Revisiting the statistical scaling of peak annual discharges with respect to the basin size in the light of rainfall climatology

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1. Introduction

Over the years, several studies have been carried out to investigate how the statistics of annual discharge maxima vary with the size of basins, with diverse findings regarding the observed type of scaling, especially in cases where the data originated from regions with significantly different hydroclimatic characteristics. In this context, an important question arises on how one can effectively conclude on an approximate type of statistical scaling of the annual discharge maxima, while accounting for the effects of local rainfall climatology.

In this work, we aim at answering this question, by investigating the effect of rainfall climatology on the spatial scaling of maximum annual floods using daily discharge data from 805 stations located in different parts of the United Kingdom. To do so, we isolate the effects of the catchment area and the local rainfall climatology, and examine how the statistics of the standardized discharge maxima vary with the basin scale.

2. Scaling of peak annual discharges with area

A. Statistical scale invariance

An attractive approach to model the statistical characteristics of maximum annual discharges at ungauged locations of a river network, is to use **scale invariance arguments** to link them to the statistical properties of maximum annual discharges measured at gauged locations within the same **statistically homogeneous** geographical region. In this case, one can obtain the statistical properties at ungauged locations *j*, as a function of those at gauged locations *i*:

$$Q^{(j)}_{max}(A_j) =^{d} G(A_i/A_j)\, Q^{(i)}_{max}(A_i) \qquad (1)$$

where ($=^{d}$) denotes equality in all finite dimensional distributions, and G is a random function that depends on the ratio $r = A_i / A_j$ and is stochastically independent from $Q^{(i)}_{max}(A_i)$. **Equation (1)** includes simple-scaling as a sub-case when G is deterministic. Also, it follows from equation (1) that the moments $E[\{Q_{max}(A)\}^q]$ of different orders q depend on the catchment size A in a log-linear way:

$$E[\{Q_{max}(A)\}^q] \propto A^{q-K(q)} \qquad (2)$$

where $K(q)$ is a non-linear (linear) function of q in the **multiscaling** (**simple-scaling**) case. **Equation (2)** is a necessary but not sufficient condition for stochastic self-similarity (or multifractality) to hold, as it describes the marginal statistics of Q_{max} as a function of scale A. More precisely it implies that:

$$Q^{(j)}_{max}(A_j) =^{md} G(A_i/A_j)\, Q^{(i)}_{max}(A_i) \qquad (3)$$

where ($=^{md}$) denotes equality in the marginal distributions of Q_{max} (i.e., obtained across independent basins of various areas A), whereas equation (1) refers to all finite dimensional distributions of the maxima field (i.e., obtained across all sub-catchments of various areas A of a basin).

Based on equation (3), a convenient way to distinguish between multiscaling and simple scaling of annual discharge maxima, is by studying the form of the **moment scaling function $K(q)$**, defined as:

$$K(q) = -\log_A E[\{Q'_{max}(A)\}^q] \quad \text{where} \quad Q'_{max}(A) = Q_{max}(A) / A \qquad (4)$$

B. Index Flood Method and local rainfall climatology

River discharges for rainfall-triggered flood events increase almost proportionally with both the **drainage area *A*** and the **rainfall depth *I***. Hence, unless all catchments fall within the same hydrologically homogeneous geographical region (i.e., in terms of rainfall accumulations), regressing Q_{max} solely against A cannot resolve the variability induced by the different hydroclimatologies and, more importantly, it may produce biased results in favor of simple scaling.

According to the main assumption of the **index flood method**, the discharge maxima at different locations within a statistically homogeneous geographical region exhibit the same probability distribution when standardized by their mean, or some other index discharge. For example, considering that the mean discharge $E[Q^{(j)}]$ at some location j increases almost proportionally with the annual rainfall depth, thus, being indicative of the local rainfall climatology, the index flood method can be written in the form:

$$\frac{Q^{(j)}_{max}}{E[Q^{(j)}]} =^{md} \frac{Q^{(i)}_{max}}{E[Q^{(i)}]} \qquad (5)$$

Assuming proportionality of discharges with rainfall intensity and some power $\theta \in (0,1]$ of the drainage area (i.e., generalized rational method), if the mean annual rainfall intensity is assumed constant over the region of interest, then the assumption: $E[Q^{(j)}]/E[Q^{(i)}] = (A_j/A_i)^{\theta}$ holds in good approximation, and the index flood approach in equation (5) reduces to a **simple-scaling rule** of annual discharge maxima with the drainage area A.

$$Q^{(j)}_{max} =^{md} \left(\frac{A_j}{A_i}\right)^{\theta} Q^{(i)}_{max} \qquad (6)$$

Evidently, in the most general case when locations *i*, *j* exhibit different hydroclimatic characteristics, as in the case when investigating data originating from different regions, the index flood method may lead to more complex types of scaling than the simple scaling rule.

It follows from the discussion above, that from a theoretical point of view, any type of moment scaling analysis of Q_{max} with A cannot be conclusive regarding the actual type of scaling of Q_{max}. This is because when regressing the moments of Q_{max} against A, one a priori assumes a constant mean rainfall intensity field over the region of interest and, consequently, a simple scaling rule for the annual discharge maxima.

3. Key idea and Data

- The main scope of the study is to investigate the effect of rainfall climatology on the spatial scaling of maximum annual floods using daily discharge data from 805 stations located in different parts of the United Kingdom; see Perdios and Langousis (2020).

This was done by studying:

- How the mean value of the standardized discharge maxima $Q'_{max} = Q_{max}/A$ depends on the catchment area A and the average precipitation in 30-year climatic periods, hereafter referred to as **SAAR** (Standard-period Average Annual Rainfall).
- How the distribution of the ratio $Q_{max}/E[Q]$, also referred to as **index flood ratio** or **amplification factor**, scales with the area A, along the lines of the index flood concept.

The **hydrologic information** used originates from NRFA (National River Flow Archive) and includes:

- Daily discharges from 805 catchments with at least 30 years of recordings.
- Catchment size information.
- SAAR values in mm.

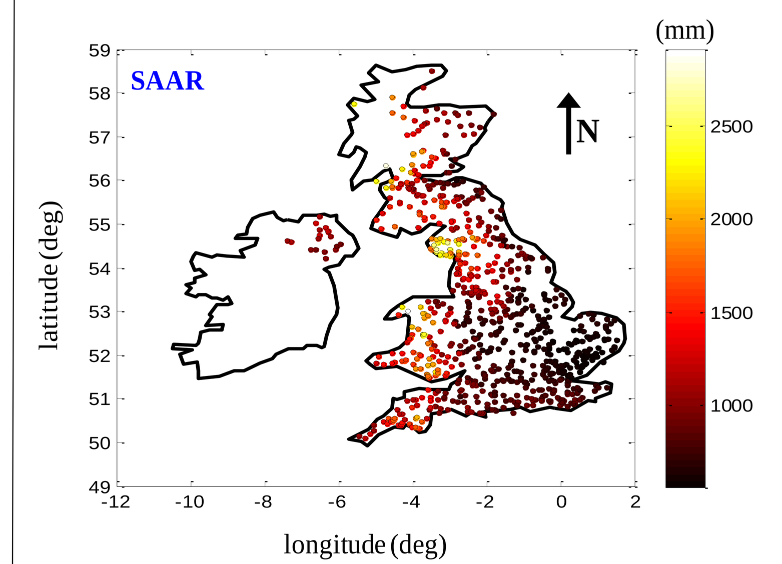

Figure 1: Spatial distribution of SAAR values for the 805 considered catchments across the United Kingdom.

- **SAAR** values tend to decrease when moving from the West to East coast
- The observed rainfall gradient is directly linked to Gulf Stream and the local topography.

4. Results – Conclusions

A. Mean value of standardized discharge maxima

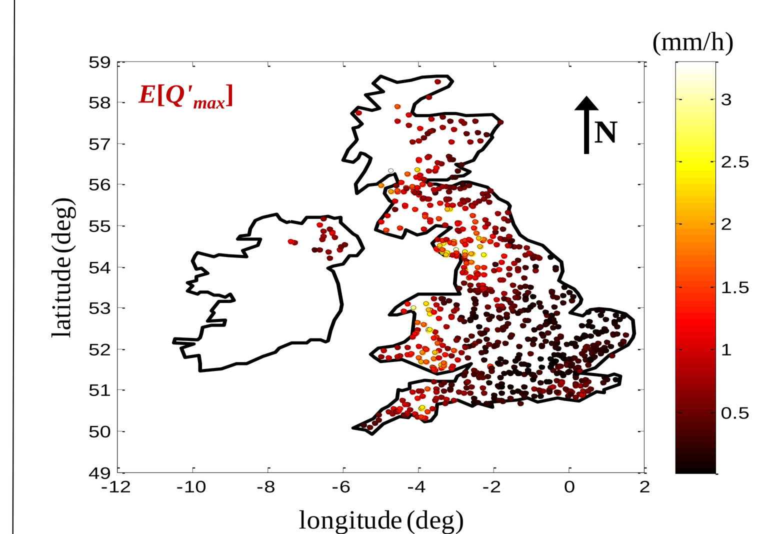

Figure 2: Spatial distribution of the mean value of the standardized discharge maxima.

- Same pattern as SAAR (see Figure 1).
- Indication of strong linkage to the local rainfall climatology.

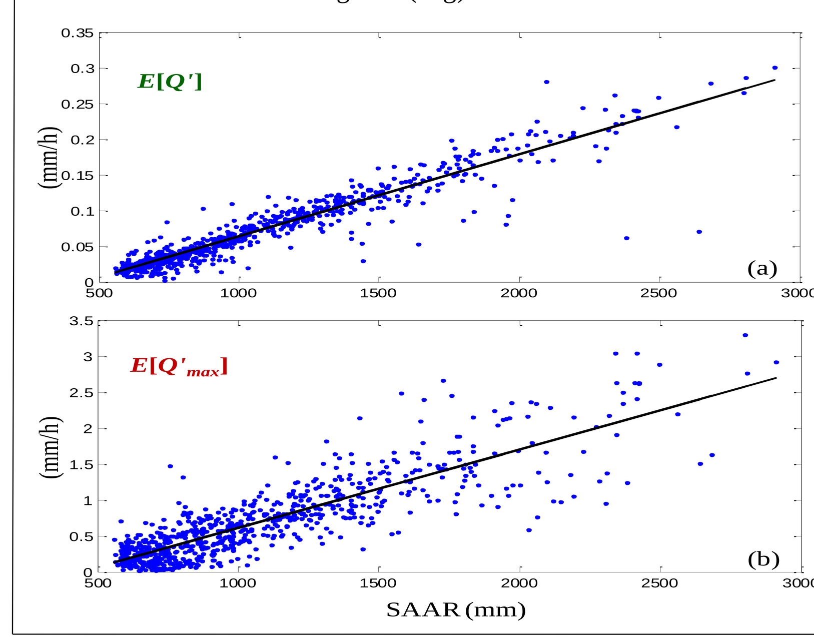

Figure 3: (a): Scatterplot (points) and linear least-squares (LS) fit (solid line) of the mean values $E[Q']$ of the standardized discharges $Q' = Q / A$ for the 805 considered catchments, with respect to their corresponding SAAR values. **(b)** Same as (a) but for the means $E[Q'_{max}]$ of the standardized annual discharge maxima $Q'_{max} = Q_{max} / A$.

Figures 3.a and **3.b** indicate that there is a strong statistical linkage between the mean values of standardized discharges and the local rainfall climatology. In addition, note that:

1. Standardization of Q and Q_{max} by the catchment area A is important for the effects of local rainfall climatology to be revealed, as river discharges increase with the basin size.
2. In the case when studying the scaling properties of annual discharge maxima, a simple way to simultaneously isolate the effects of the area of the basin and the local rainfall characteristics is to **follow the exact version of the index flood method** and study the distribution of the amplification factor $\gamma_{max} = Q_{max} / E[Q]$.

B. Scaling of standardized annual maxima and amplification factor.

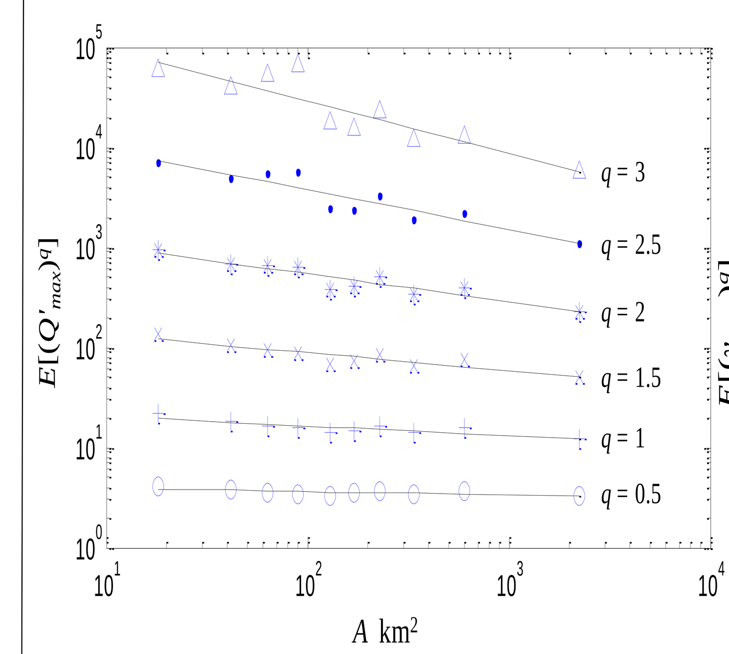

Figure 4: Log-log plots of the empirical moments $E[(Q'_{max})^q]$ as a function of the catchment area A, for different moment orders. Black solid lines correspond to least-squares (LS) fits.

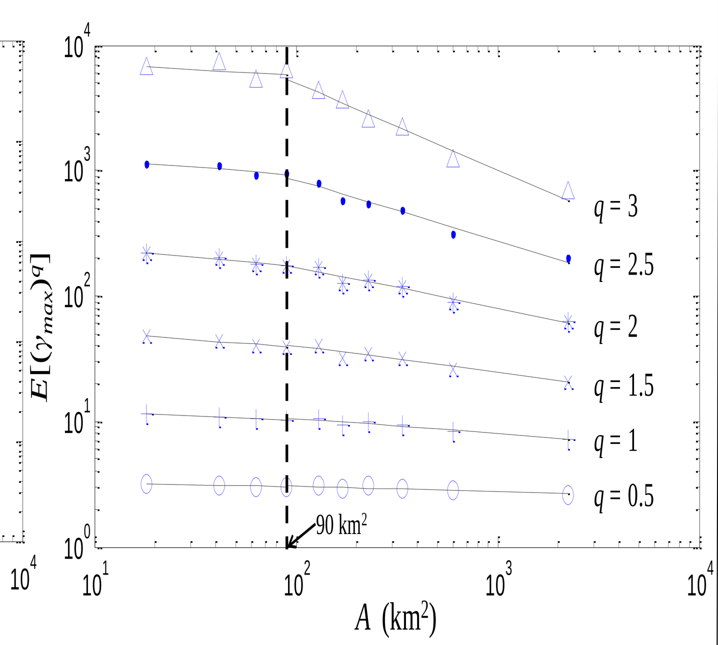

Figure 5: Log-log plots of the empirical moments $E[(\gamma_{max})^q]$ as a function of the catchment area A, for different moment orders. Black solid lines correspond to least-squares (LS) fits.

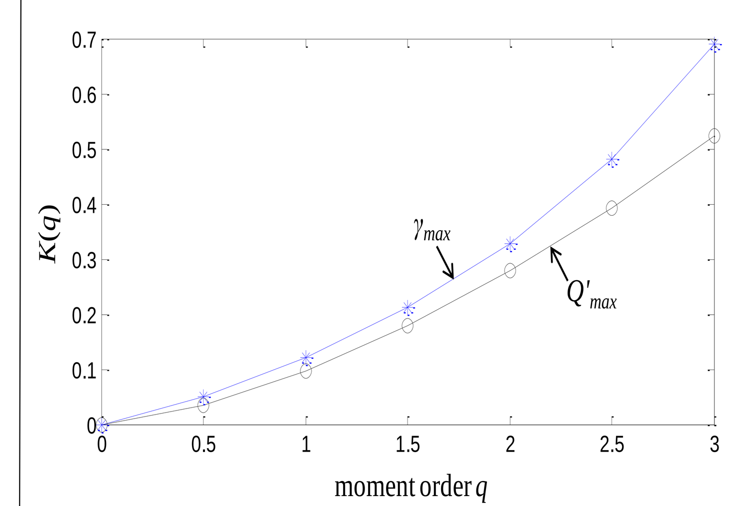

Figure 6: Empirical moment scaling function $K(q)$ for the standardized discharge maxima $Q'_{max} = Q_{max} / A$ (circles), and the amplification factor $\gamma_{max} = Q_{max} / E[Q]$ (stars).

Standardized discharge maxima Q'_{max}

- Log-linearity with the drainage area A.
- $K(q)$ function is **non-linear** → indication of significant deviations from simple scaling.
- Observed deviations from simple scaling can be attributed to the **multifractal** structure of actual rainfields.

Amplification factor γ_{max}

- **Break of log-linearity** for spatial scales below approximately 100 km² → concurs with the observed break of scaling in spatial rainfall
- $K(q)$ function remains non-linear, close to that of the standardized discharge maxima.

Reference

Perdios, A. and A. Langousis (2020) Revisiting the Statistical Scaling of Annual Discharge Maxima at Daily Resolution with Respect to the Basin Size in the Light of Rainfall Climatology, Water, 12(2), 610; https://doi.org/10.3390/w12020610