GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

EGU General Assembly 2020

Partitioning of preferential flows in fracture networks:

Smoothed Particle Dynamics simulations and analytical modeling of infiltration dynamics

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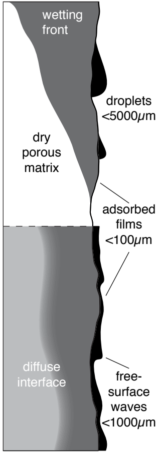
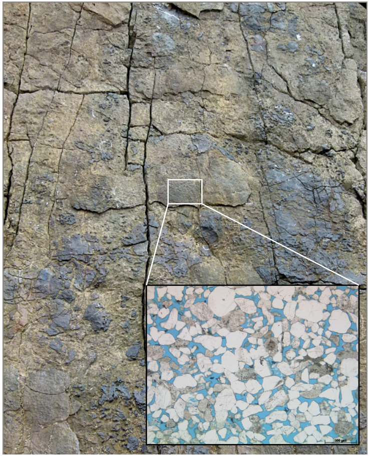
Preferential flow in porous-fractured media

Fracture scales: Overview

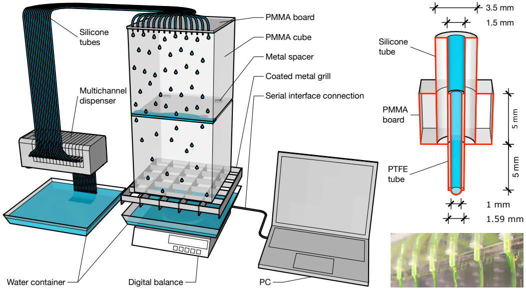




- Scales: mm vs. μm
- Breakdown of classical volume-effective approaches
- Challenging: Rapid and erratic flow dynamics
- Flowrate dependent flow modes: Droplets $\rightarrow$ rivulets $\rightarrow$ films
- Common force balance:
  viscous $<$ inertial $<$ capillary
- Apertures above $\sim$0.8mm: Transition into inertial regime (Wood et al 2005, WRR)

What is happening at fracture intersections?

Analogue percolation experiments



Laboratory setup (Kordilla et al. 2017, Noffz et al. 2018)

- Total flow rate $Q_0 = 15\,\mathrm{ml/min}$
- Flow regimes: Droplet flow ($15 \times 1\,\mathrm{ml/min}$) and rivulet flow ($3 \times 5\,\mathrm{ml/min}$)
- Cascade of cubes: $20\,\mathrm{cm} \times 20\,\mathrm{cm} \times 20\,\mathrm{cm}$
- Aperture width $d_f = 1\,\mathrm{mm}$ and $2.5\,\mathrm{mm}$
- Static contact angle $\theta_0 \approx 65^\circ$

Analytical model

Fracture inflow: $$Q_f(t) \equiv \frac{dM_f(t)}{dt} = Q_0 - \frac{dM_b(t)}{dt} \tag{1}$$

Washburn: $$\frac{dl(t)}{dt} = \frac{c_f}{l(t)} \qquad c_f = \frac{\Delta P_c}{\mu}\frac{d_f^2}{4} \tag{2}$$

Parallel plate : $$\Delta P = \frac{2\sigma cos(\theta)}{d_f} \tag{3}$$

$$l(t = t_0) = l_0 \qquad l(t) = \sqrt{l_0^2 + 2c_f(t - t_0)}. \tag{4}$$

$$M_f(t) = A_f l(t) \qquad Q_f(t) = A_f \frac{dl(t)}{dt} = \frac{Q_0}{\sqrt{1 + 2k_f(t - t_0)}} \tag{5}$$

$$k_f = c_f / l_0^2 \tag{6}$$

Regime transitions

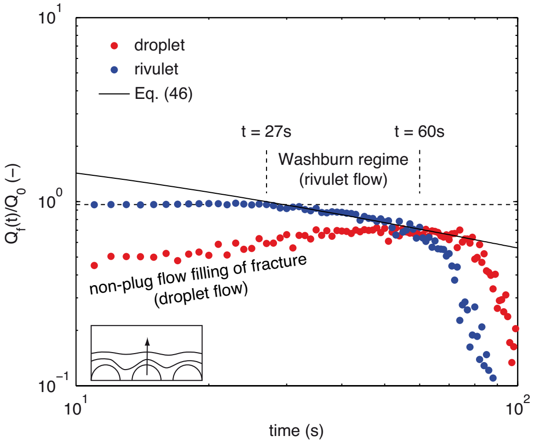


Figure 1: Kordilla et al. (2017)

- Both boundary conditions: Recovery of classical Washburn $t^{-1/2}$ scaling
- Transition times depend on intersection partitioning dynamics
- Rivulet flow: Initially rapid plug flow filling of horizontal fracture, then transition into Washburn regime
- Droplet flow: Individual droplets bypass until fluid front is closed. Transition into Washburn regime occurs later

What about “upscaling”?

Transfer-function

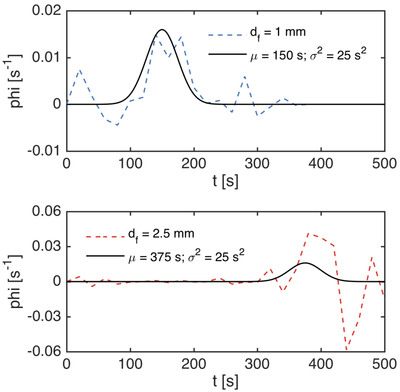


Figure 2: Transfer function φ vs. time (Noffz et al. 2018).

Output signal: $$Q_{nf}(t) = \int_0^t \varphi_{nf}(t-t')Q_{nf-1}(t')$$

Transfer-function: $$\varphi(t) = \frac{dQ_1(t)}{dt} = -\frac{dQ_f(t)}{dt}$$

Gaussian: $$\varphi(t) \propto \frac{exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}}$$

$$\int_0^\infty dt\varphi(t) = 1$$

Inflow: $$Q_{f,nf}(t) = Q_0\left[1 - \int_0^t dt_{nf-1}\varphi(t-t_{nf-1})\ldots\right.$$

$$\left.\int_0^{t_3} dt_2\varphi(t_3-t_2)\int_0^{t_2} dt_1\varphi(t_1)\right]$$

Transfer-function

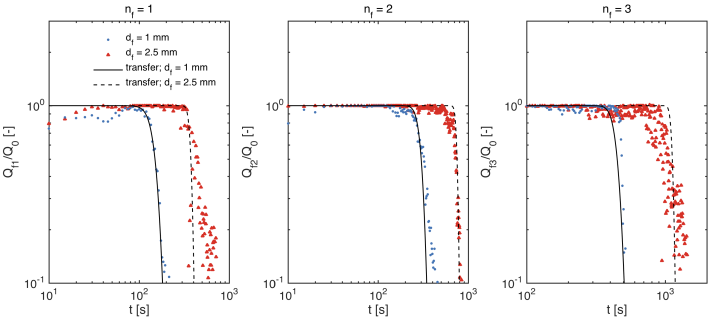


Figure 3: Normalized fracture inflow rate vs. time (Noffz et al. 2018).

- Predictive modeling of unsaturated flow dynamics during rivulet flow by Gaussian transfer-function
- Better recovery of tailing requires better process understanding

Can we parameterize $\varphi(\ldots)$ to model the outflow?

Process-based transfer function

$$\varphi_{pw}(t) = \frac{1}{Q_0}\frac{dQ}{dt} = \delta(t_c - t) - \frac{W(t)a}{Q_0}\left[\delta(t - t_{max}) - \delta(t - t_c)\right] + \frac{W'(t)a}{Q_0}\left[\mathcal{H}(t - t_{max}) - \mathcal{H}(t - t_c)\right]$$

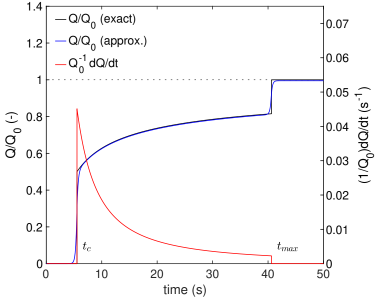


Figure 4: Normalized outflow rate and transfer function $\varphi_{pw}(t) = Q_0^{-1}dQ/dt$ (Kordilla et al. 2020, under revision).

- Process-basdd normalized outflow via an analytical approach
- Transfer function takes into account the switch from plug-flow to Washburn-type flow at a critical time t_c
- W is a Washburn-type penetration function applied to a horizontal fracture with aperture a

Process-based transfer function

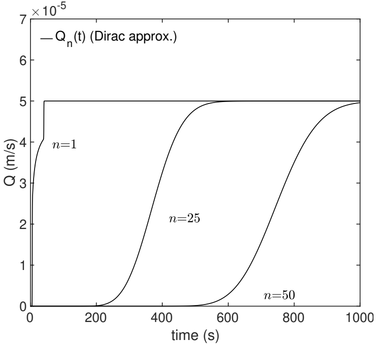


Figure 5: Flow through a system of $n = 1$, 25 and 50 fractures and $L_{max} = 0.3\,\mathrm{m}$ where $t_{max} > t_c$ (Kordilla et al. 2020, under revision).

- Modeling of outflow through a system of n fracture intersections

$$Q_n(t) = \int_0^t Q_{n-1}(t')\varphi_{pw,p}(t - t')dt' \qquad (7)$$

- Simple yet effective analytical approach
- Applicable to systems with low matrix porosity and/or short time-scales
- Effect of porous matrix imbibition not yet included

What about the porous matrix?

Multiscale SPH

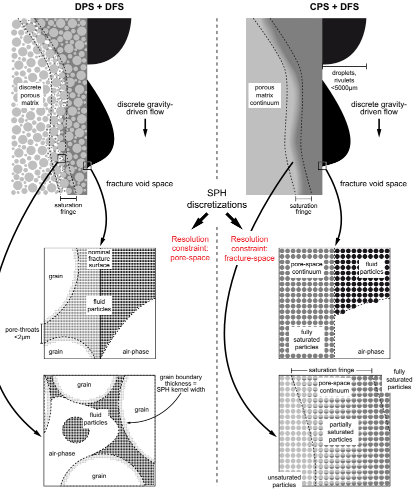


Figure 6: Multiscale SPH coupling scheme (Kordilla 2017)

Governing Equations

Navier-Stokes equation: $\dfrac{d\mathbf{v}}{dt} = -\dfrac{1}{\rho}\nabla P + \dfrac{\mu}{\rho}\nabla^2\mathbf{v} + \mathbf{g}$

Richards equation: $\dfrac{\partial\Theta(\psi)}{\partial t} = (C_m + \rho\mathbf{g}SeS_s)\dfrac{\partial\psi}{\partial t} = \nabla\cdot\mathbf{K}_s k_r(\psi)\nabla\psi + \dfrac{\partial K(\psi)}{\partial z}$

Van Genuchten Parameters:

$$\left.\begin{aligned} Se &= \tfrac{1}{\left[1+|\alpha\psi^n|\right]^m} \\ \Theta &= \Theta_r + Se(\Theta_s - \Theta_r) \\ k_r &= Se^l\left[1-\left(1-Se^{\frac{1}{m}}\right)^m\right]^2 \\ C_m &= \tfrac{\alpha m}{1-m}(\Theta_s-\Theta_r)Se^{\frac{1}{m}}\left(1-Se^{\frac{1}{m}}\right)^m \end{aligned}\right\}\text{if } \psi < 0 \qquad \left.\begin{aligned} Se &= 1.0 \\ \Theta &= \Theta_s \\ k_r &= 1.0 \\ C_m &= 0.0 \end{aligned}\right\}\text{if } \psi \geq 0$$

SPH discretization

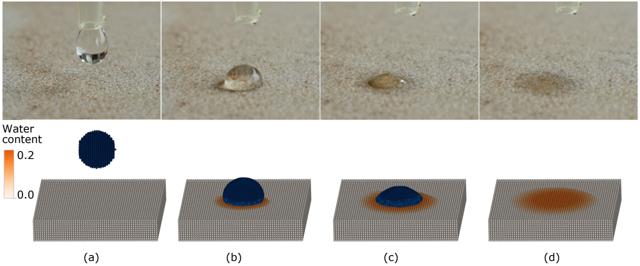
SPH - discretization of Navier-Stokes equation:

$$\frac{d\mathbf{v}_i}{dt} = -\sum_{j=1}^{N} m_j \left(\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}\right)\frac{\mathbf{r}_{ij}}{r_{ij}} \cdot \frac{dW(r_{ij},h)}{dr_{ij}} + 2\mu \sum_{j=1}^{N} m_j \frac{\mathbf{v}_{ij}}{\rho_i \rho_j r_{ij}} \cdot \frac{dW(r_{ij},h)}{dr_{ij}} + \mathbf{g} + \frac{1}{m_i}\sum_{j=1}^{N} s_{ij}(A_{ij}\tilde{W}(r_{ij},\frac{h}{2})\frac{\mathbf{r}_{ij}}{r_{ij}} - \tilde{W}(r_{ij},h)\frac{\mathbf{r}_{ij}}{r_{ij}})$$

SPH - discretization of Richards equation:

$$\frac{d\Theta_i}{dt} = (C_{m_i} + \rho_i \mathbf{g} Se_i S_i)\frac{d\psi_i}{dt} = \sum_{j=1}^{N} 2\frac{m_i m_j}{m_i + m_j}\frac{\rho_i + \rho_j}{\rho_i \rho_j} \cdot \mathbf{K}_s k_{r_i}(d\psi_{ij} + dz_{ij}) \cdot \frac{dW(r_{ij},h)}{dr_{ij}}$$

Droplet infiltration



The experimental (top) and simulation (bottom) results of droplet imbibition at different times: (a) $t_0 = -0.004\,\mathrm{s}$, (b) $t_1 = 0.396\,\mathrm{s}$, (c) $t_2 = 1.836\,\mathrm{s}$, and (d) $t_3 = 2.676\,\mathrm{s}$ (Shigorina et al. 2020, under revision).

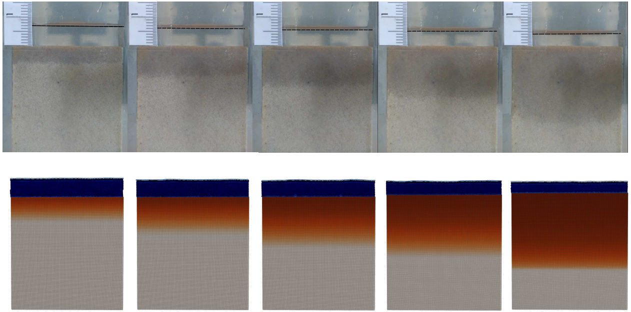
- Saturation of solid by water particles (no penetration)
- Water particles are removed if their virtual saturation falls below critical threshold
- Mass conservation:

$$\frac{\partial \Theta}{\partial t} = \nabla \cdot \left(\sum q_{in} - \sum q_{out}\right) = 0$$

- Total water content:

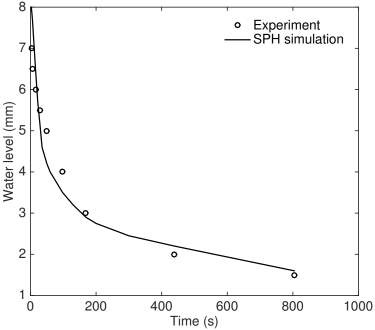
$$\Theta_{total} = const$$

Validation experiment



(a) (b) (c) (d) (e)

Experimental (top) and simulation (bottom) results of infiltration into a sandstone: (a) $t_1 = 3\,\mathrm{s}$; (b) $t_2 = 16\,\mathrm{s}$; (c) $t_3 = 30\,\mathrm{s}$; (d) $t_4 = 50\,\mathrm{s}$; (e) $t_5 = 100\,\mathrm{s}$ (Shigorina et al. 2020, under revision).



Water level height above sandstone

What about reality?

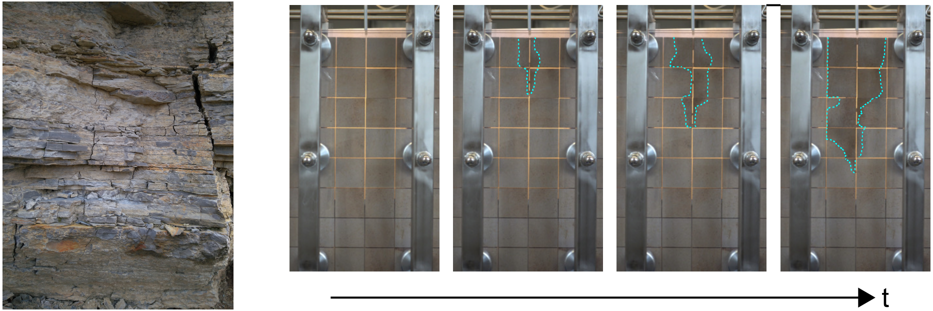


Figure 7: Flow through fracture networks and well-known topology/geometry (Rüdiger et al. 2020, under revision).

- Effect of adjacent porous matrix
- Connection between fracture topology/geometry, matrix properties and infiltration dynamics
- Relation to field site experiments?
- Effects of dimension reduction, 2D vs. 3D?

Conclusion

Conclusion

- Fracture networks provide rapid bypass capacities
- Fracture intersections are critical relay points for rapid infiltration
- Transfer function can be obtained/enhanced via small-scale process analysis
- The porous matrix plays a crucial role in the redistribution and large-scale dispersion dynamics
- Field experiments and analogue fracture setups are required to cross-validate findings and explore the quality of analytical abstraction

Stay healthy and see you next year!

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Questions? Drop me a mail or join our live chat:

Tuesday, 5 May 2020, 08:30–10:15.

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