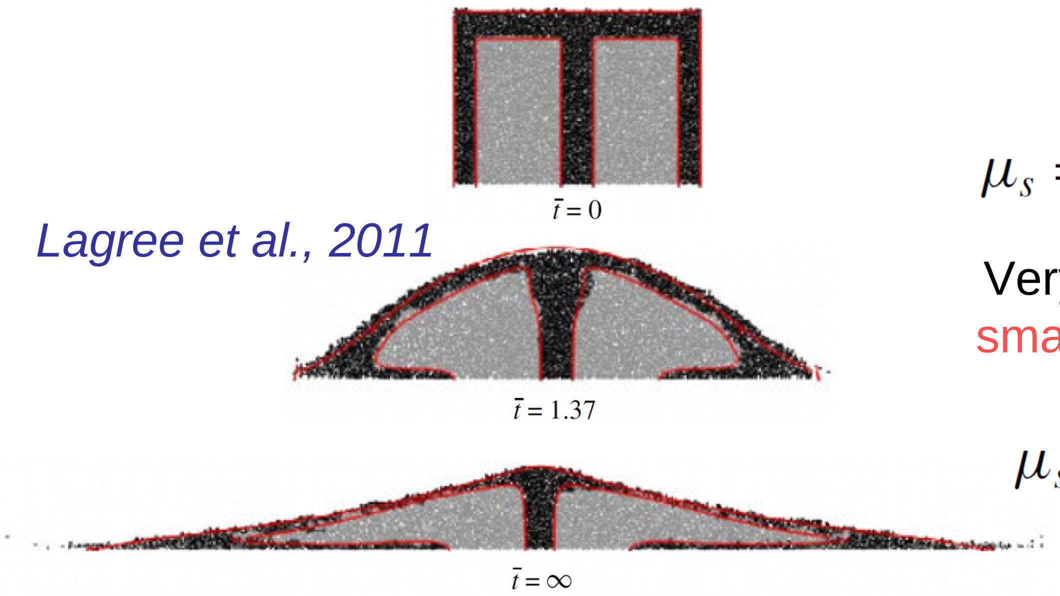
A
B
C
P discrete
σN Basilisk
P Hydro Basilisk
P Basilisk
σN Shaltop
P Hydro discrete
σN (Pa)
time (s)
N = 65 909 discs
Computation time : 47h 31mn
2 Intel Xeon E5-2650 2.00 GHz (2x8 cores)
Initial time

Quantitative comparison with other models and experiments

Comparaison **Contact Dynamics** and **Navier-Stokes** simulations

$\mu = 0.5$ and $e = 0.5$

Lagree et al., 2011



$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0 + I} I$$

$\mu_s = 0.32,\ \Delta\mu = 0.28$ and $I_0 = 0.4$

Very good agreement BUT with parameters smaller than those measured experimentally

$\mu_s = 0.38,\ \Delta\mu = 0.26$ and $I_0 = 0.279$

Jop et al., 2005

DEM goes further than experiments if no additionnal dissipation is accounted for !

Molecular Dynamics & Contact Dynamics

$$m_i \frac{\mathrm{d}\ \mathbf{u}_i}{\mathrm{d}\ t} = m_i\ \mathbf{g} + \sum_{j=i}^{N} \mathbf{f}_{i,j}$$

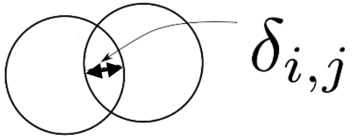
volume forces

contact forces

- **Molecular Dynamics**

Contact forces are functions

Hertz's contact law



normal stiffness

$$\mathbf{f}_{i,j} \cdot \mathbf{n}_{i,j} = \kappa\ \delta_{i,j}^{\frac{3}{2}}$$

unit normal vector

numerical overlap

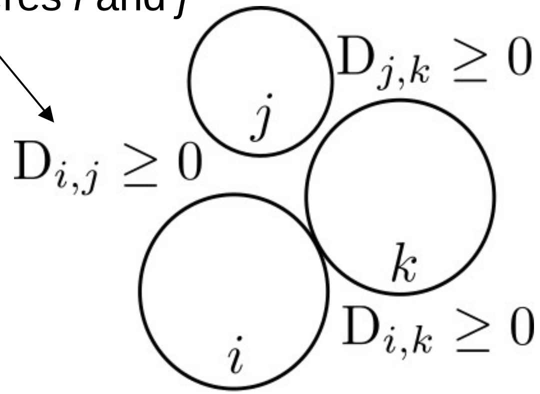
- **Contact Dynamics**

Constrained optimization problem

Contact forces verify contact laws

Find contact forces such that beads do not overlap…

Distances between spheres *i* and *j*



Larger time steps

Molecular Dynamics & Contact Dynamics

$$m_i \frac{\mathrm{d}\,\mathbf{u}_i}{\mathrm{d}\,t} = m_i\,\mathbf{g} + \sum_{j=i}^{N} \mathbf{f}_{i,j}$$

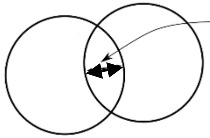
volume forces

contact forces

- **Molecular Dynamics**

Contact forces are functions

Hertz's contact law



$\delta_{i,j}$

normal stiffness

$$\mathbf{f}_{i,j} \cdot \mathbf{n}_{i,j} = \kappa\ \delta_{i,j}^{\frac{3}{2}}$$

unit normal vector

numerical overlap

- **Contact Dynamics**

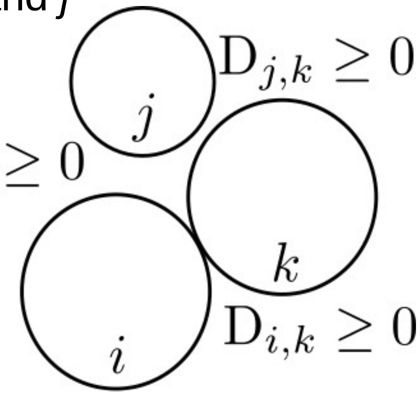
Constrained optimization problem

Contact forces verify contact laws

Find contact forces such that beads do not overlap…

Distances between spheres *i* and *j*

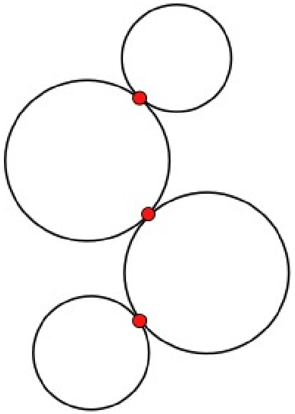
$\mathrm{D}_{i,j} \geq 0$



Larger time steps

DEM with global computation of friction effects

Constrained optimisation problem *Stewart 2000, Moreau 2003*



Term from frictionless model (normal forces)

$$\max_{\mathbf{u},\ \boldsymbol{\lambda},\ \boldsymbol{\tau}} \mathcal{F}(\ \mathbf{u},\ \boldsymbol{\lambda}\ ) - \boldsymbol{\tau} \cdot \mathbf{v}_{\mathrm{t}}(\ \mathbf{u}\ )$$

Global dissipated power of tangential forces

such that Newton's laws are verified and :

$| \boldsymbol{\tau} | \leq \mu \boldsymbol{\lambda}$ Coulomb's friction law

$\boldsymbol{\lambda} \geq 0$ Repulsive normal forces

- Convex functional (numerically solved MOSEK)
- Global computation of contact forces
  stable implicit-scheme
- No iteration on the contact network !

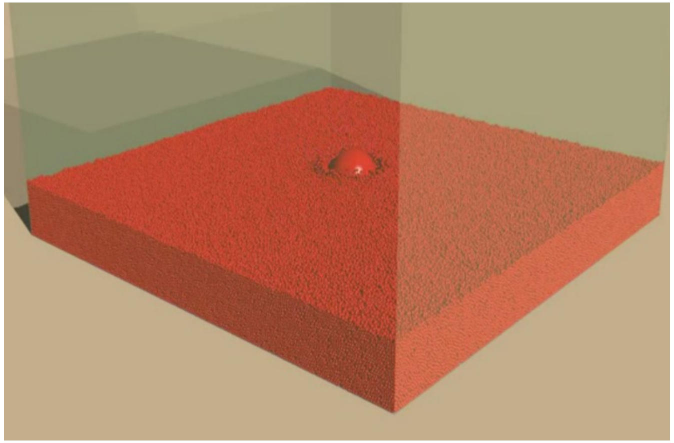
- The choice of the functional implies the prevention of overlaps:

$\mathrm{D}_{i,j}^{n+1} \geq 0$ is direclty obtained as an optimality condition

Global friction effects in Contact Dynamics

Objective : Numerical scheme dealing with friction that can handle **large time-step values**

Add friction forces to the global frictionless model SCoPI

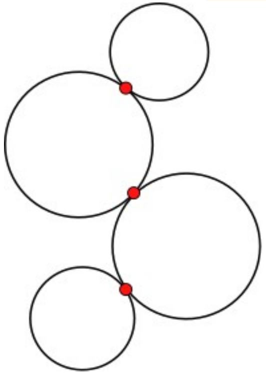


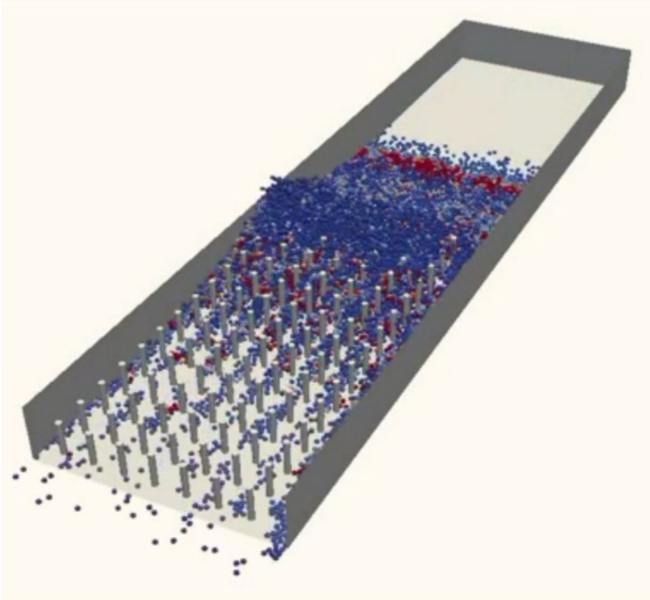
1 000 000 spheres
S. Faure, A. Lefebvre-Lepot

- Global computation of the contact forces at the same time: no iteration

Implicit scheme

Large time steps





40 000 spheres
S. Faure, A. Lefebvre-Lepot

Maury 2006, Lefevre 2009