CLOUDY
with a chance of steaming bombs
Modelling Steaming Surtseyan Ejecta

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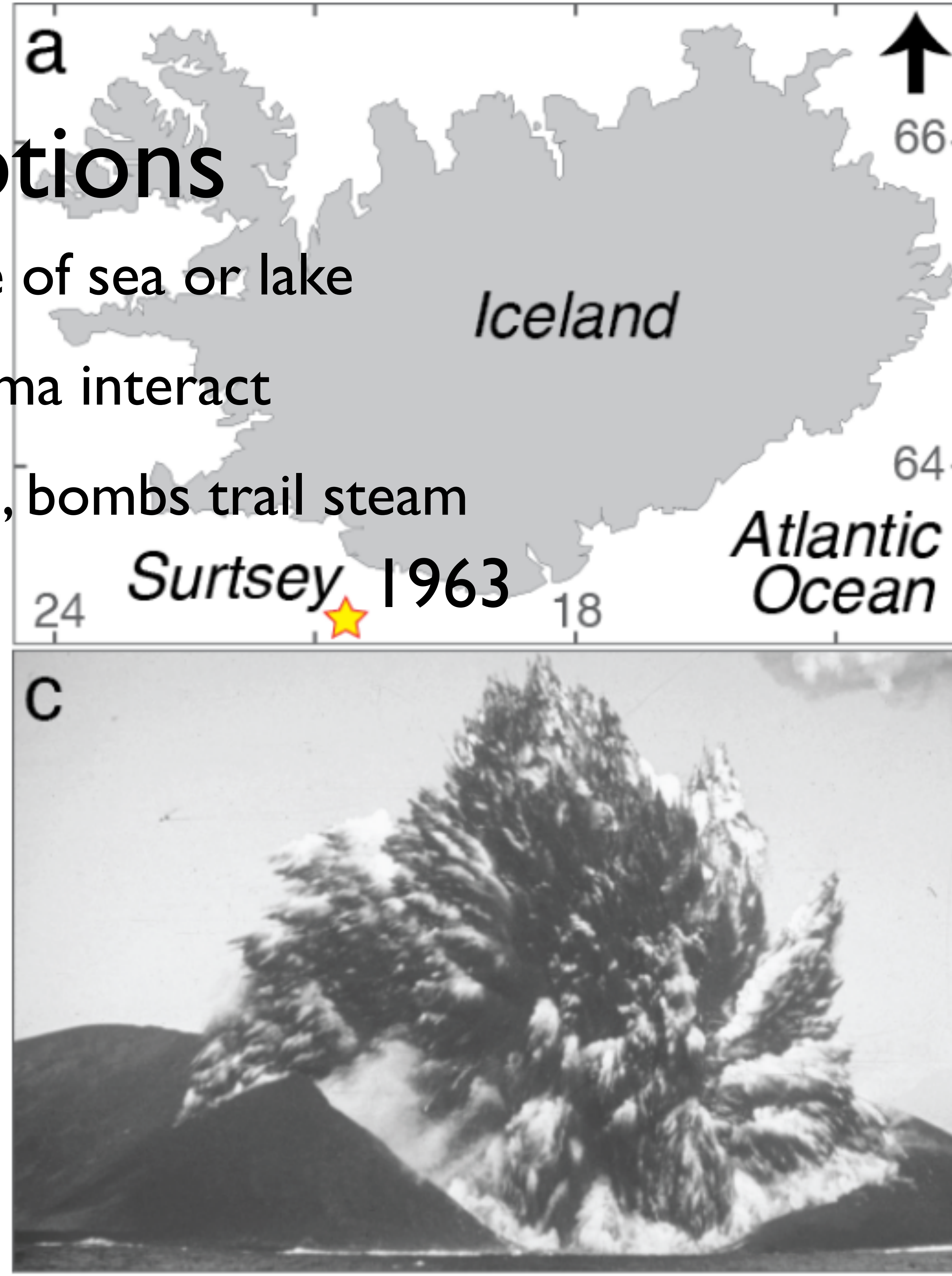
EGU 2020

Surtseyan Eruptions

- underwater near surface of sea or lake
- water and vesicular magma interact
- lots of steam, cock's tails, bombs trail steam
- relatively silent
- re-entry of slurry mix



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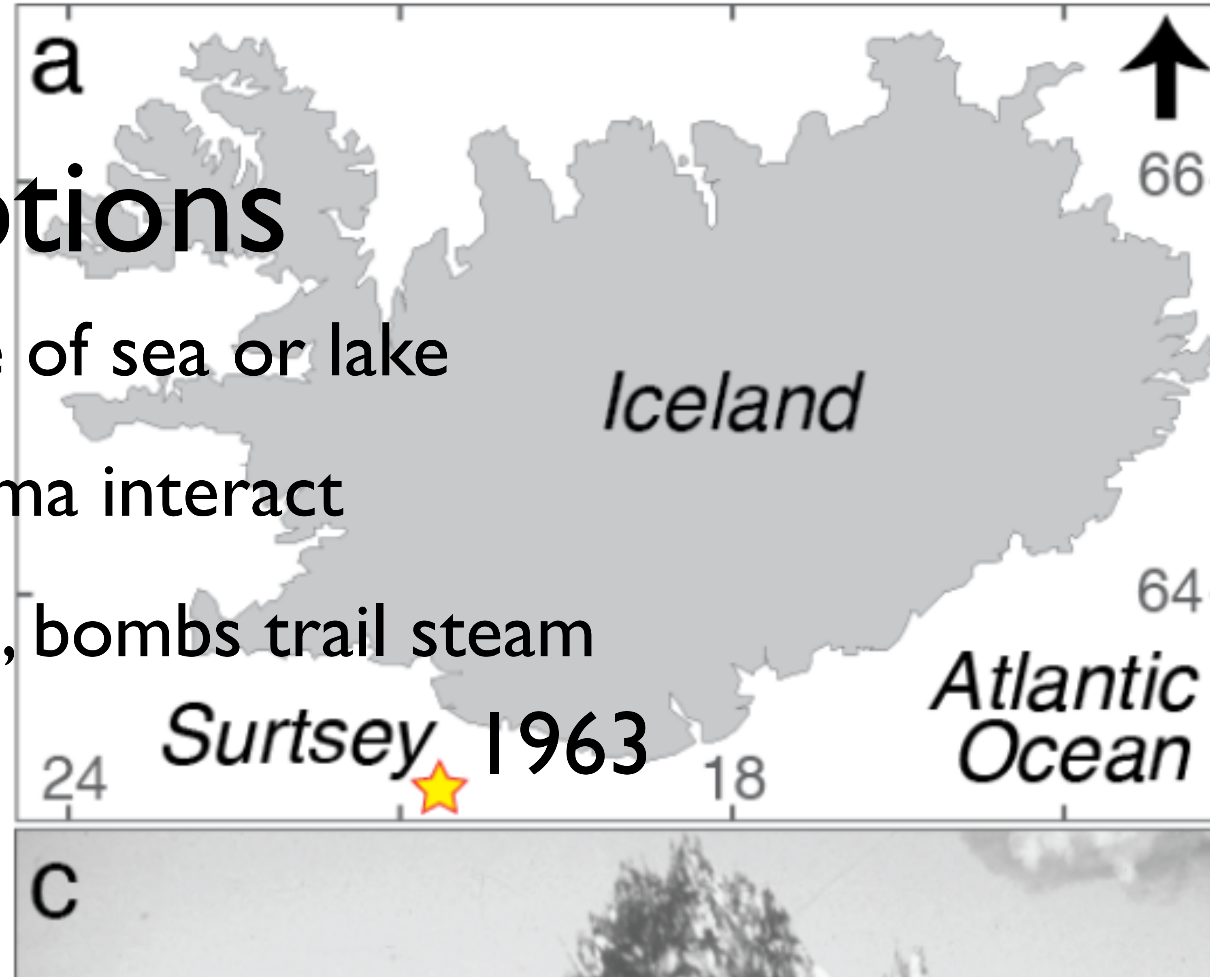

Magma-water interactions in subaq
volcanism

Bull Volcanol (1986) 48: 275–289

Peter Kokelaar

Emergent volcanoes are characterized by distinctive steam-explosive activity that results primarily from a bulk interaction between rapidly ascending magma and a highly mobile slurry of clastic material, water, and steam. The water gets into the vent by flooding across or through the top of

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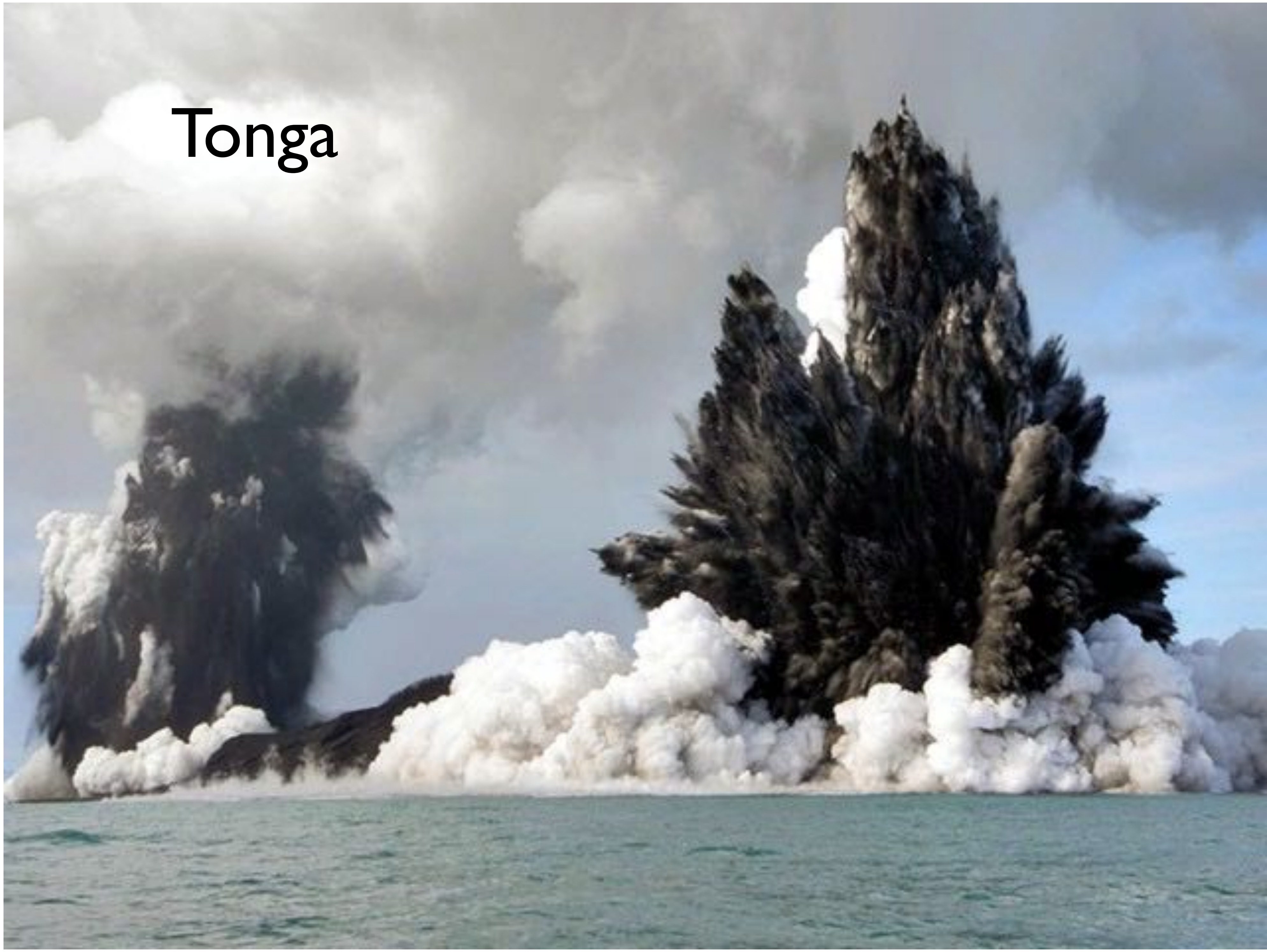
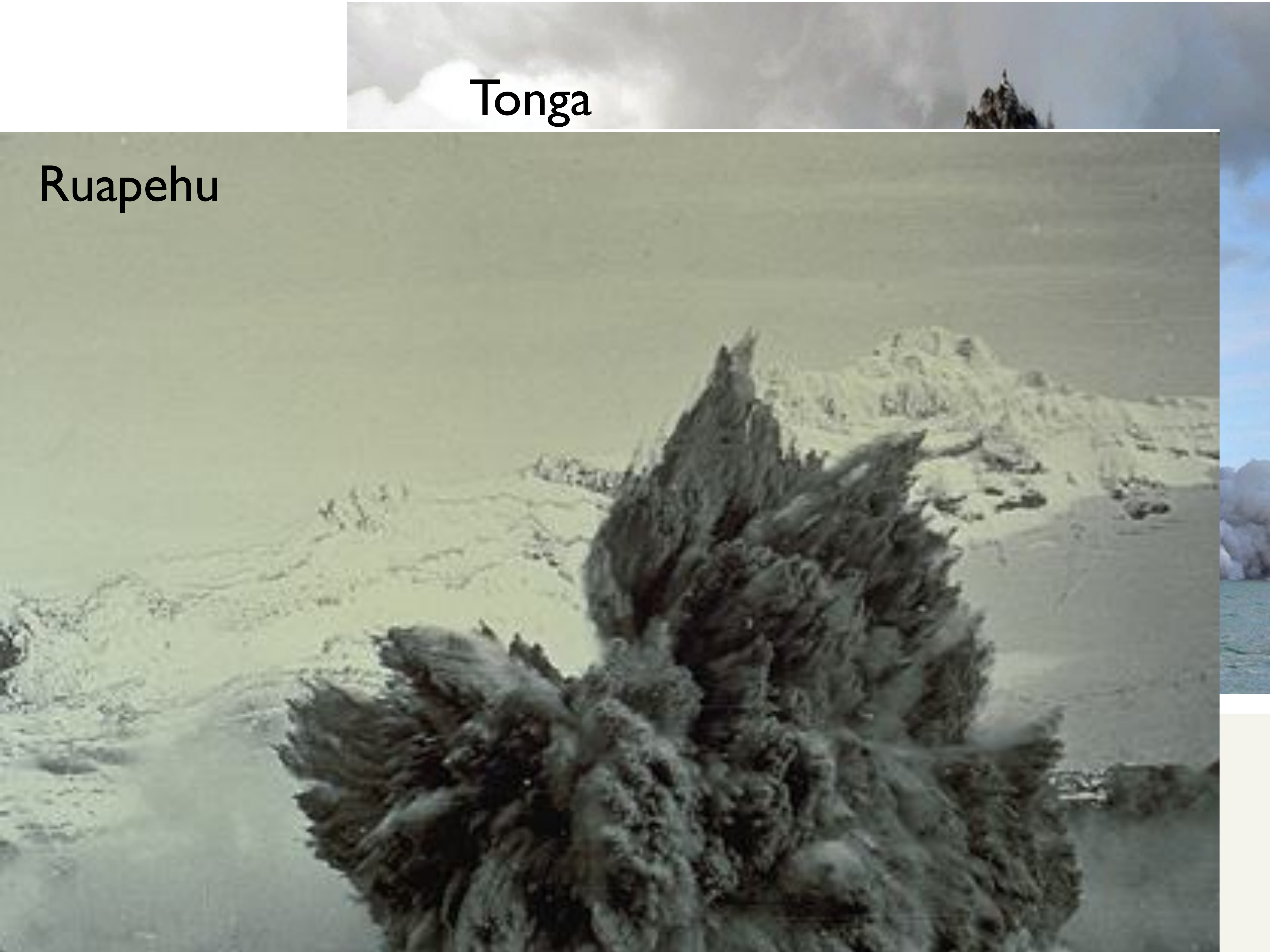

Tonga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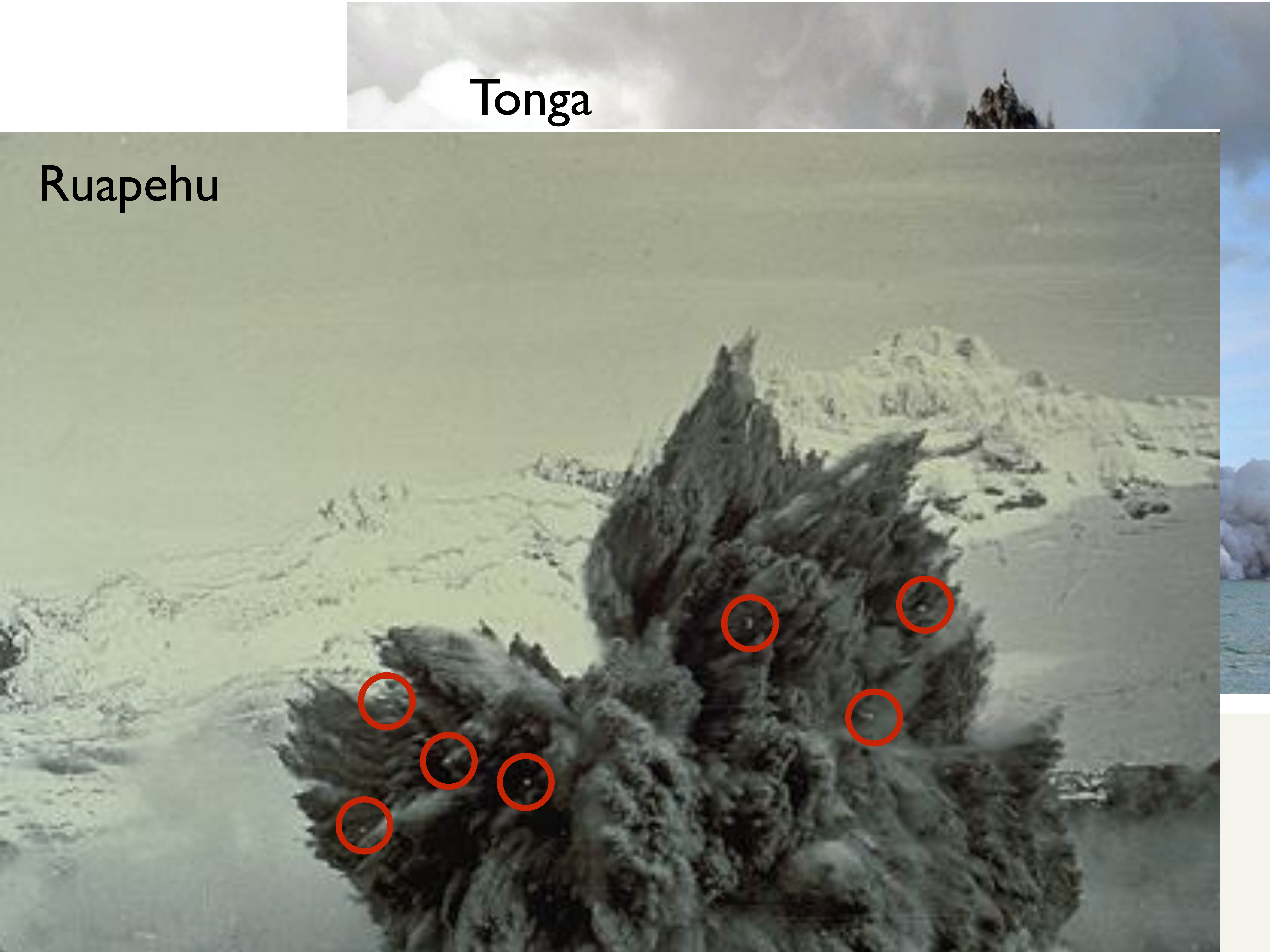
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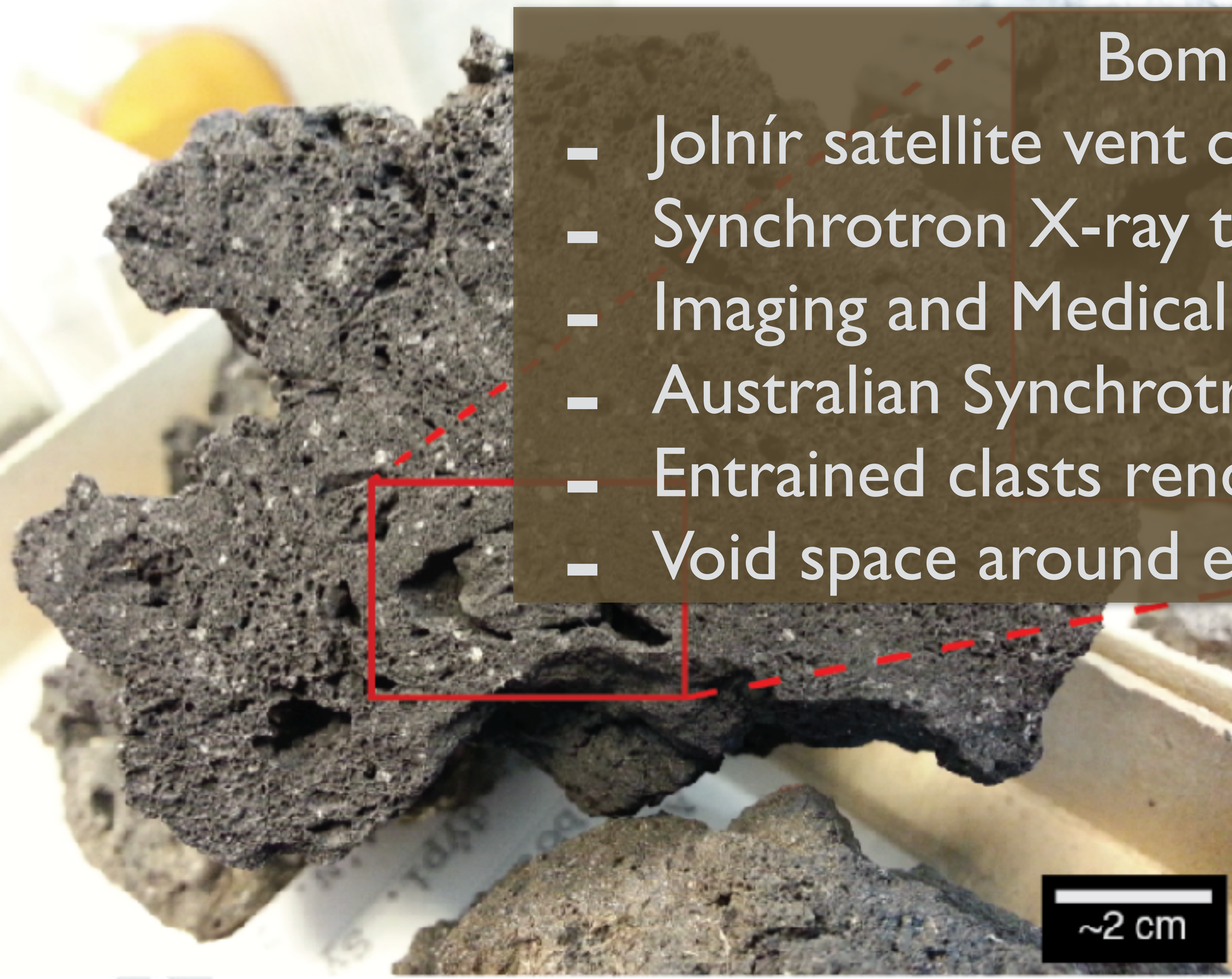
Bomb

- Jolnír satellite vent of Surtsey
- Synchrotron X-ray tomography
- Imaging and Medical Beamline (IMBL)
- Australian Synchrotron, Melbourne.
- Entrained clasts rendered blue
- Void space around entrained clasts



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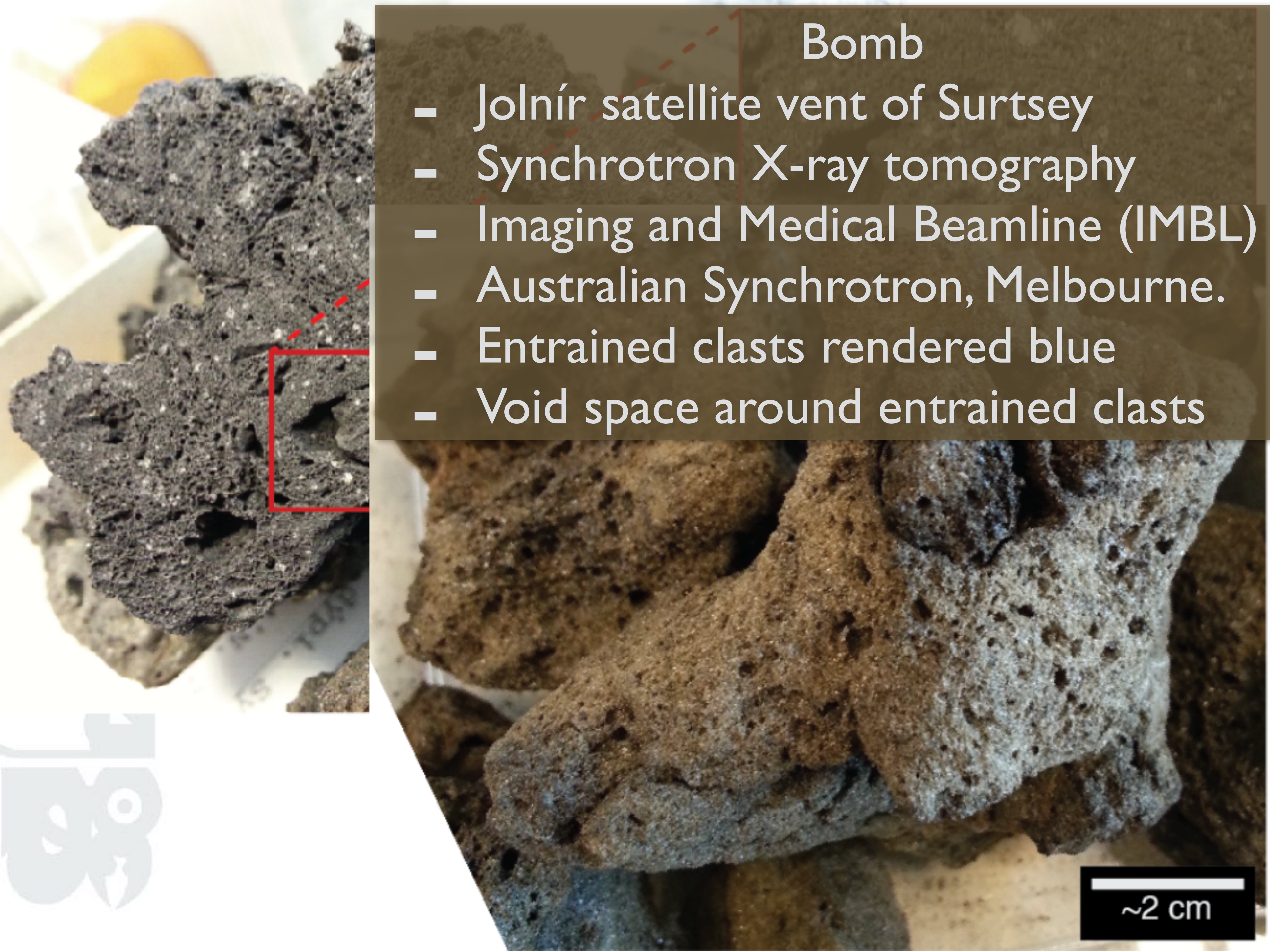


~2 cm


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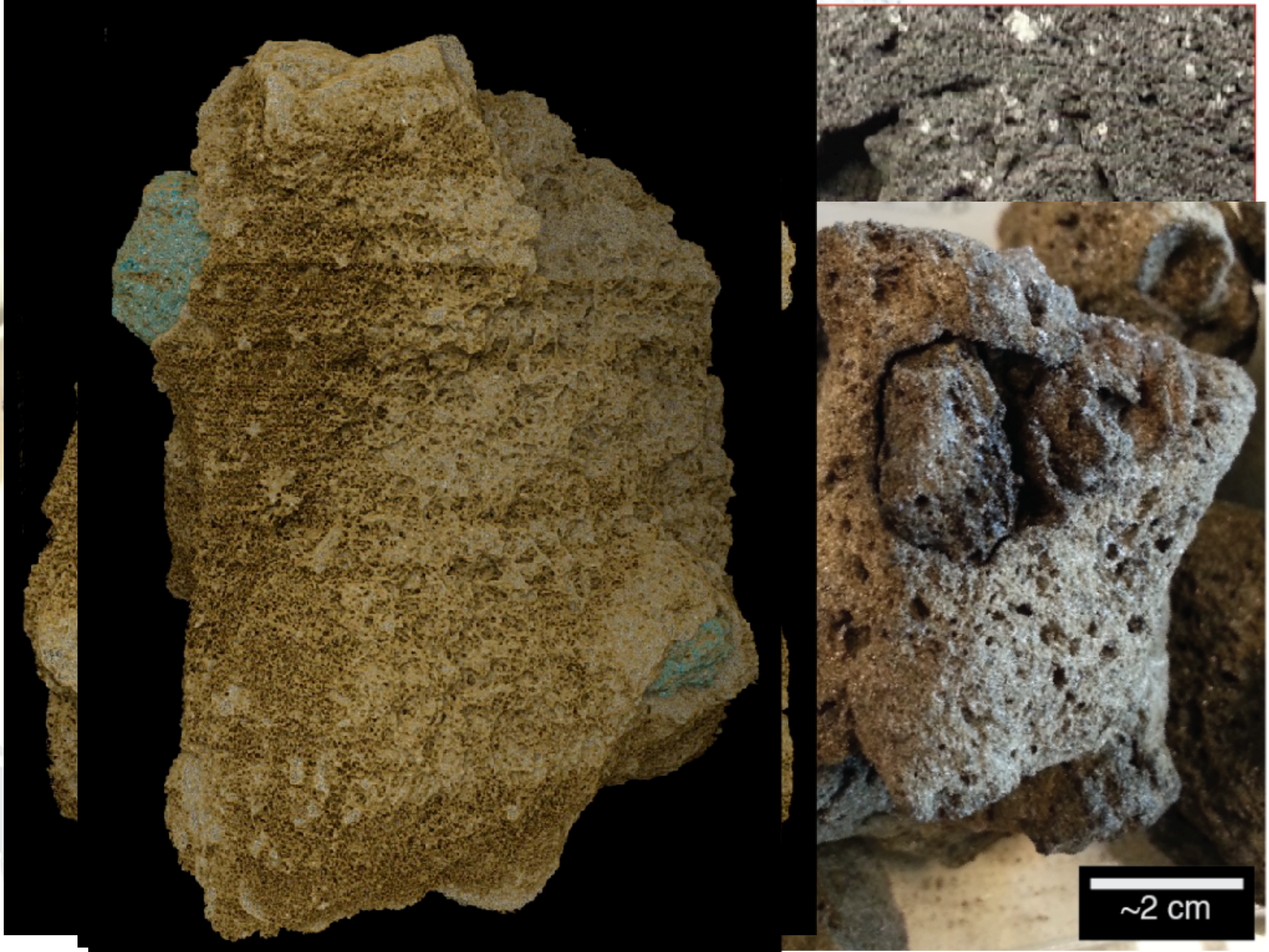
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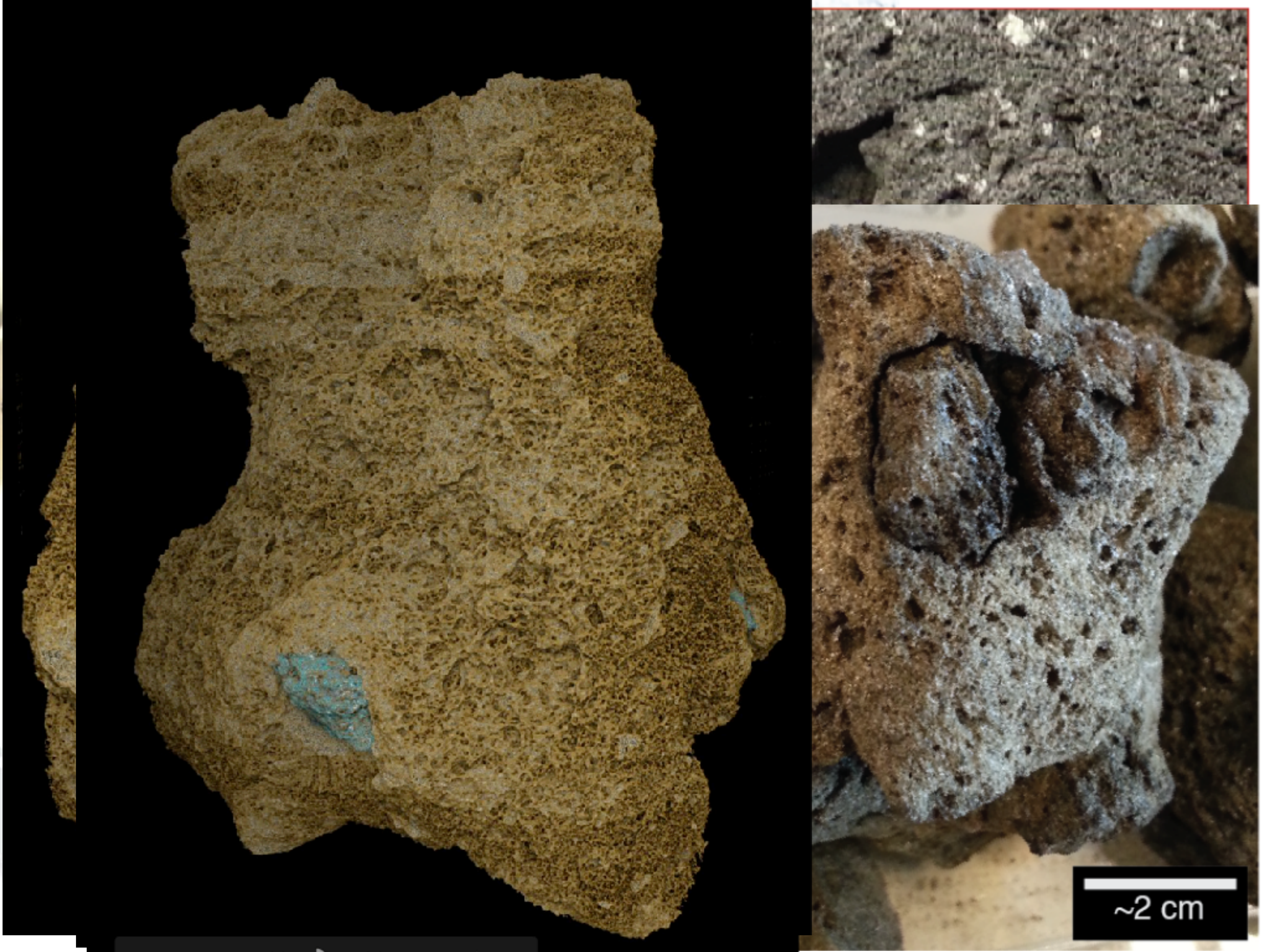
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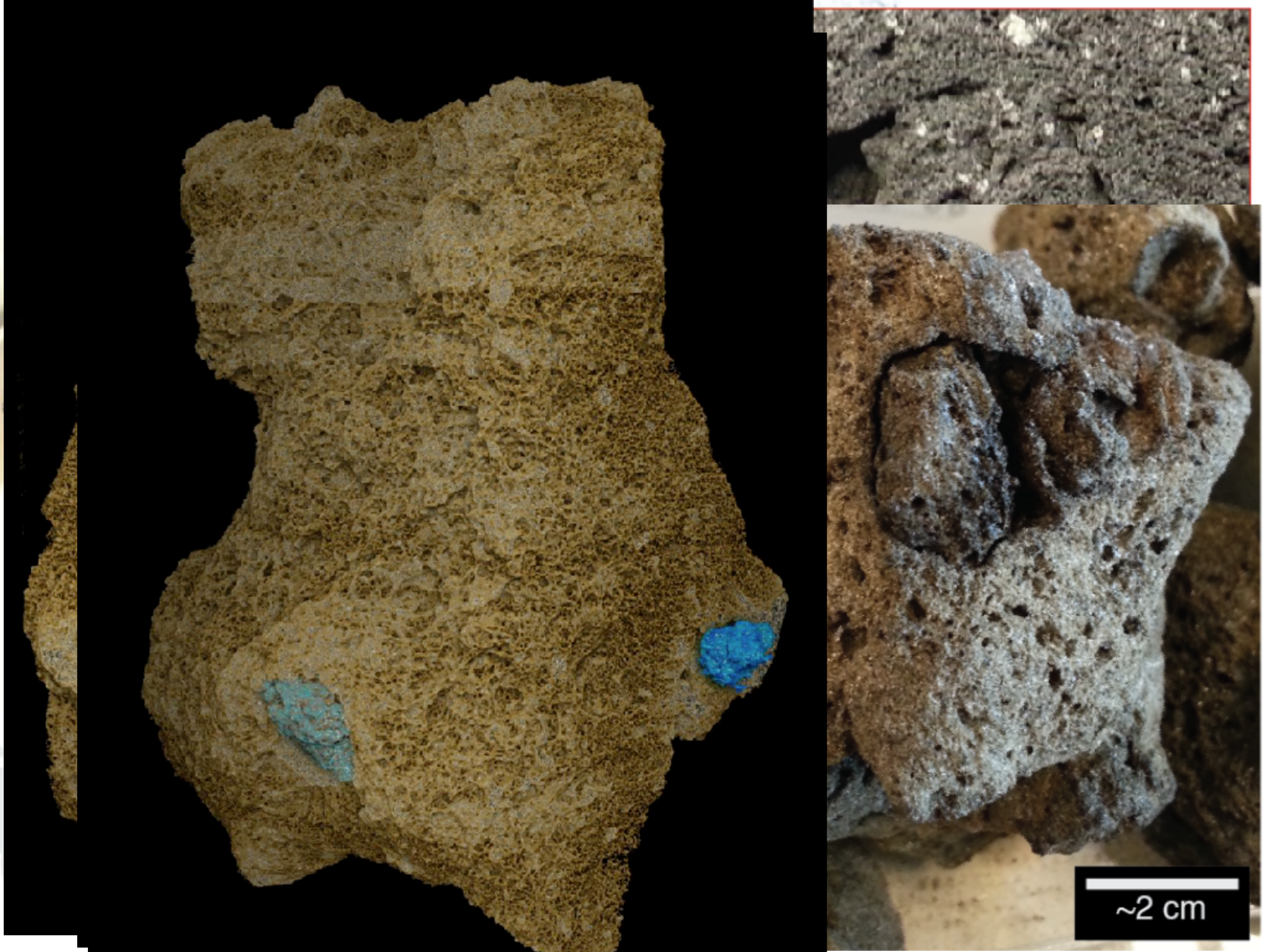
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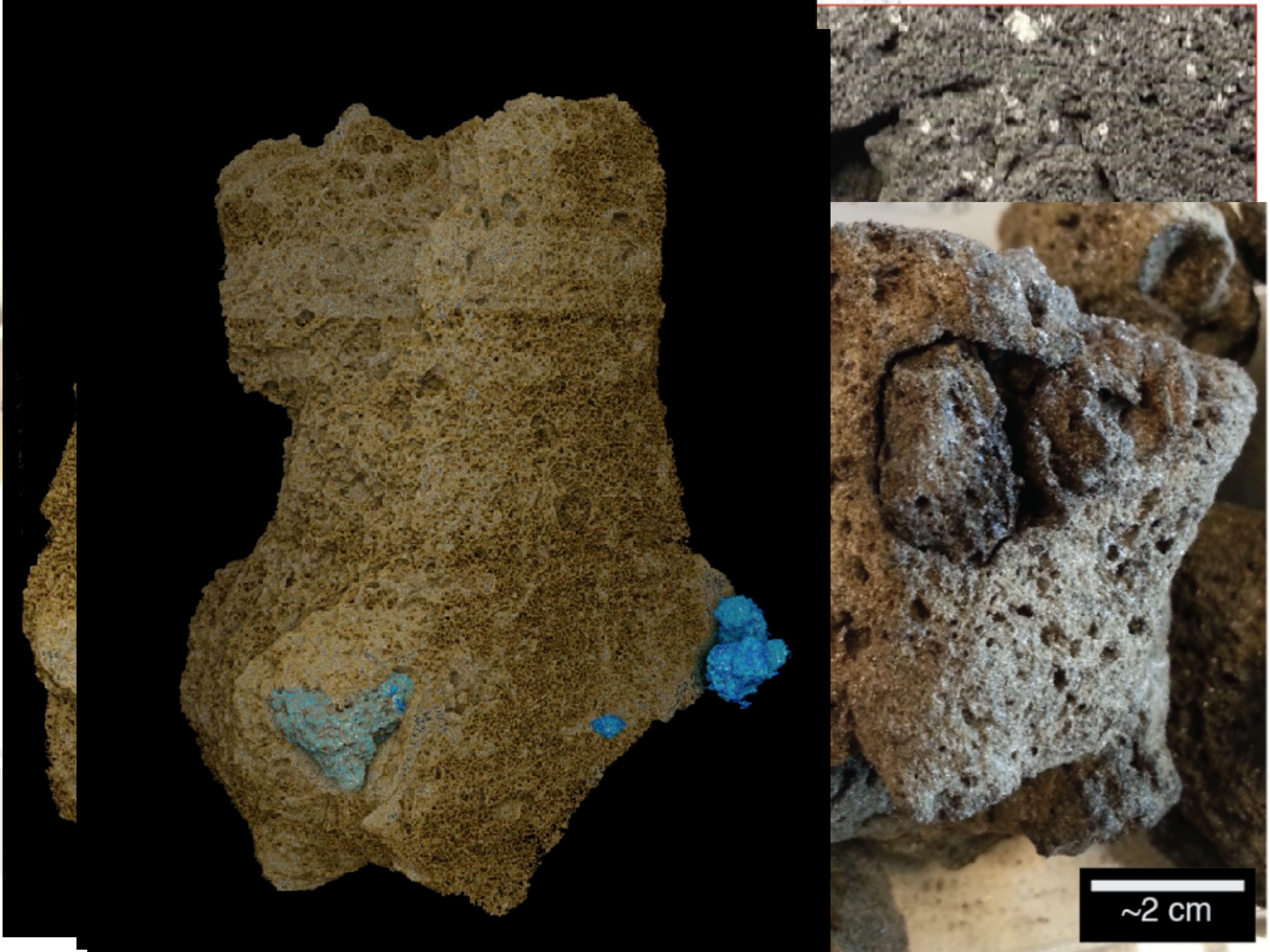
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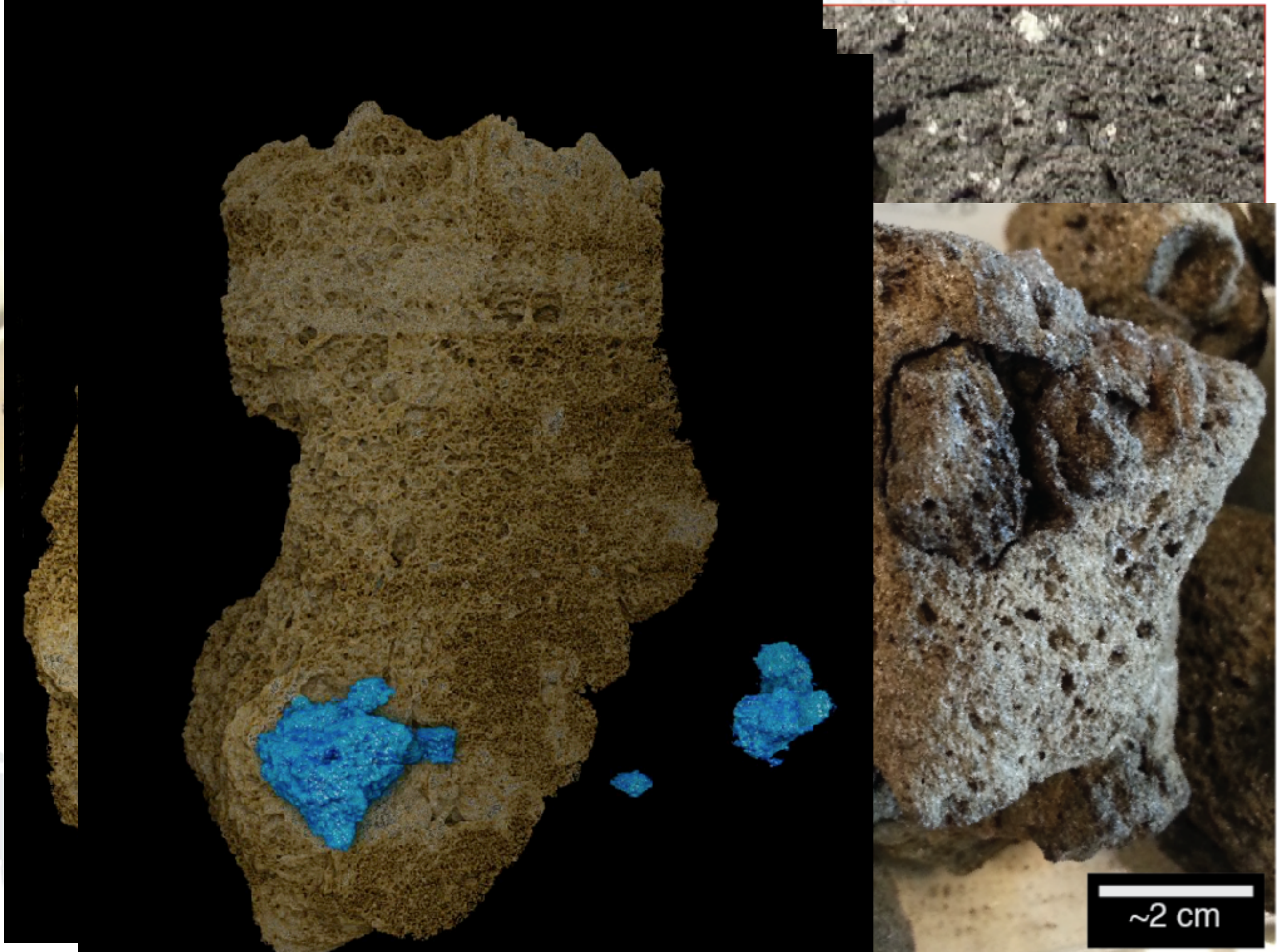
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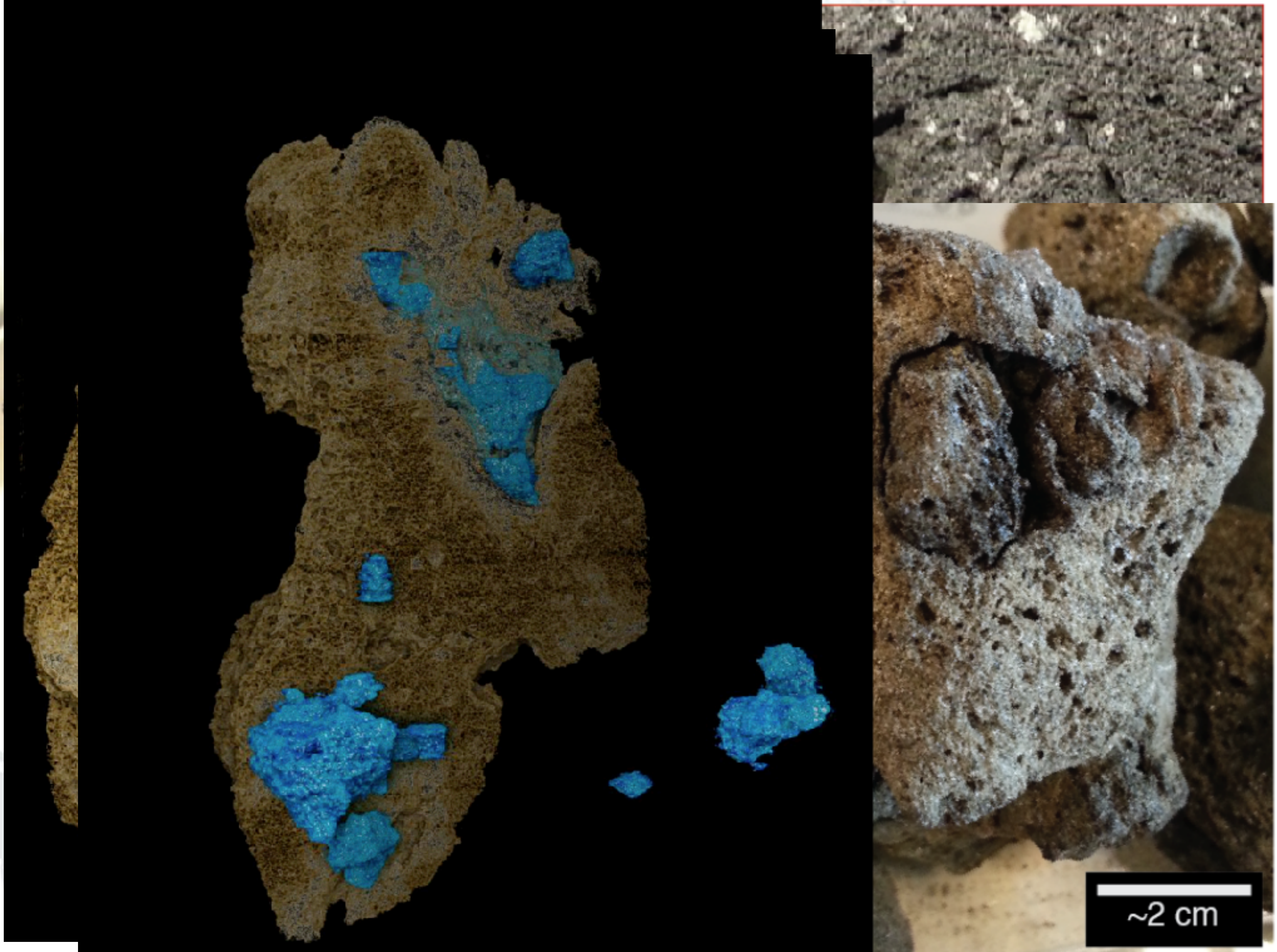
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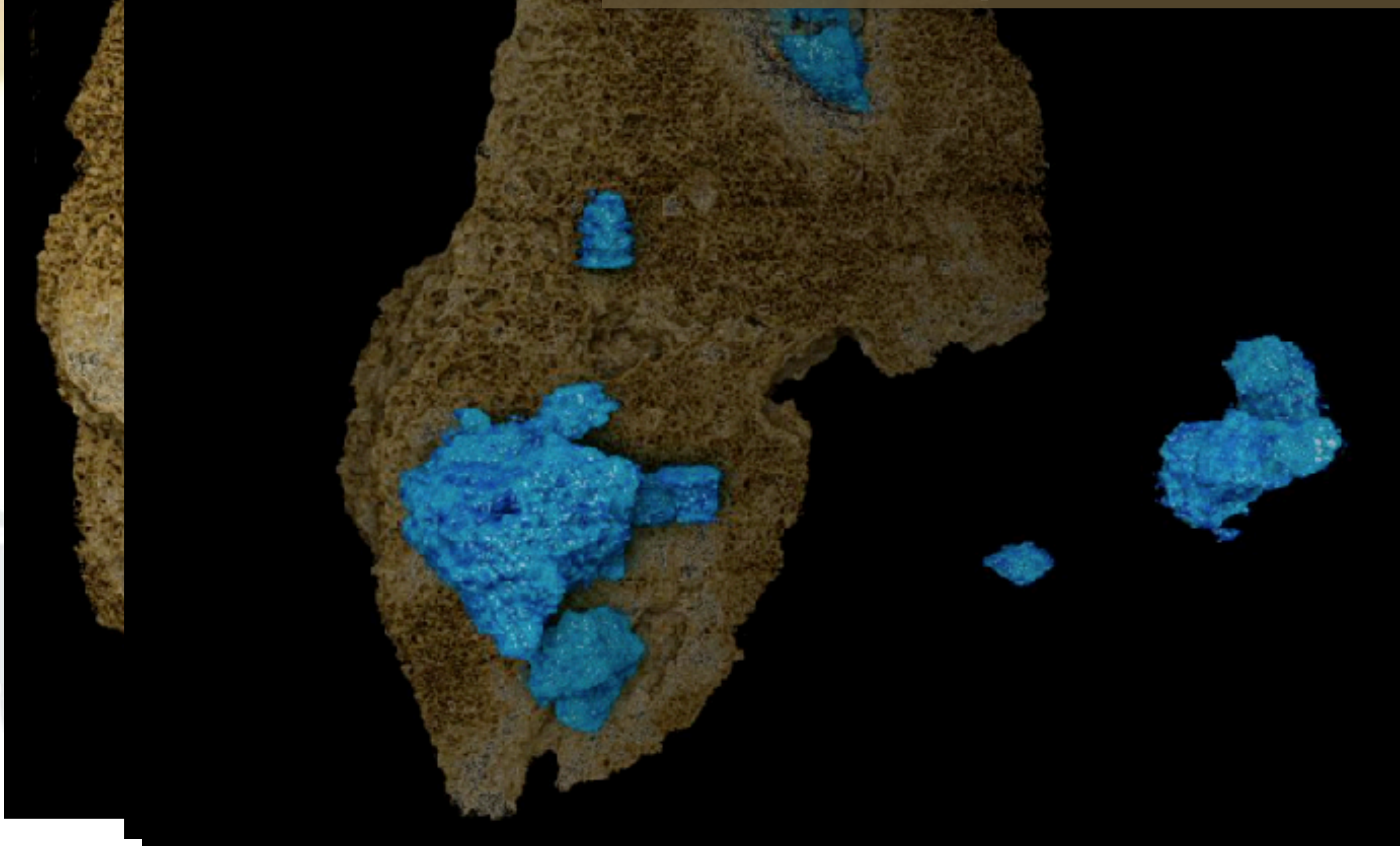
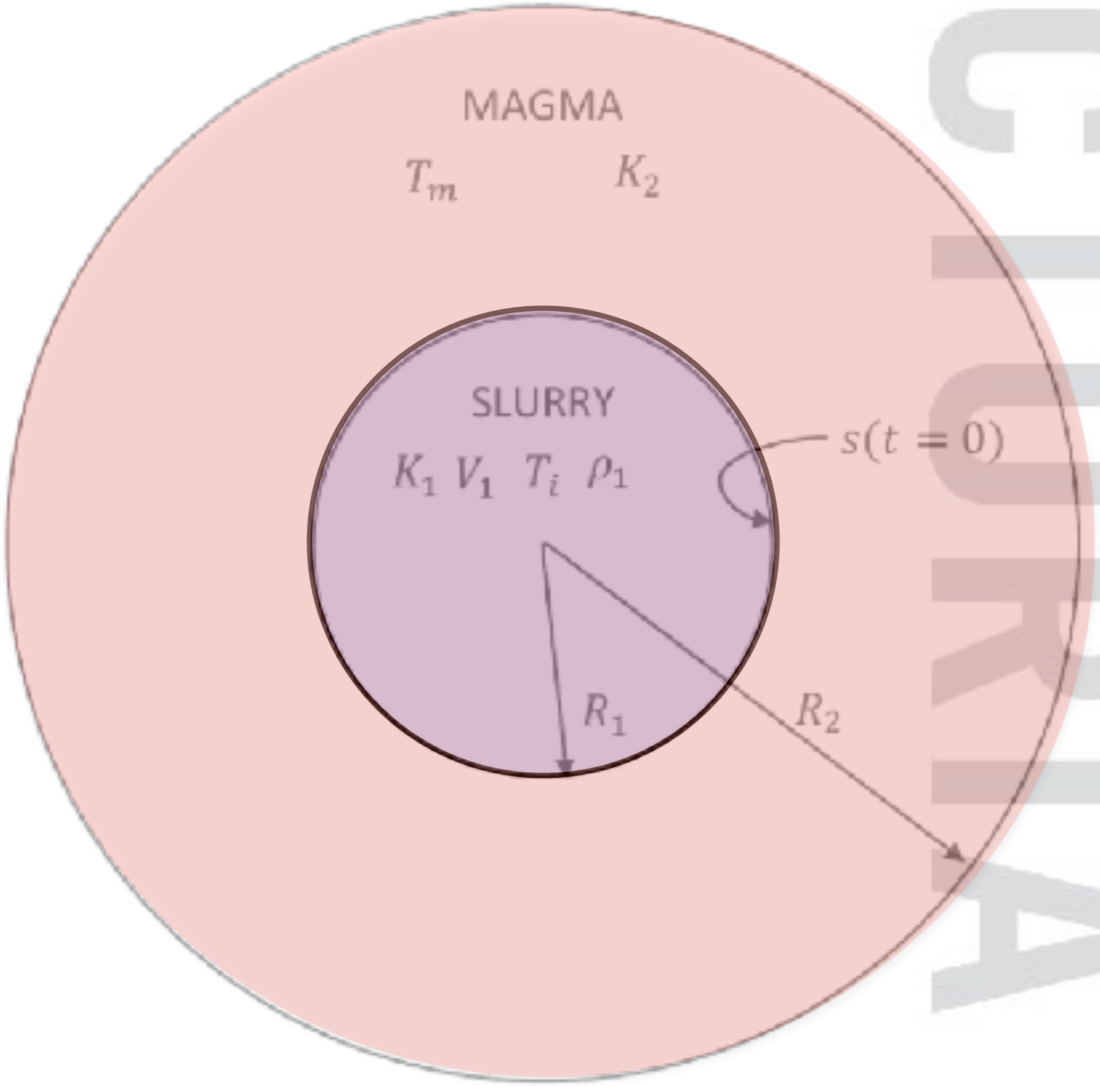


Mathematical Model

Conceptual model:

- spherical ejecta



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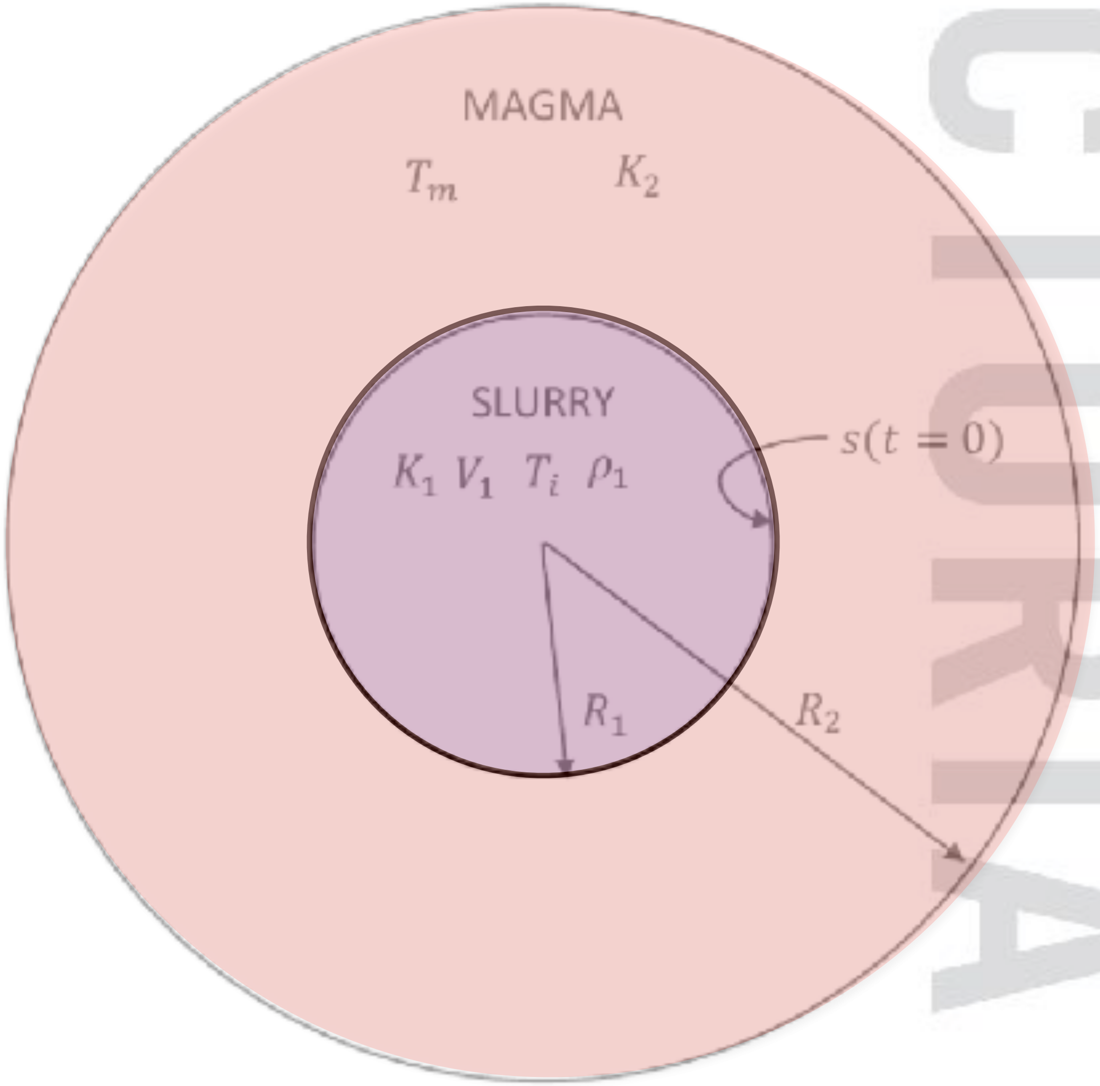
Conceptual model:

- spherical ejecta
- viscous bubbly basalt



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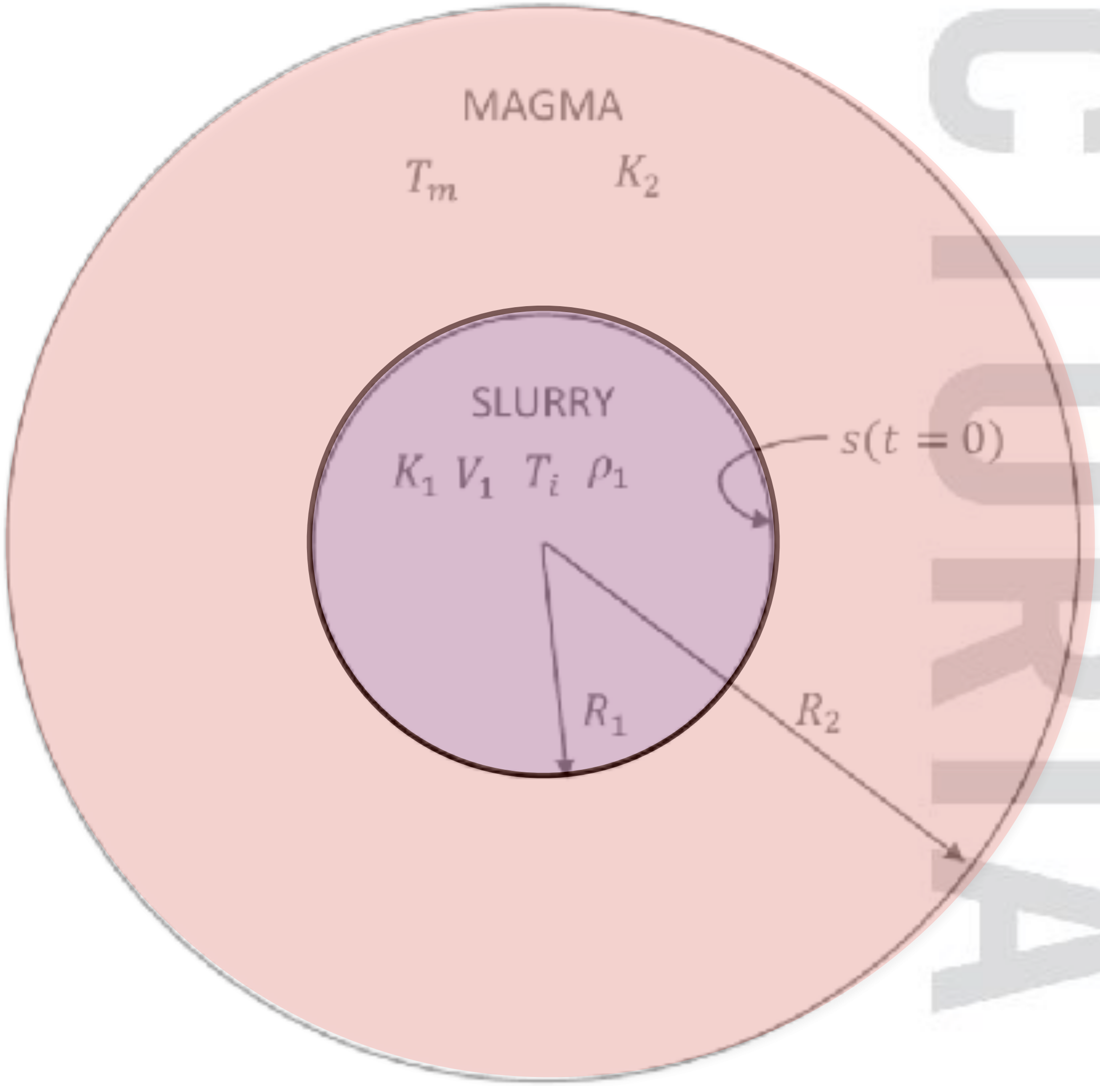

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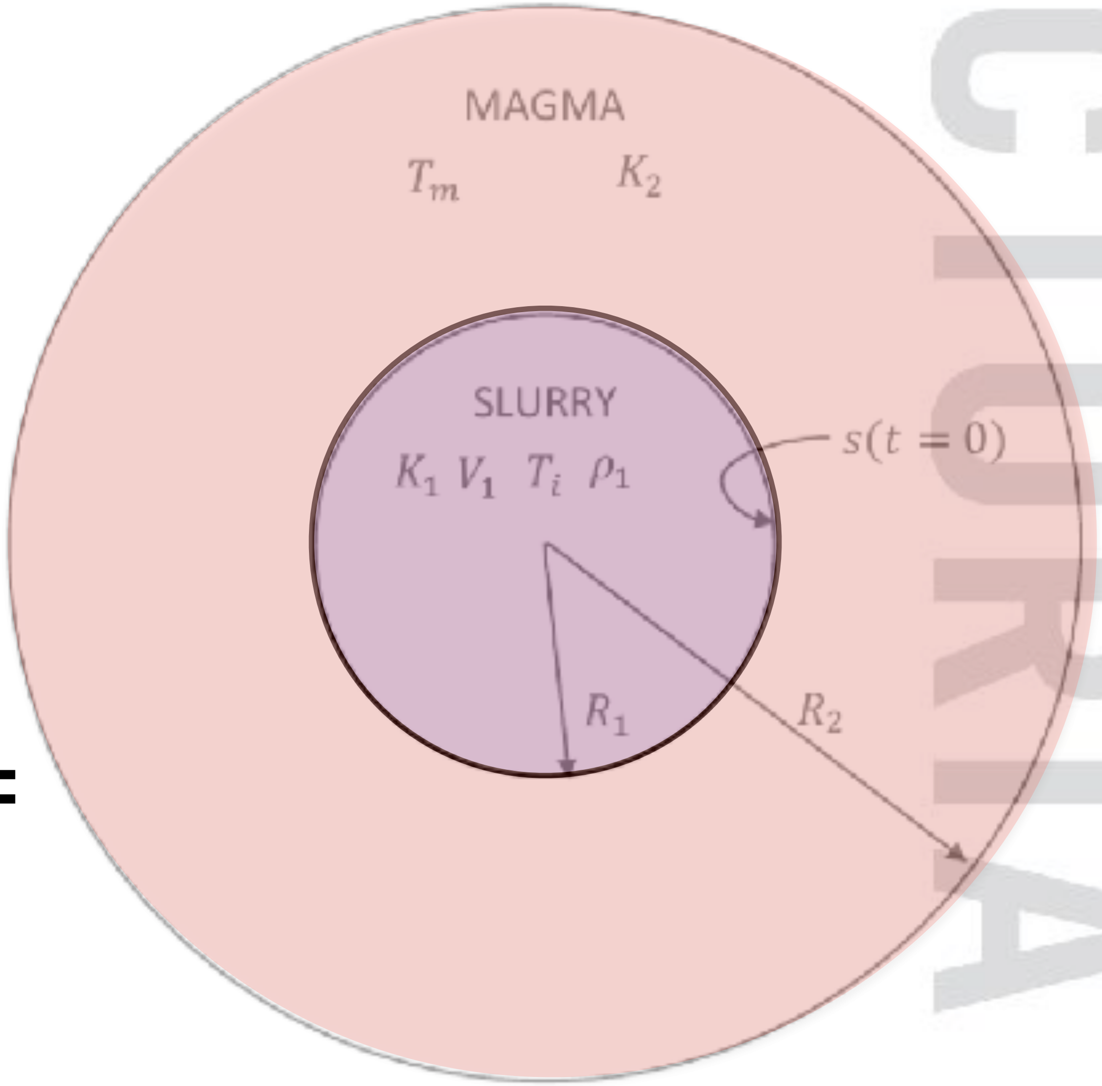

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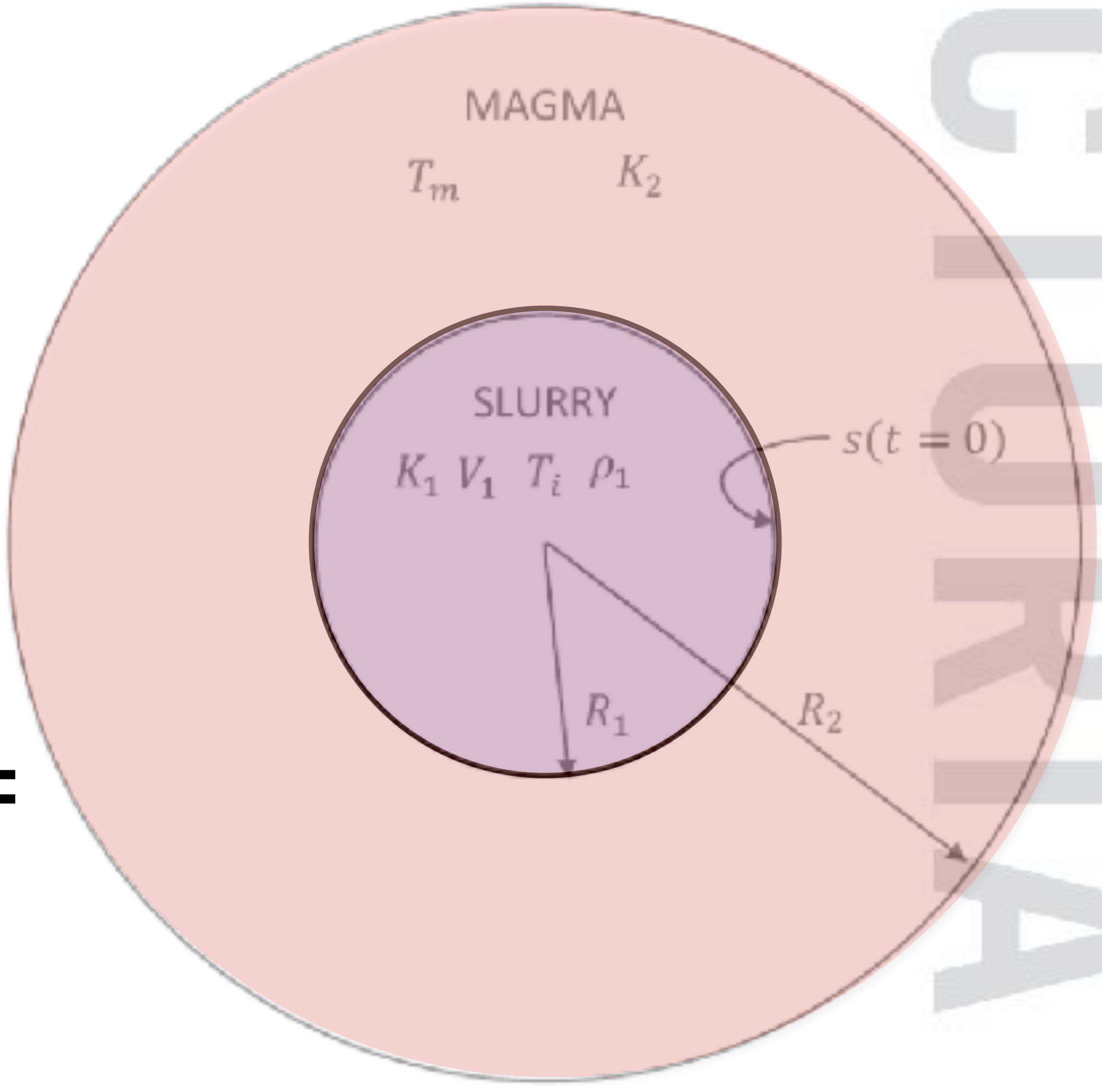

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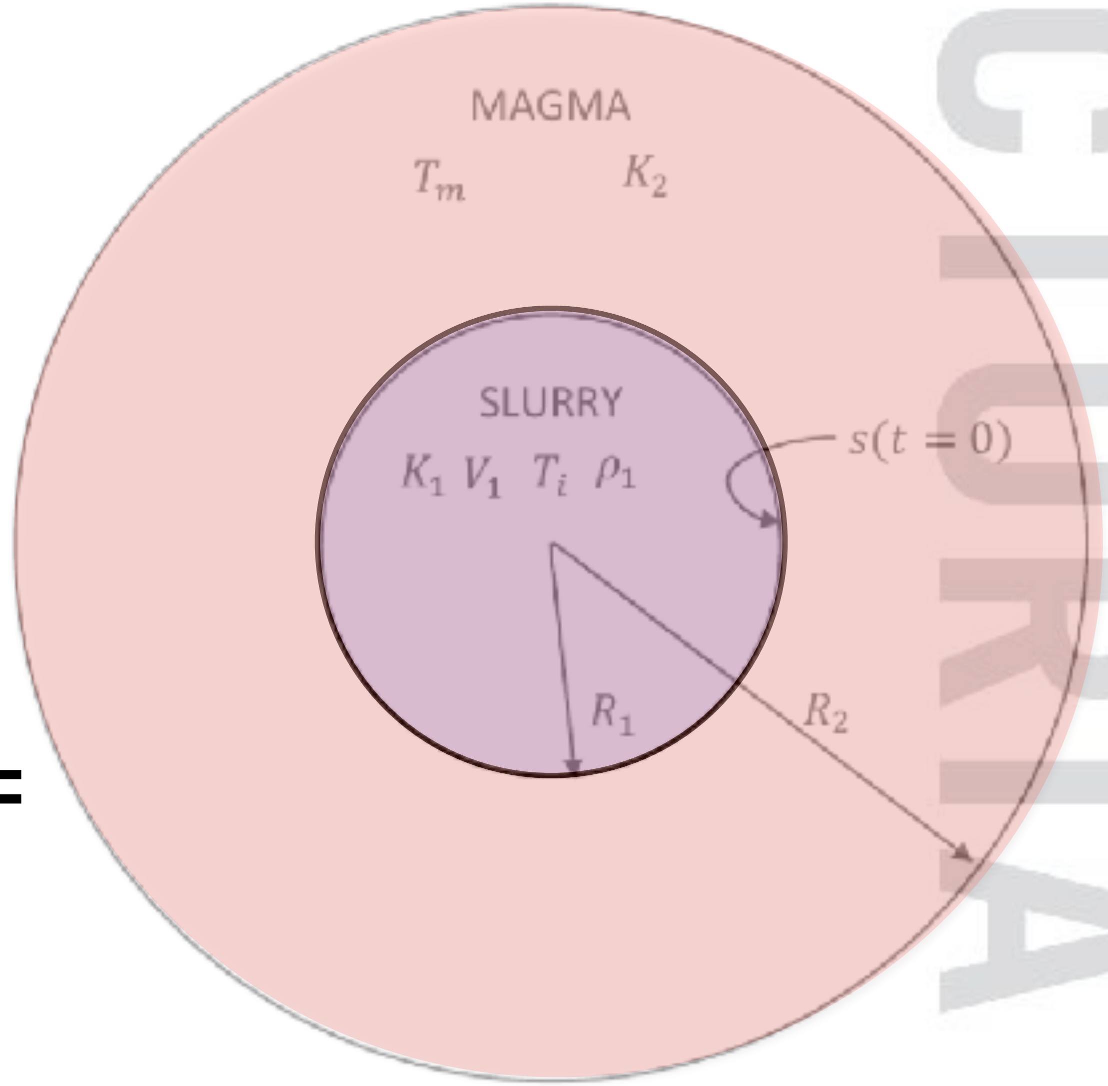

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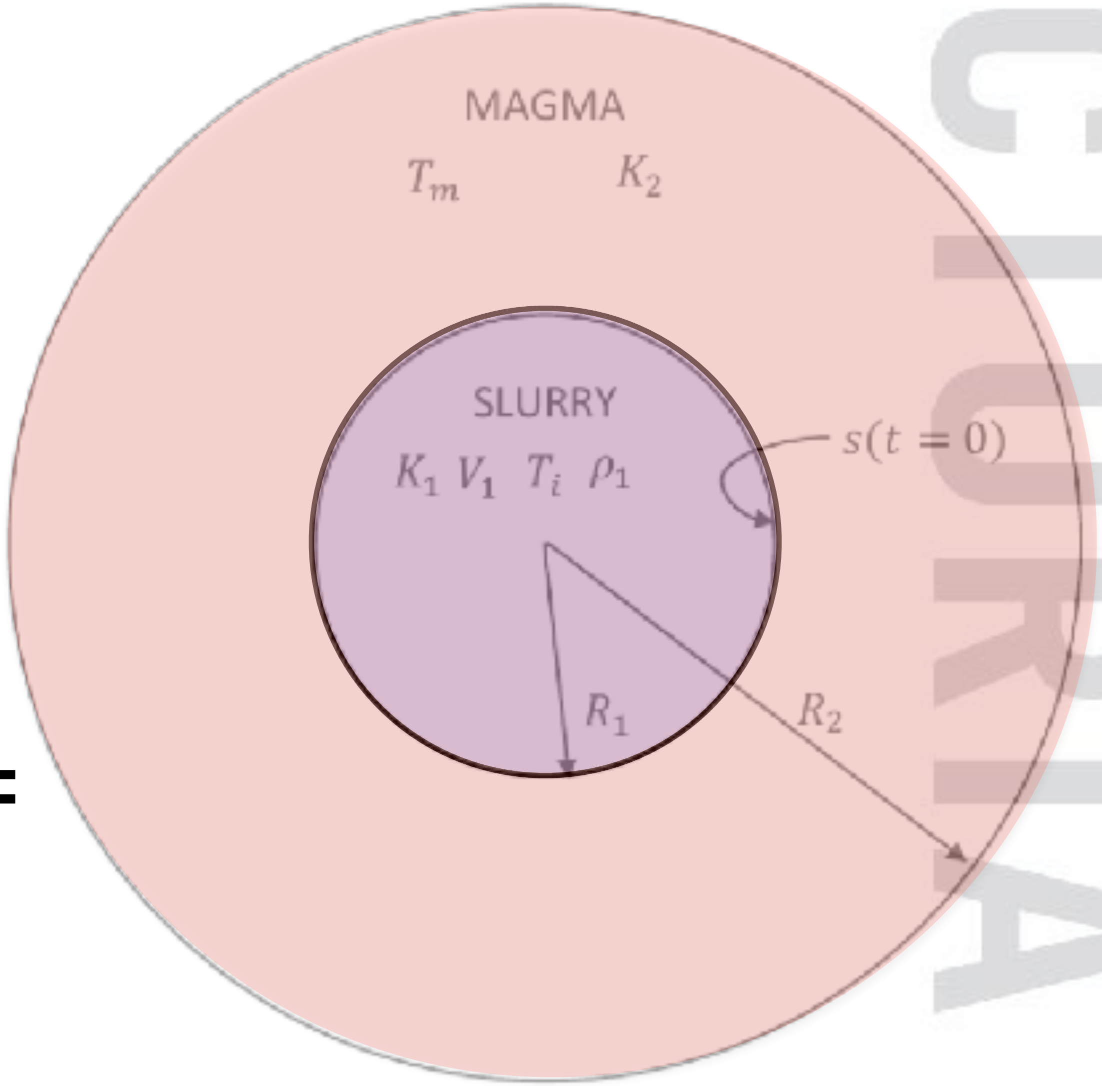

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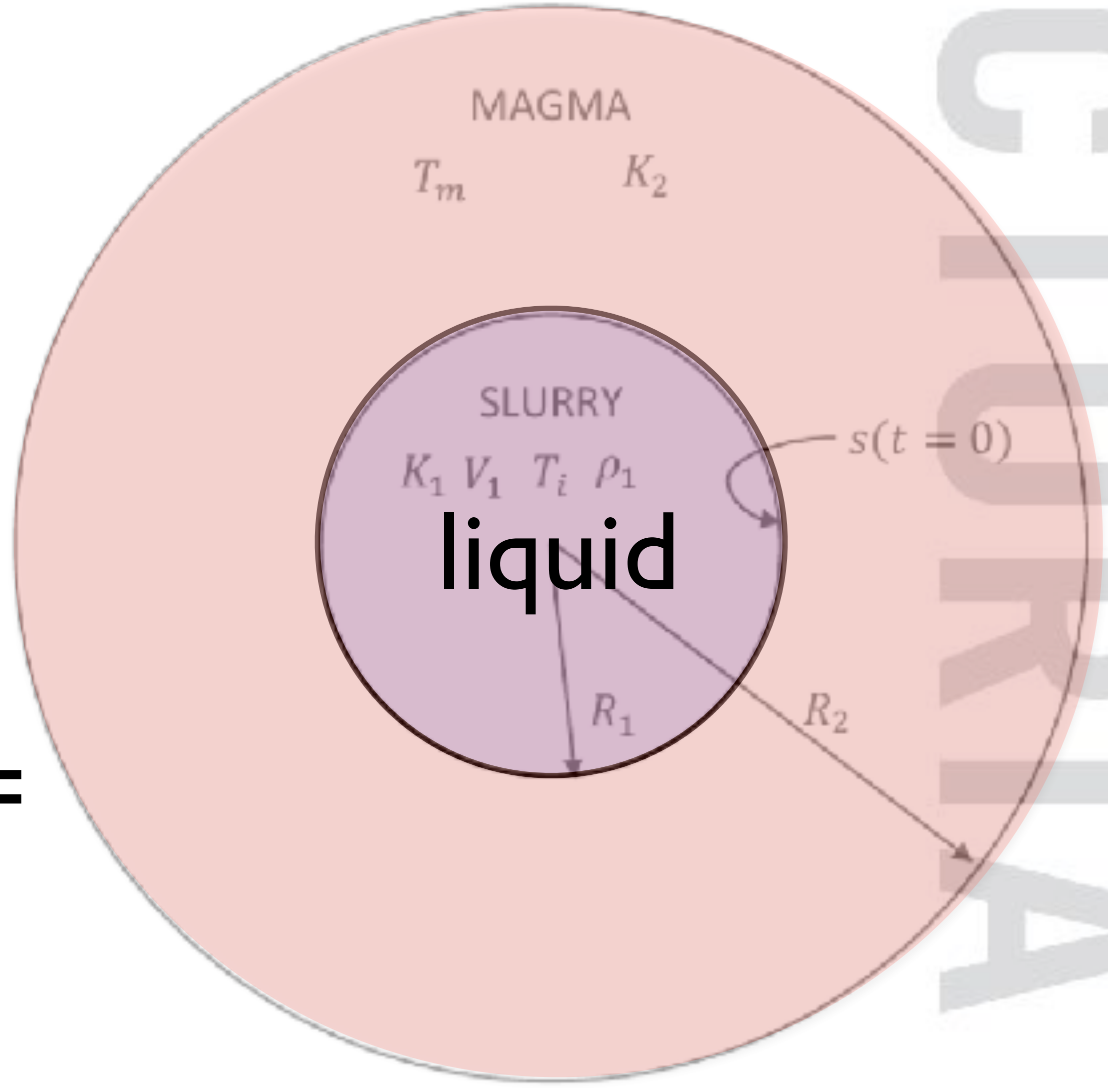

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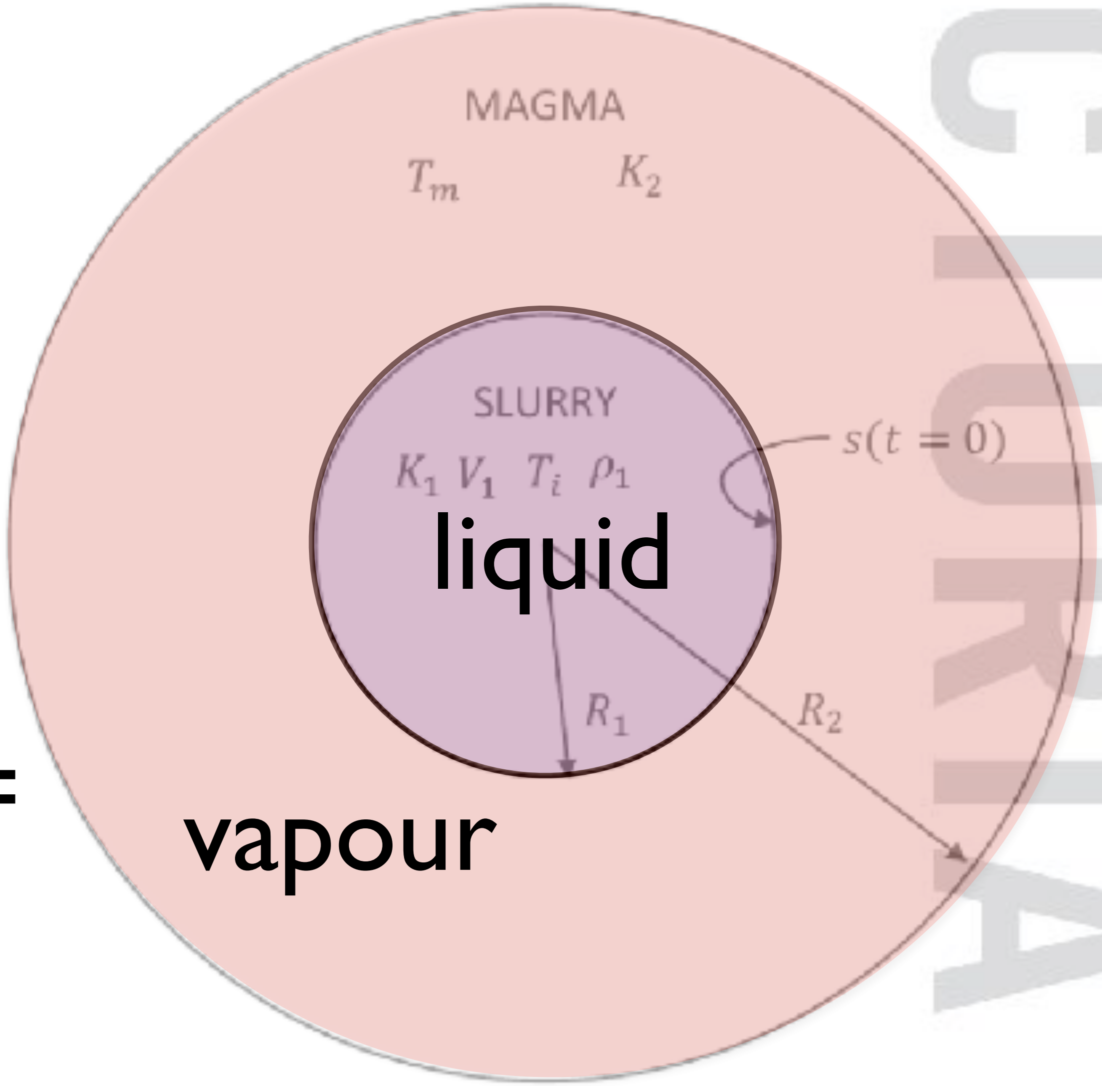

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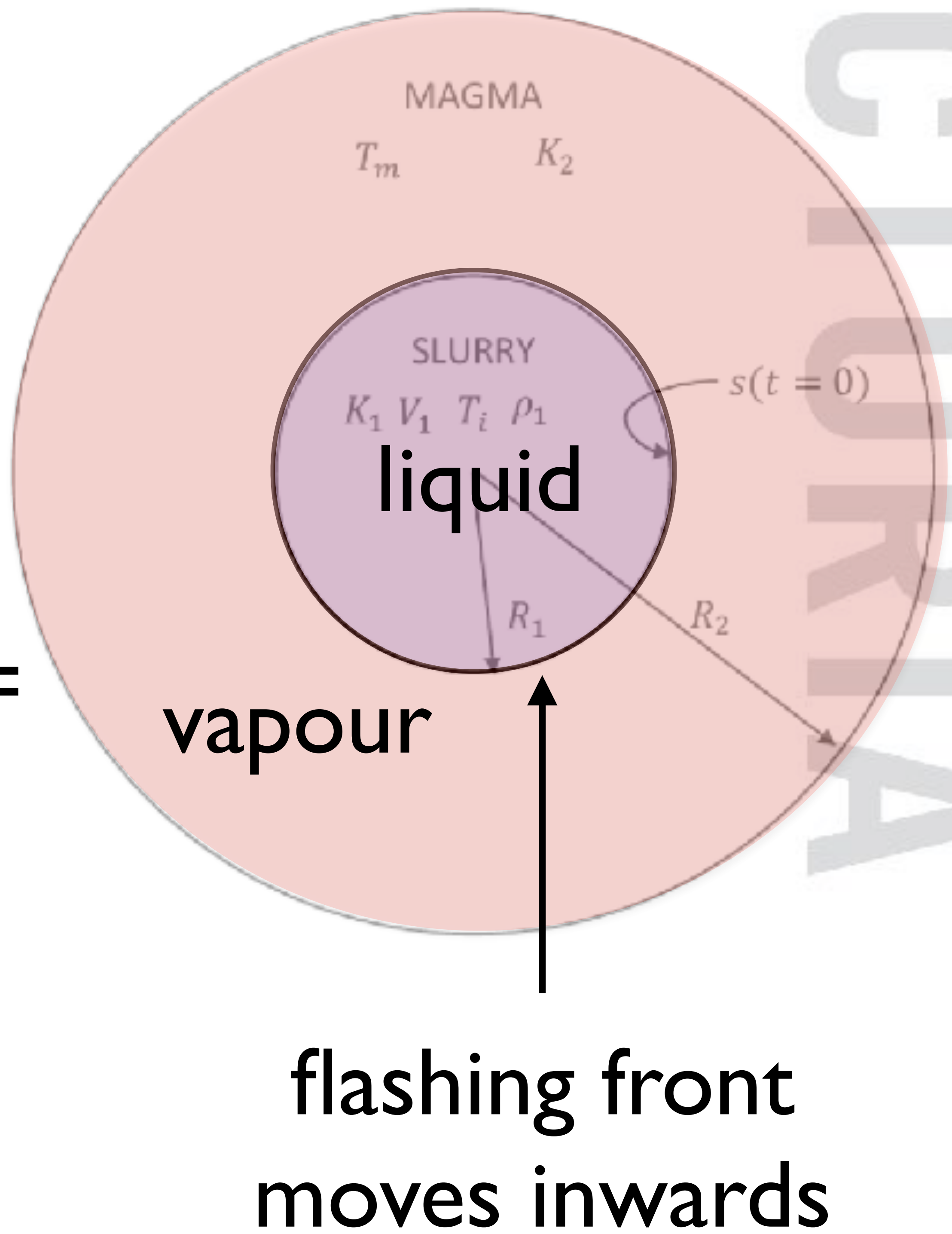

Mathematical Model

in vapor region

Mathematical Model

in vapor region

energy conservation

$$(1-\phi)\rho_m c_{pm}\frac{\partial T}{\partial t} + \rho_v c_{pv}\left[\phi\frac{\partial T}{\partial t} + u\cdot\nabla T\right] - \left[\phi\frac{\partial p}{\partial t} + u\cdot\nabla p\right]$$
$$= K_e\nabla^2 T.$$

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mass conservation

$$\phi\frac{\partial \rho_v}{\partial t} + \nabla\cdot(\rho_v u) = 0$$

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momentum (Darcy's law)

$$u = -\frac{k}{\mu_v}\nabla p$$

$$\mathbf{u} = \phi\mathbf{v}$$

Mathematical Model

in vapor region

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momentum (Darcy's law)

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ideal gas law

$$\rho_v=\frac{M_v p}{RT}$$

$$\mathbf{u}=\phi\mathbf{v}$$

in liquid region

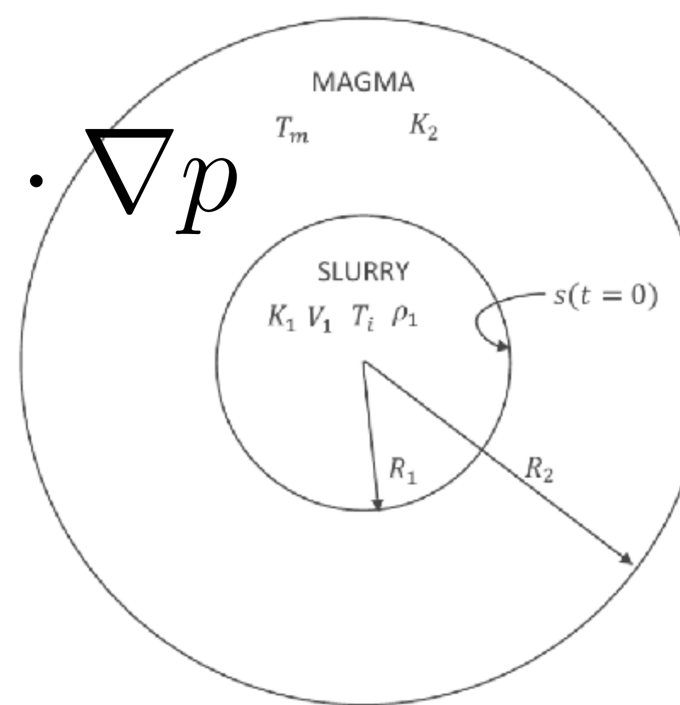
symmetry and low compressibility implies no flow

saturated porous medium

$$\phi \frac{\partial \rho_l}{\partial t} + \nabla \cdot (\rho_l \mathbf{u}_l) = 0$$

$$\mathbf{u}_l = -\frac{k}{\mu_l} \nabla p$$

$$\varrho' c' \frac{\partial T}{\partial t} + \rho_l c_{pl} \mathbf{u}_l \cdot \nabla T - \phi \beta T \frac{\partial p}{\partial t} - \beta T \mathbf{u}_l \cdot \nabla p$$
$$= K_{el} \nabla^2 T$$



at interface

on saturation curve $$p_{sv} = p_0 e^{\frac{M_v L}{RT_0}\left[\frac{T_s - T_0}{T_s}\right]}$$

transform to a moving frame,
integrate mass, energy across flash front:

$$\phi\rho_s h_{sl}(v - \dot{s}) = \phi\rho_l h_{sl}(v_l - \dot{s}) = [K\nabla T]_-^+ + \phi(v - v_l)p$$

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boundary and initial conditions

$$p(R_2) = p_a \ , \quad \frac{\partial p}{\partial r} = 0 \text{ at } r = 0$$

initial p

initial T: hot in magma, at boiling in inclusion

$$\frac{\partial T}{\partial t} = \frac{\epsilon_3}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) , \quad r < s(t) ,$$

$$\dot{s} = \epsilon_4 \rho_s \frac{\partial p}{\partial r} = -\frac{1}{\mathrm{St}}\left[\frac{\partial T}{\partial r}\right]_-^+ , \quad r = s(t) ,$$

$$p = \exp\left[H\left(\frac{T - T_0}{T}\right)\right] , \quad r = s(t)$$

$$\frac{\partial T}{\partial t} = \frac{\delta_5}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) , \quad r > s(t)$$

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Nondimensionalise

reject smallest parameters

$$T = T_0 , \ r = 1 ; \ p = 1 , \ r = 1 ; \quad \frac{\partial T}{\partial r} = 0 , \ r = 0 ;$$

initial conditions $T = T_0 , \ r < s(0) ; \quad T = 1 , \ r > s(0) ;$

$$p = 1 ; \quad s(0) = R_1/R_2 .$$

0.002

$$\frac{\partial T}{\partial t} = \frac{\epsilon_3}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right), \quad r < s(t),$$

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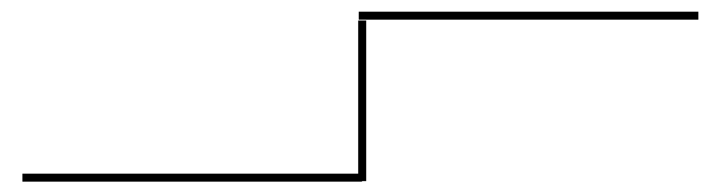

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- boiling driven by magma temperature gradient
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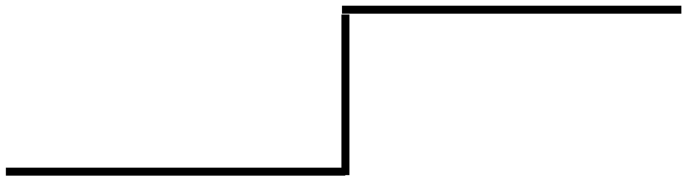
- moving boundary: freeze with Landau transformations



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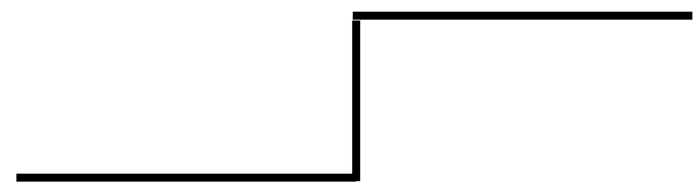

$$\zeta = \frac{r}{s} \quad \text{in slurry} \qquad \xi = \frac{r-s}{1-s} \quad \text{in hot magma}$$

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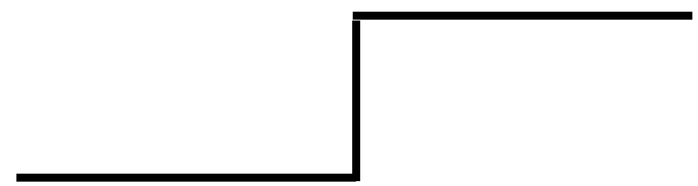

$$\zeta = \frac{r}{s}$$ in slurry

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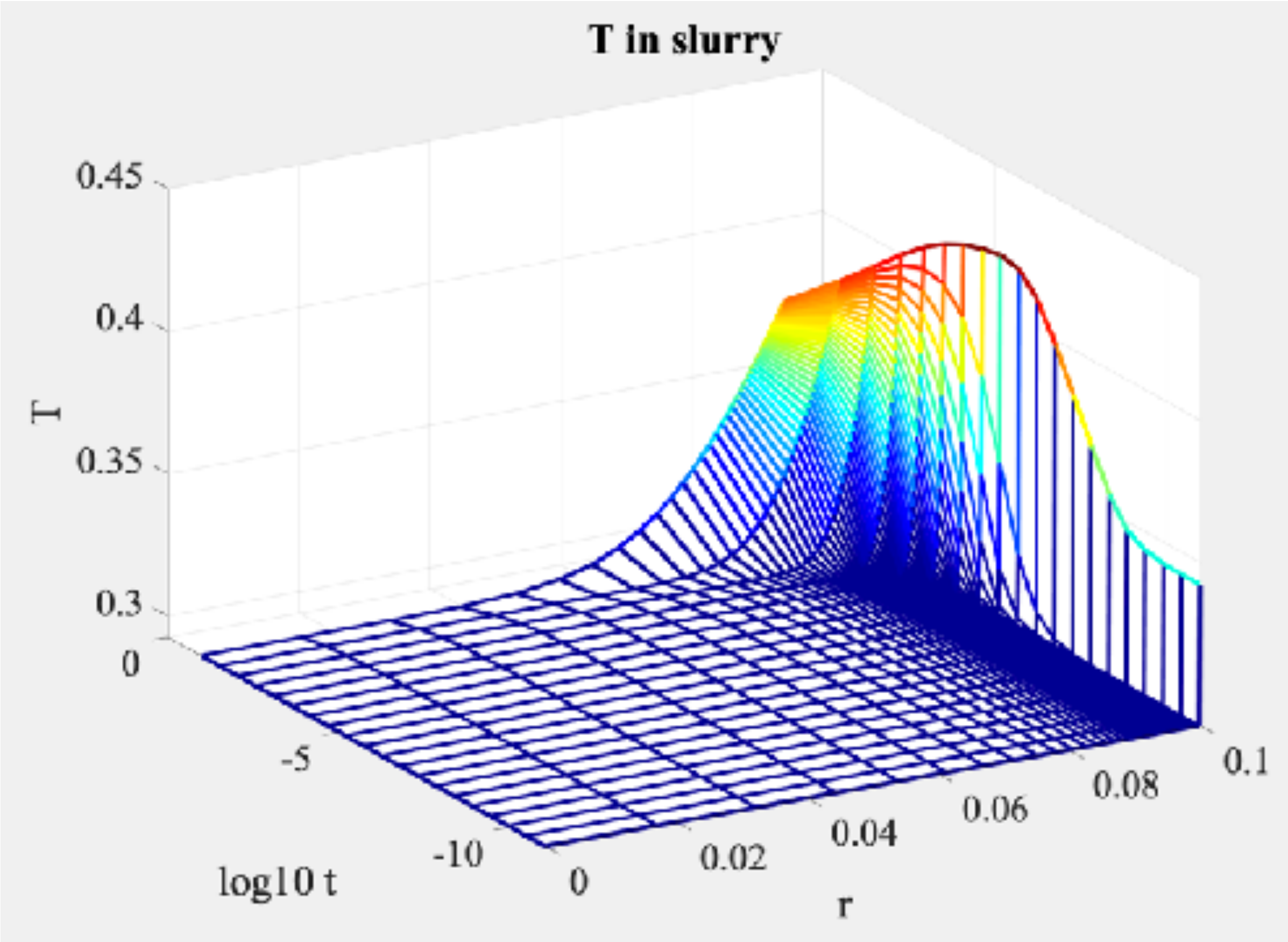
Then use method of lines. Upwind advection terms. Transform to non-uniform mesh, to resolve thermal boundary layer.

Numerical solutions

1cm inclusion
10cm bomb
moving flash front



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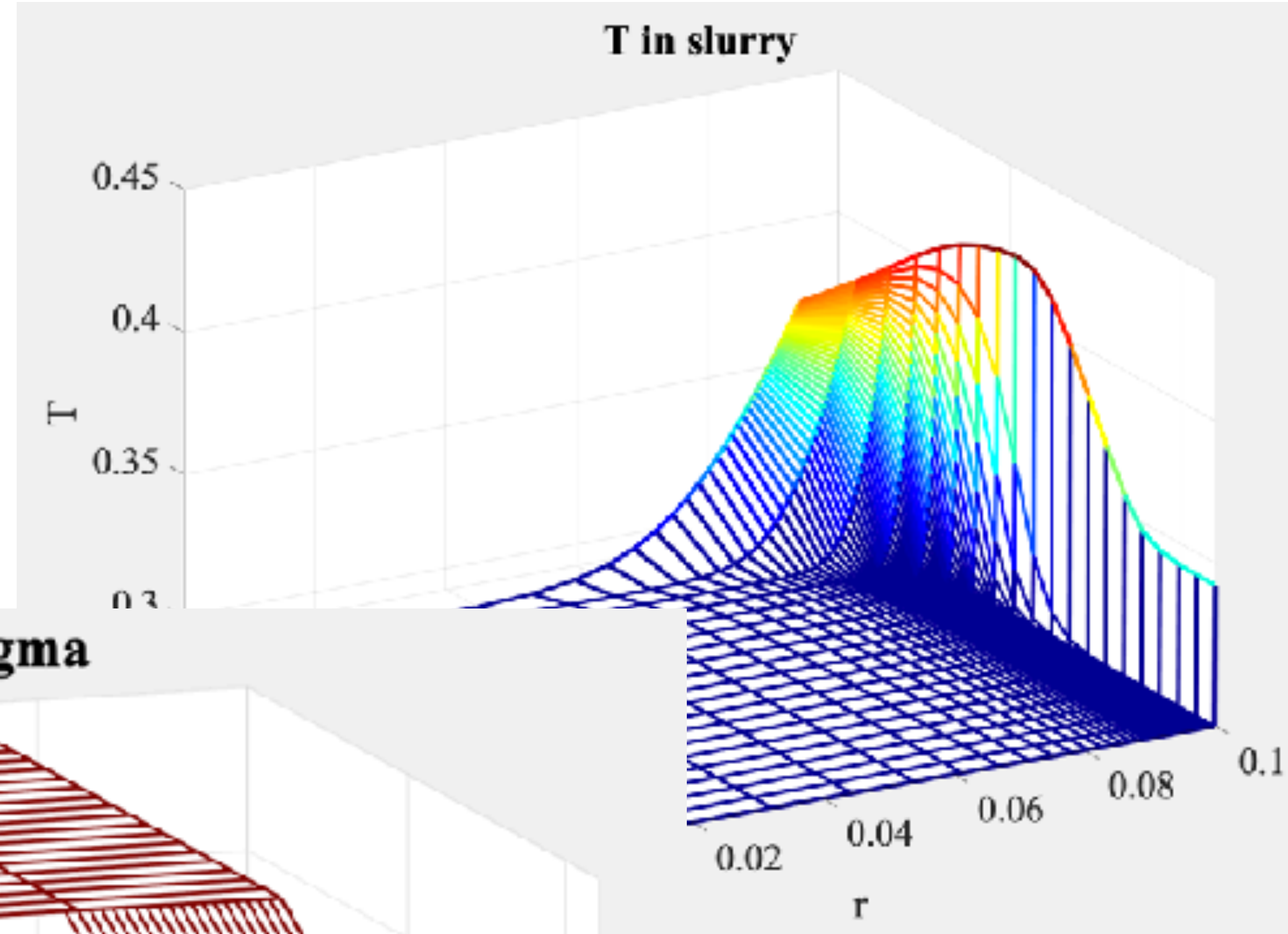

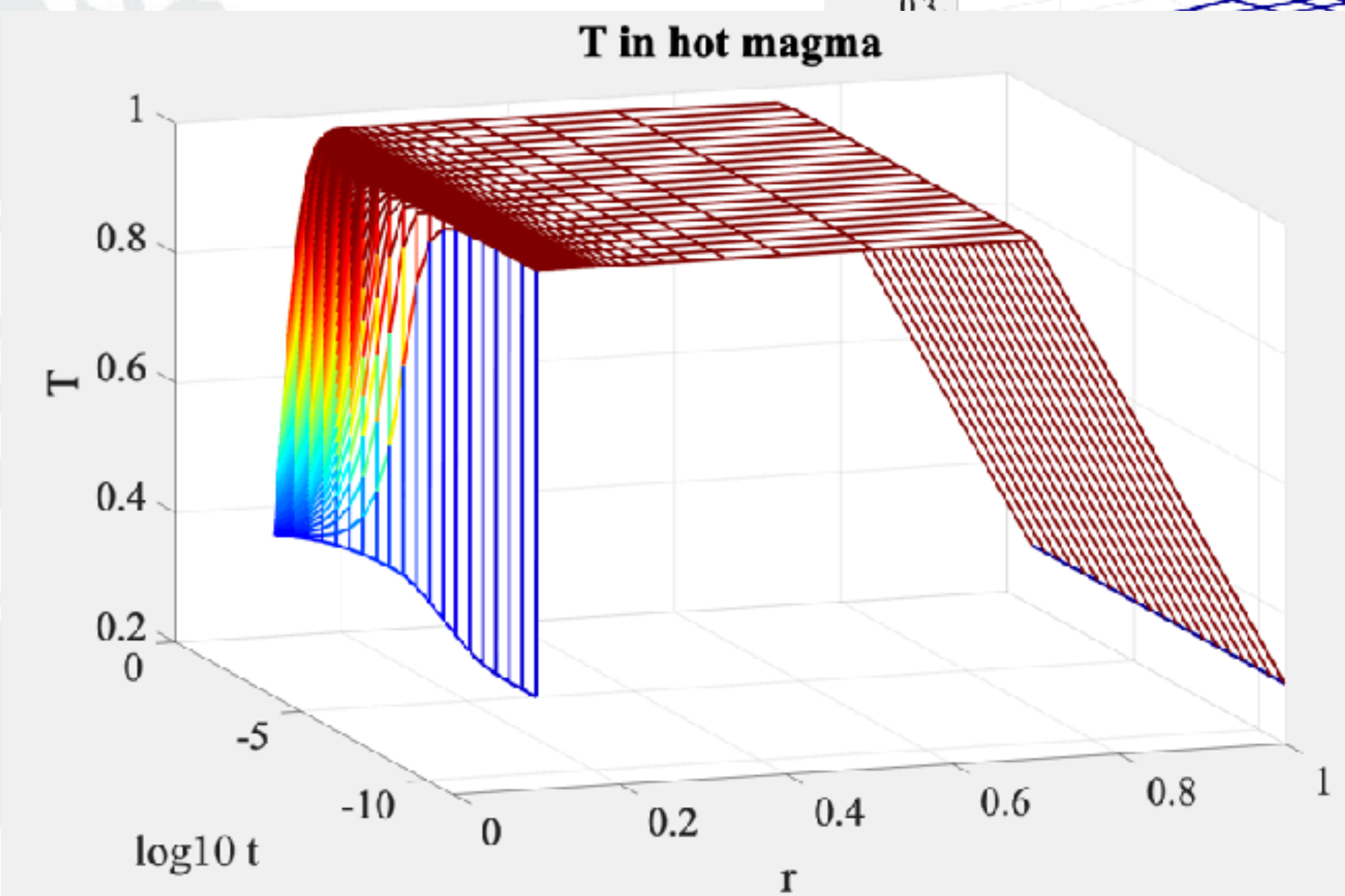

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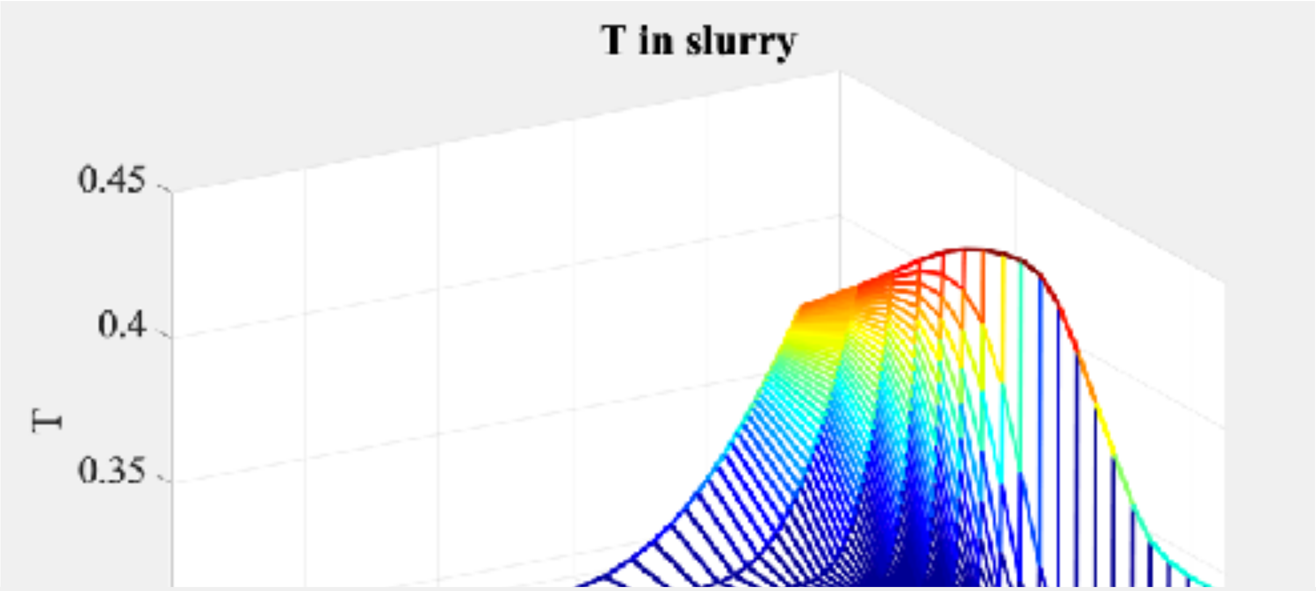

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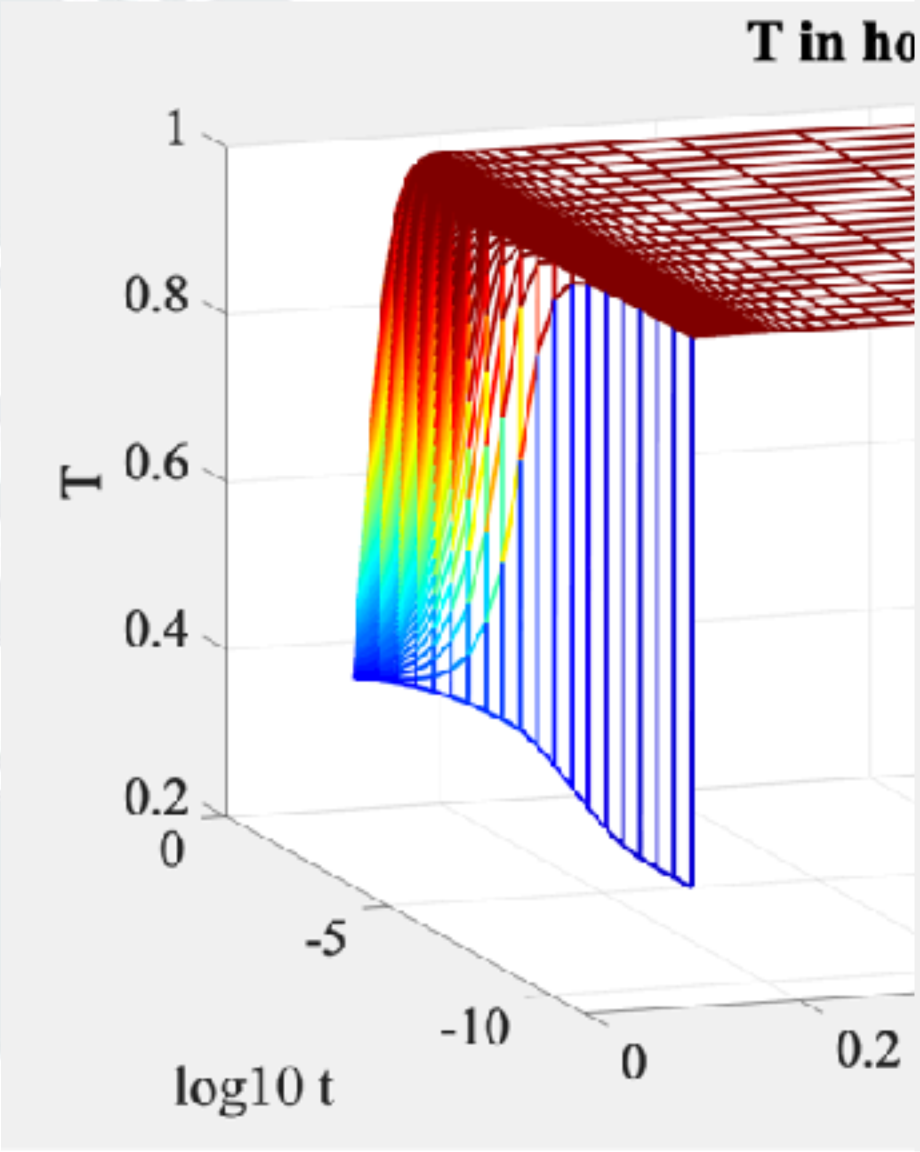
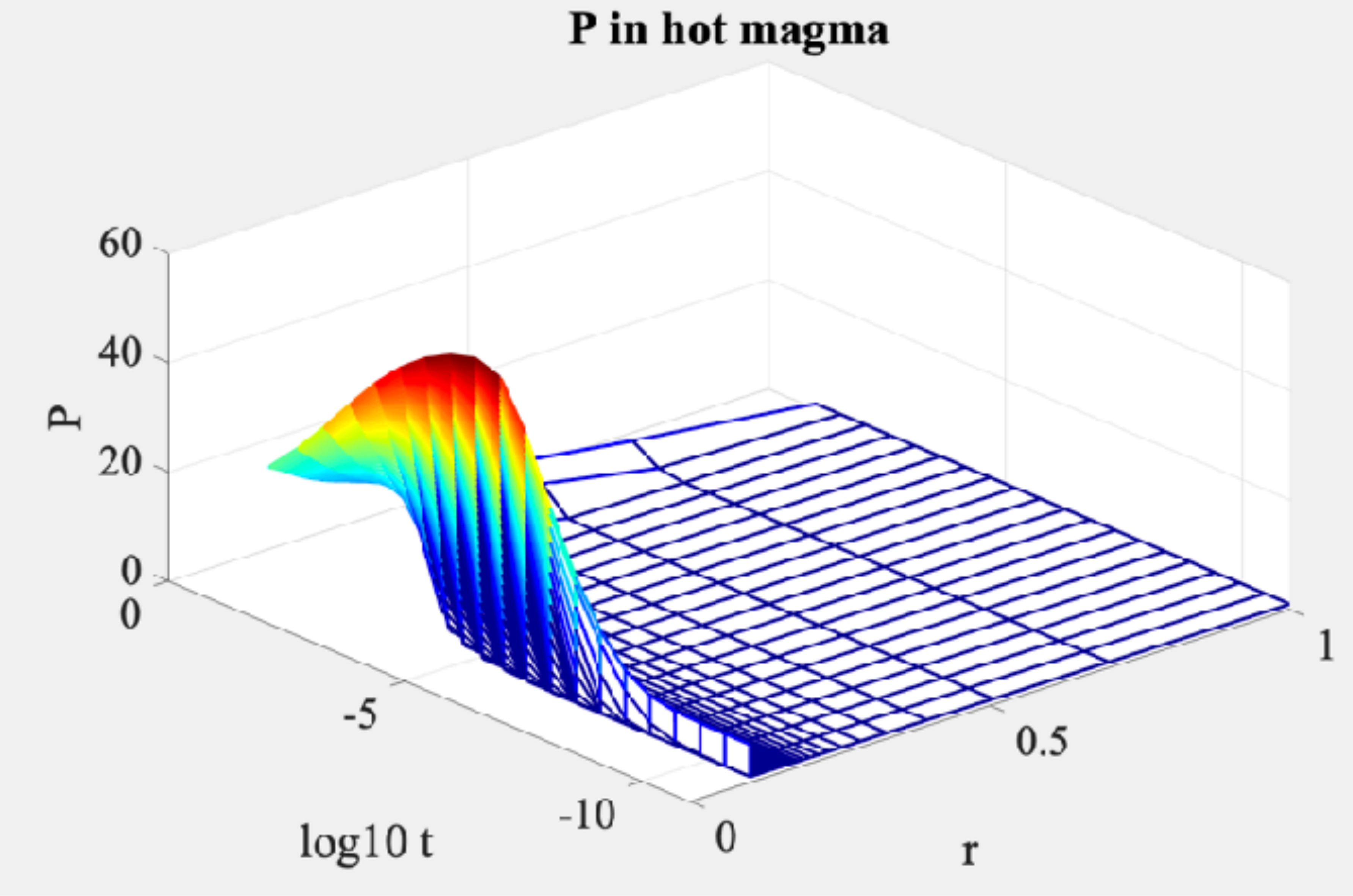
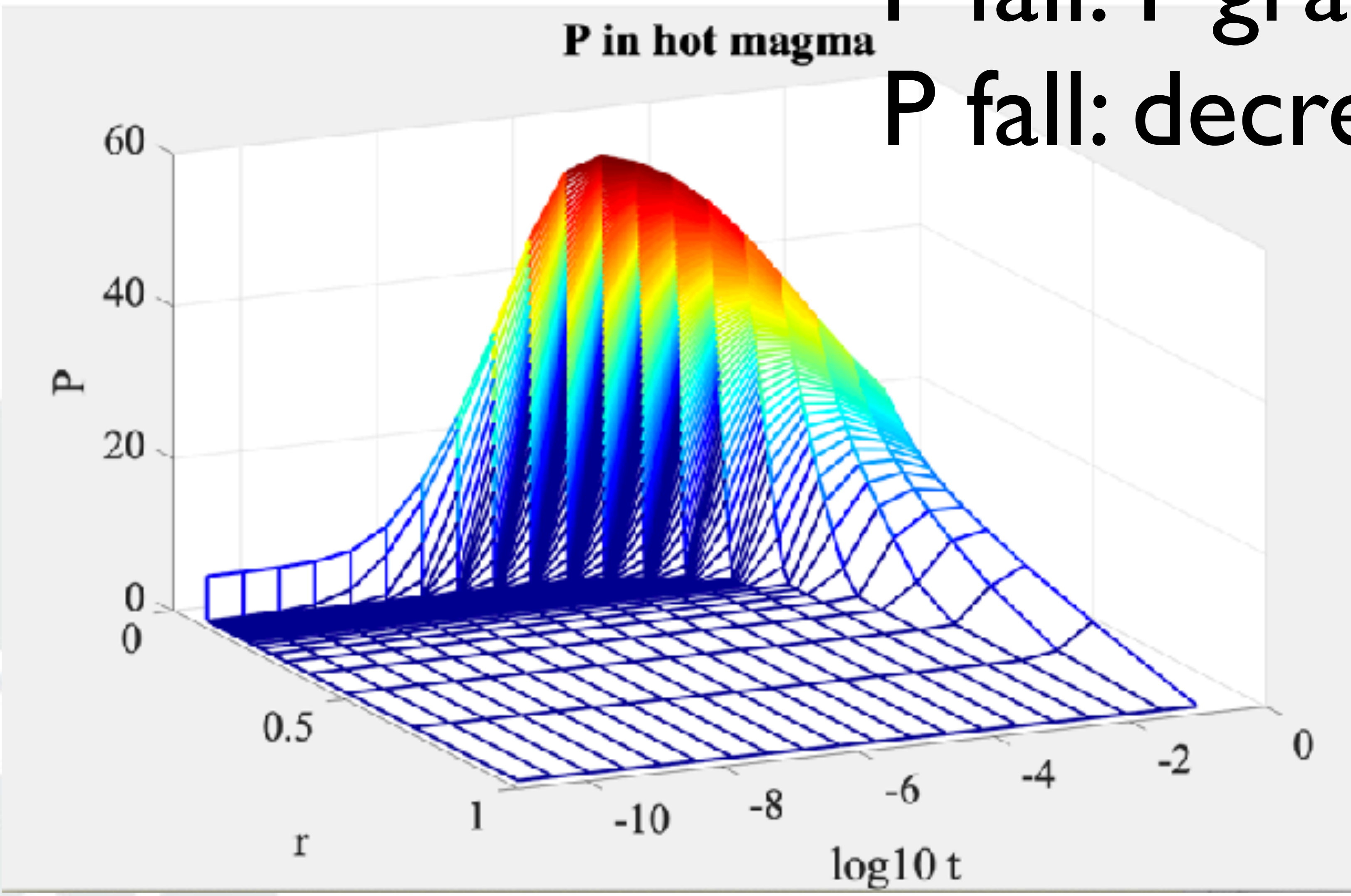
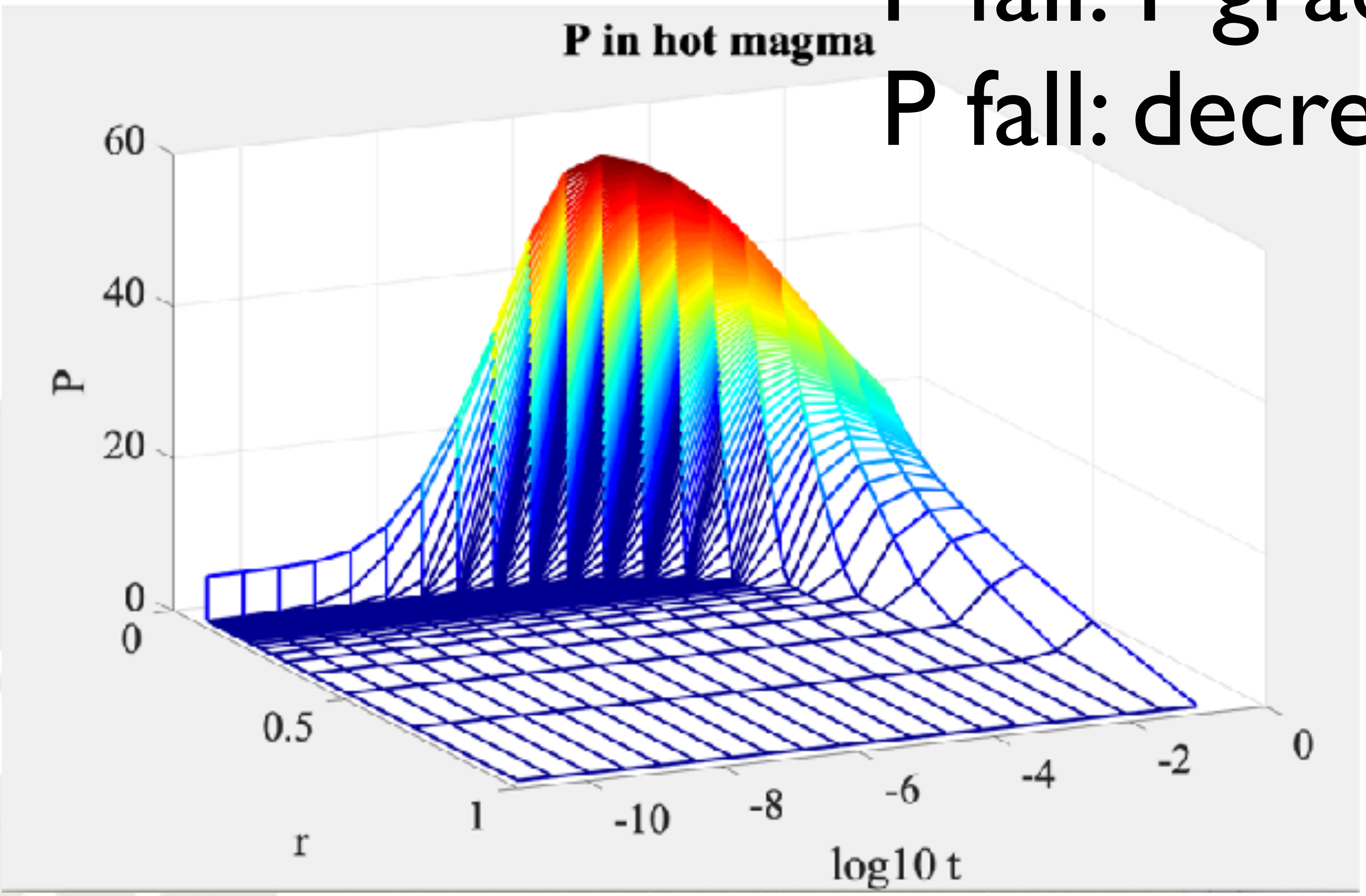




P rise: steam flux prescribed
P fall: T gradient easing off
P fall: decreasing slurry area



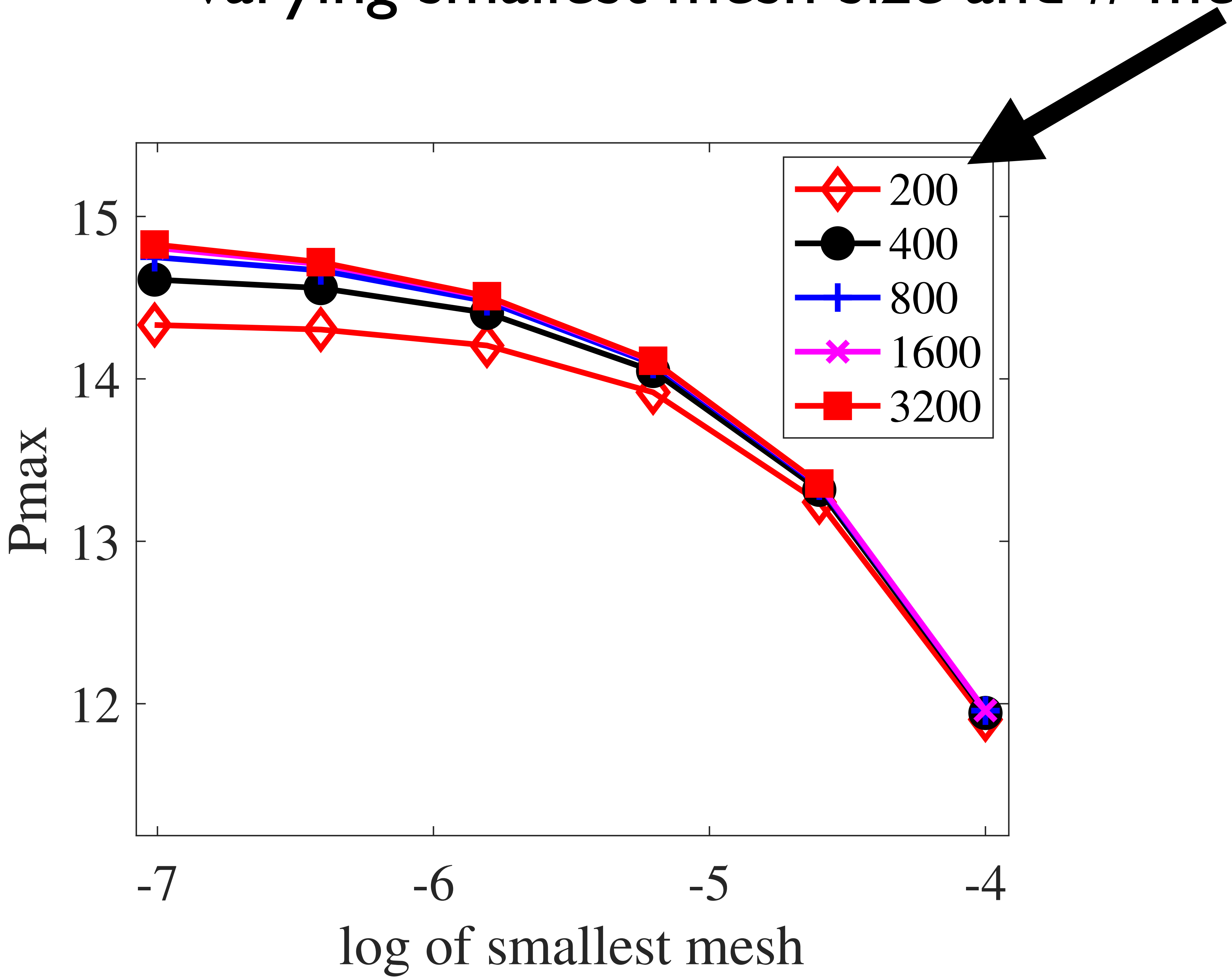
P rise: steam flux prescribed
P fall: T gradient easing off
P fall: decreasing slurry area



T~1.0 in magma
T~0.3 at flash
initial T gradient: unbounded?

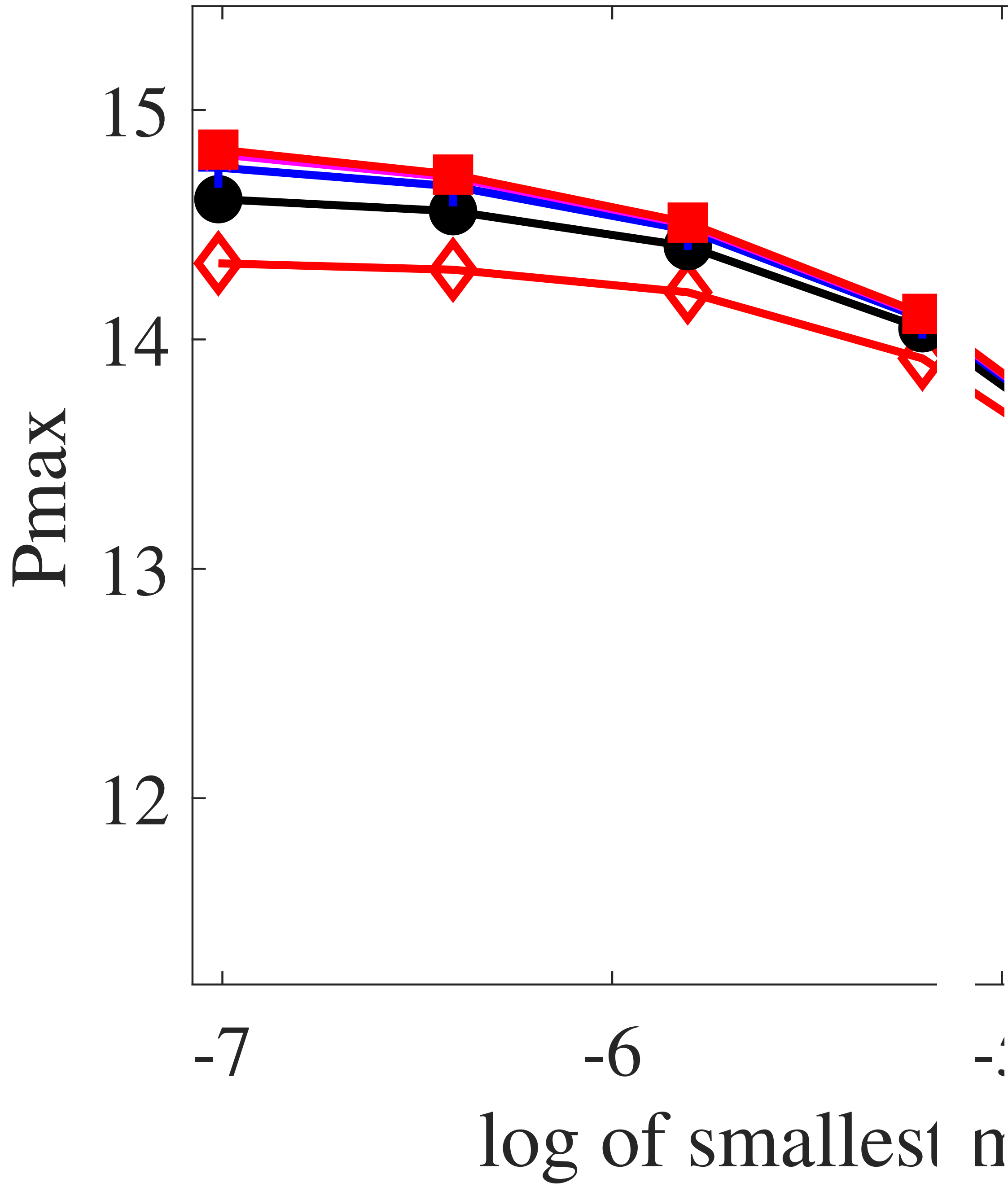
numerical convergence

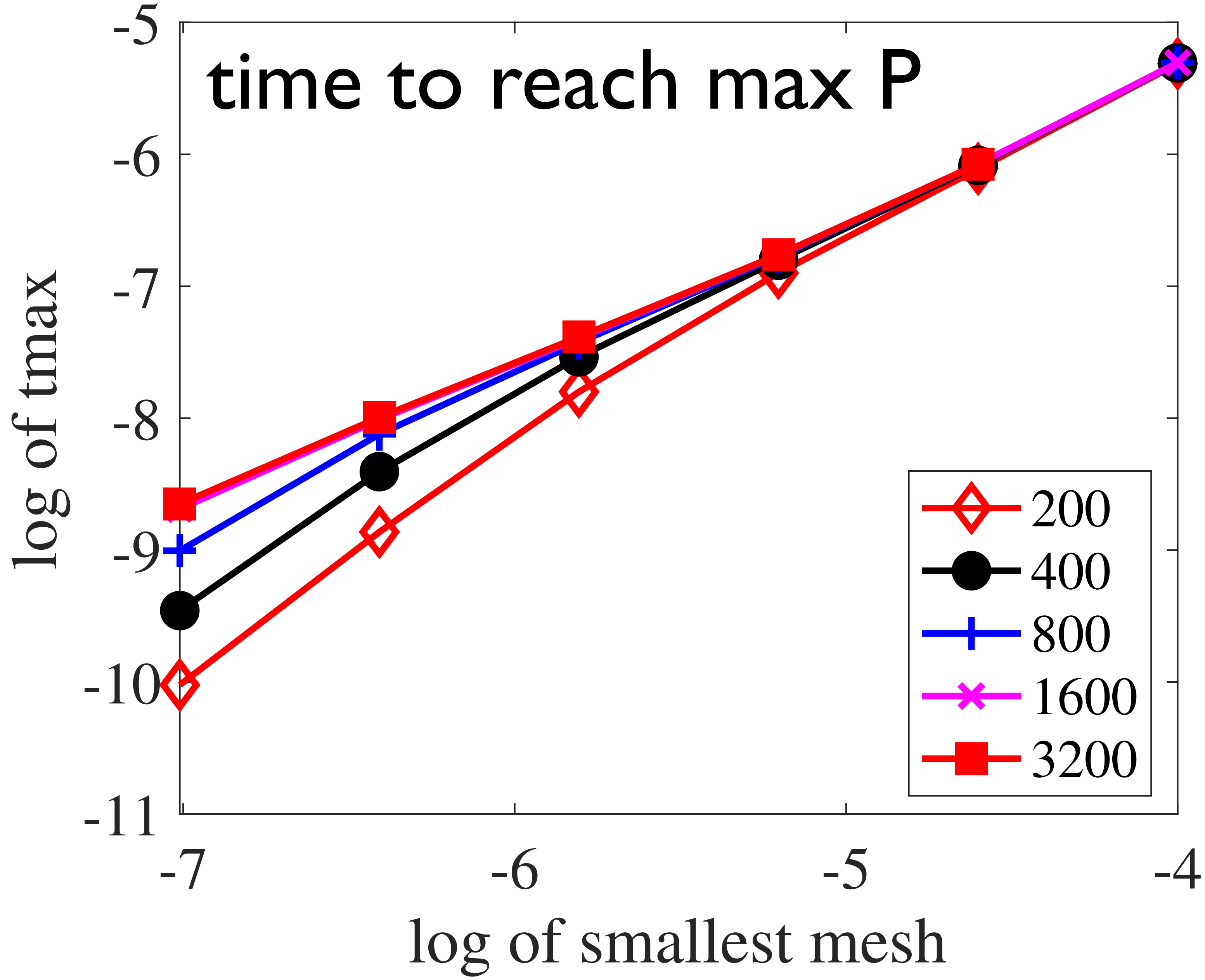
varying smallest mesh size and # mesh points



numerical convergence

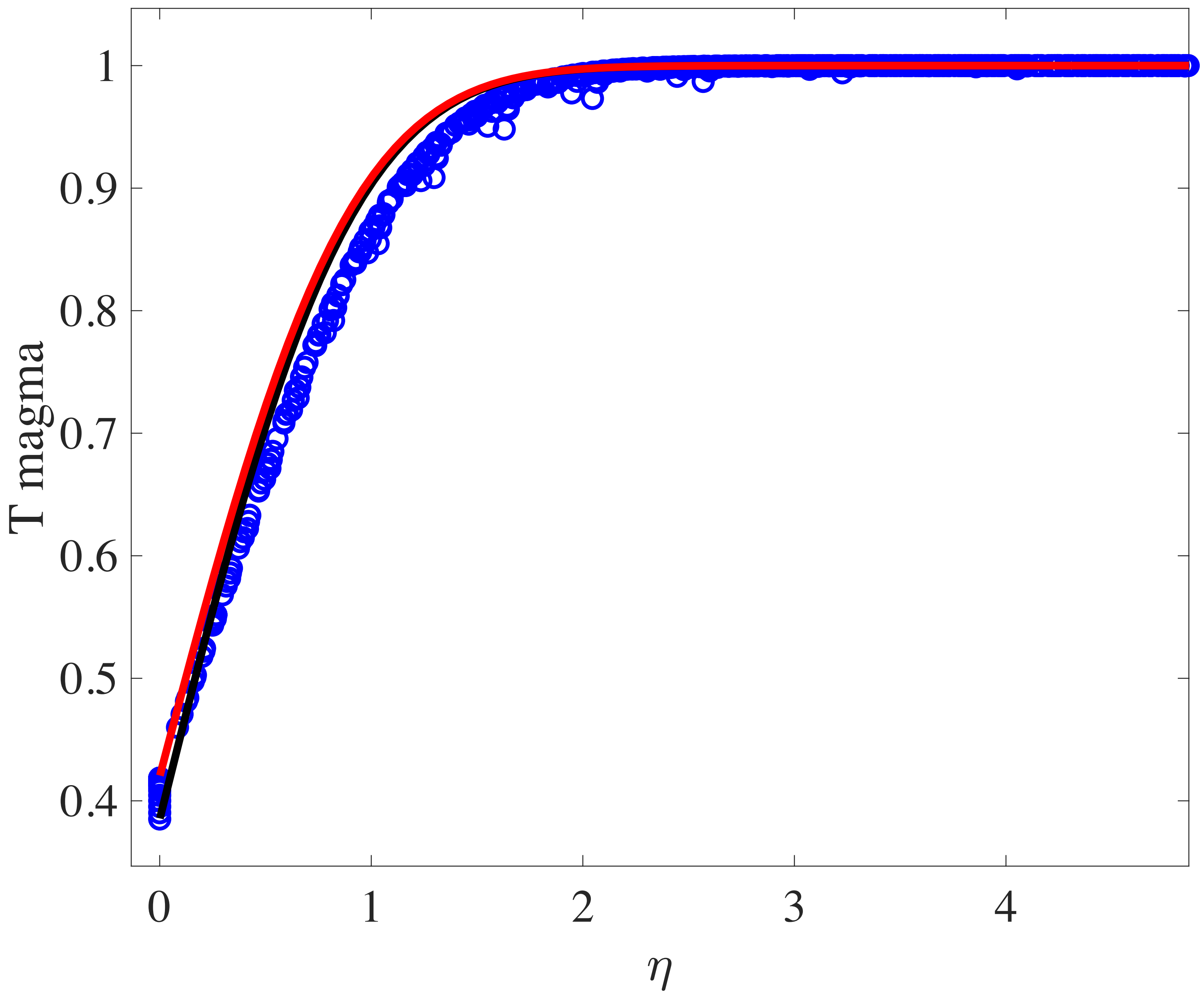
varying smallest mesh size and # mesh points



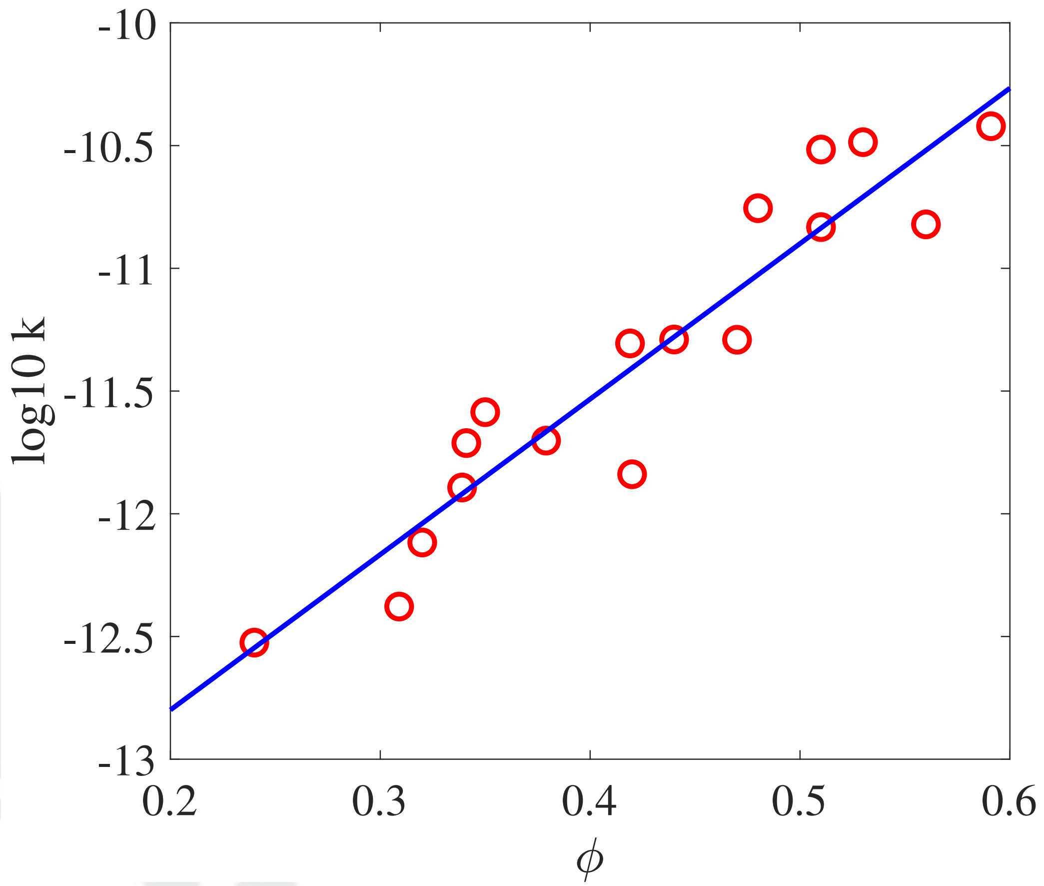


Thermal Boundary Layer

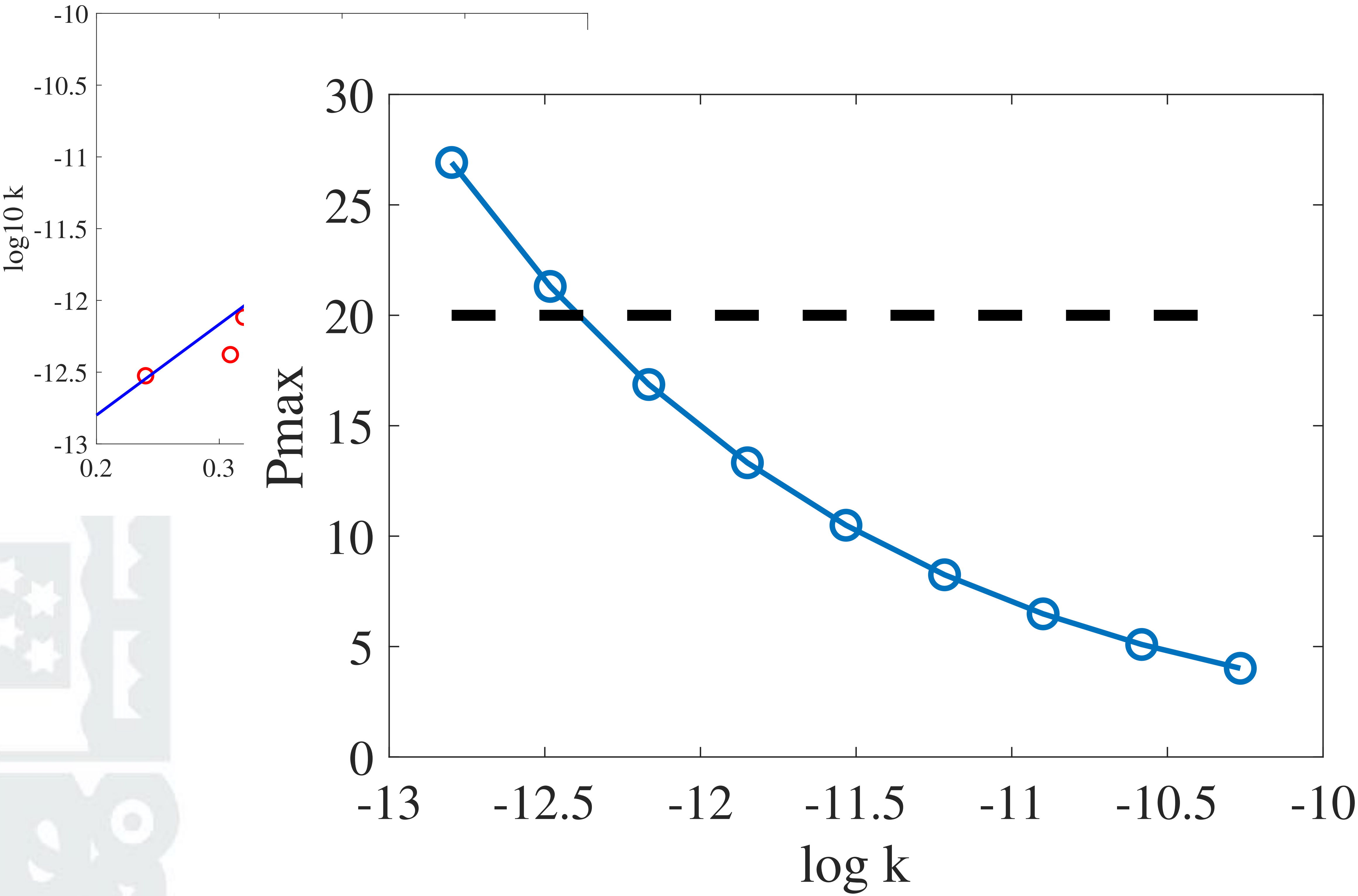
$$T = (1 - T_f)\ \operatorname{erf}\left(\frac{\sigma}{2\sqrt{t}}\right) + T_f$$



Data - bomb samples

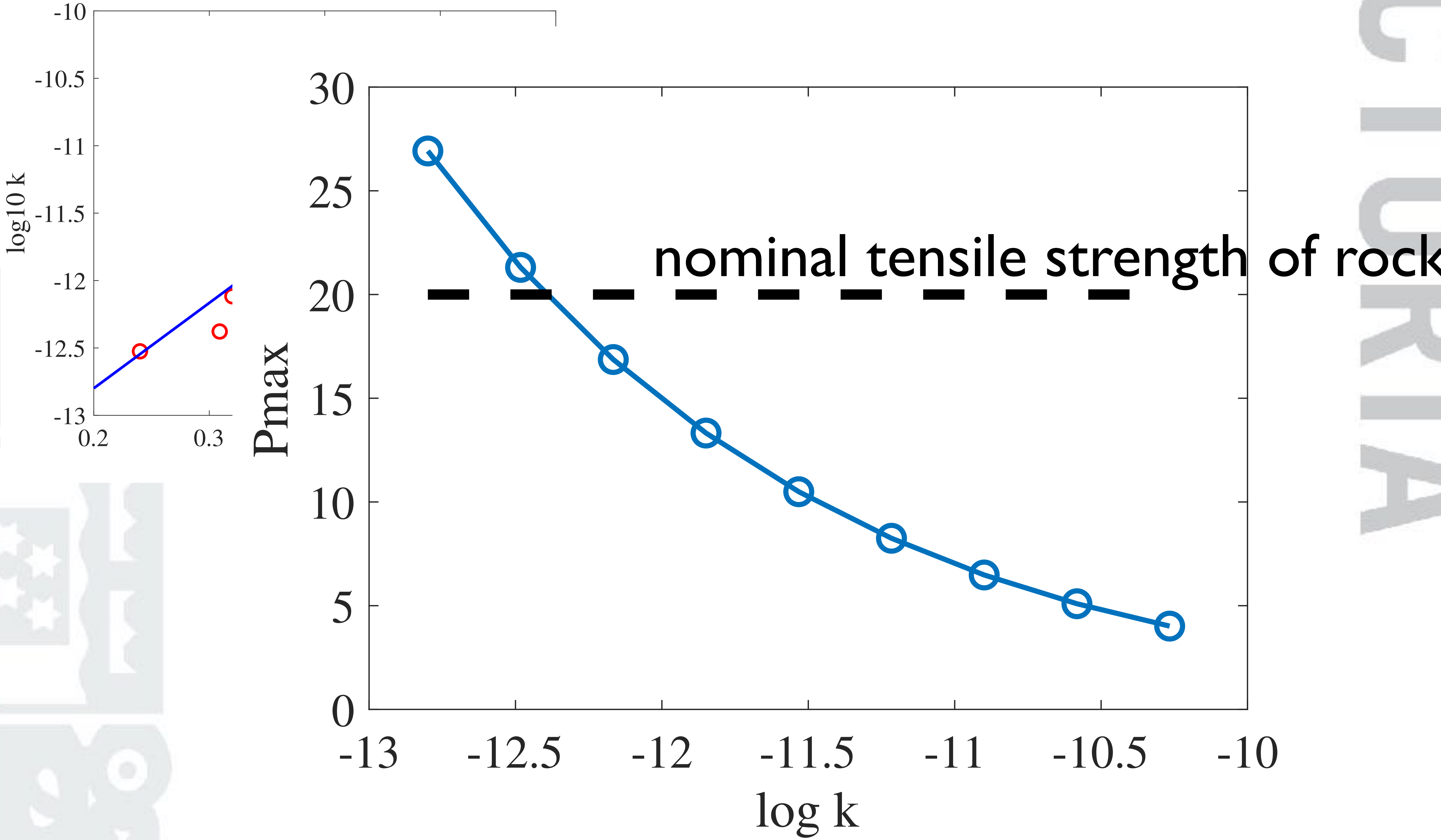


Data - bomb samples



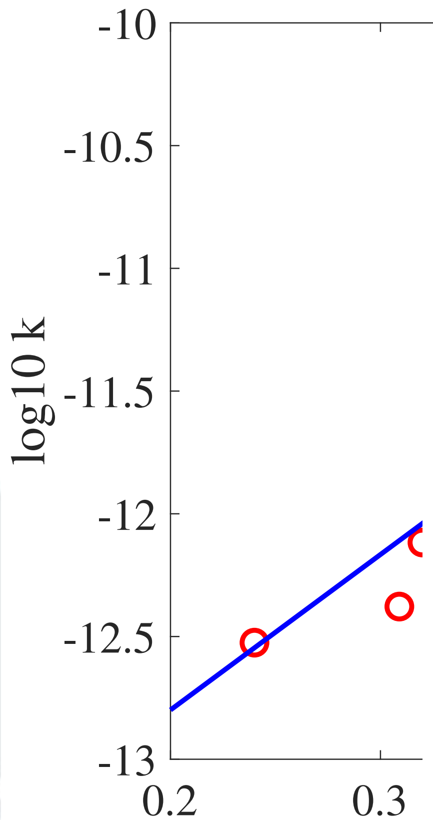
The maximum pressure simulated, using $\log k = 6.33\phi - 14.1$

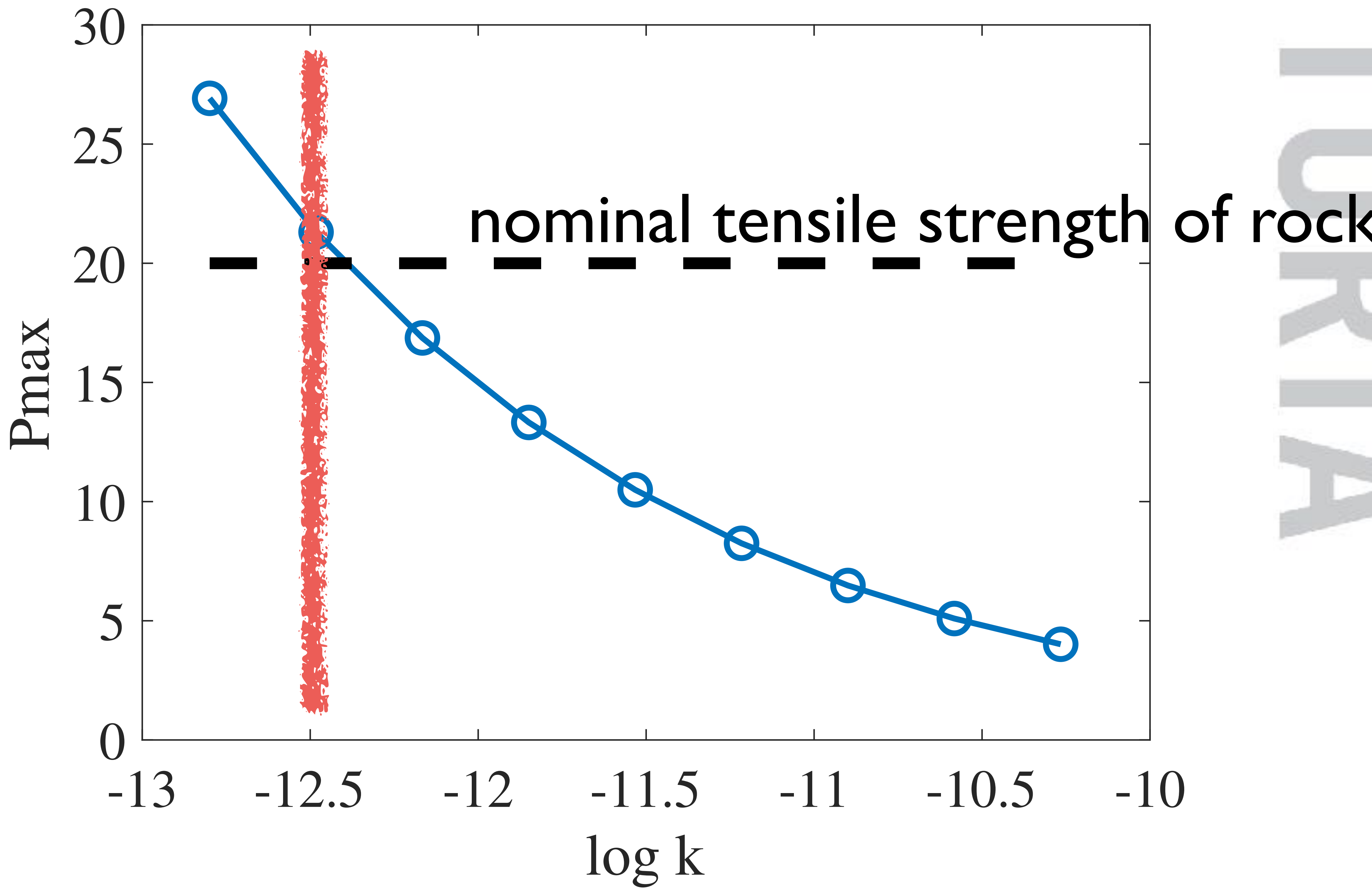
Data - bomb samples



The maximum pressure simulated, using $\log k = 6.33\phi - 14.1$

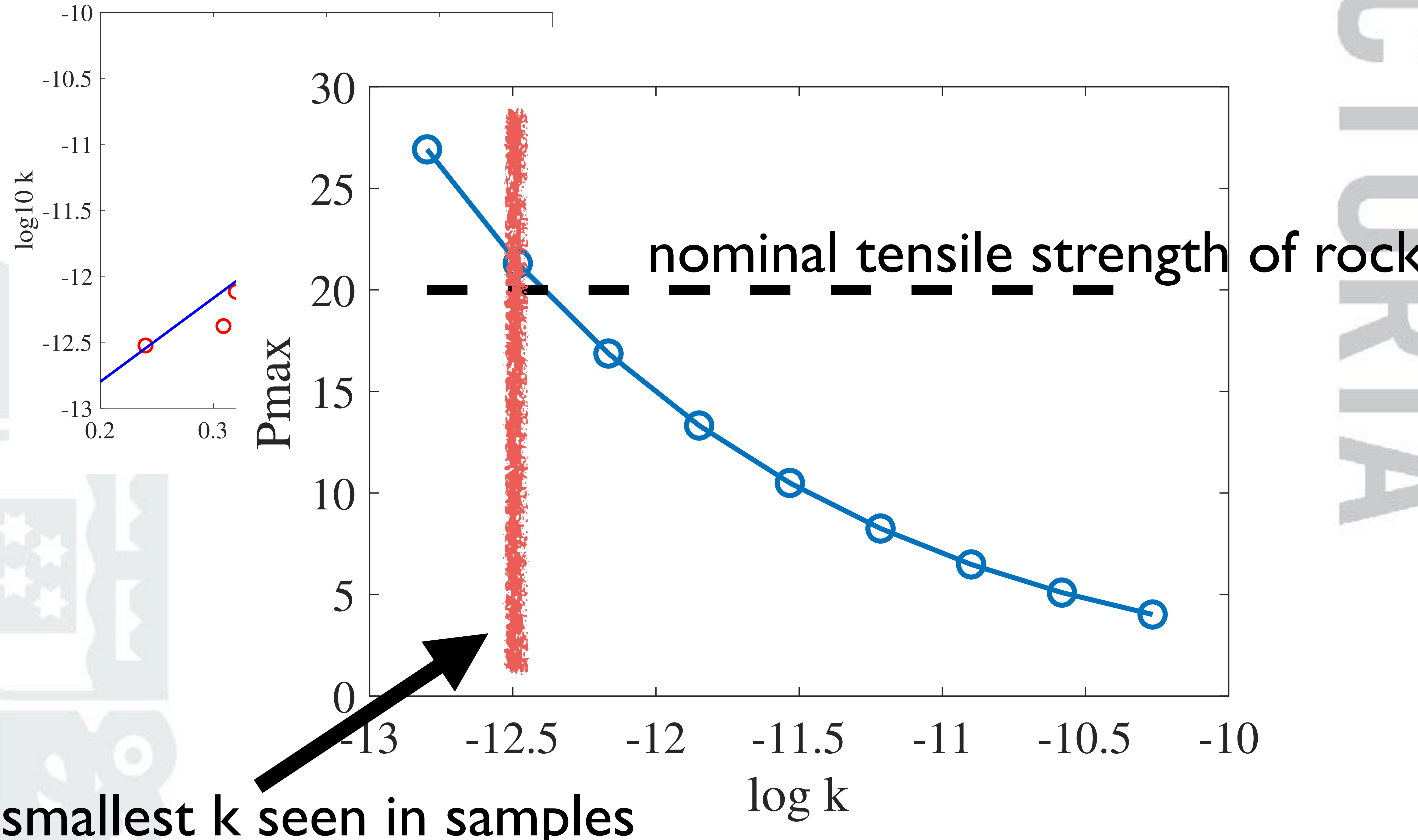
Data - bomb samples





The maximum pressure simulated, using $\log k = 6.33\phi - 14.1$

Data - bomb samples



The maximum pressure simulated, using $\log k = 6.33\phi - 14.1$

Conclusions

Hunga Tonga
Before & After

Conclusions

numerics and asymptotics
=> fragmentation criterion

Conclusions

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numerics and asymptotics

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pressure, temperature:
different timescales

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hope to use steady state pressure solution to
bound the maximum pressure

Thank you!