



Analysis of long-term catchment data: a nonlinear perspective

Holger Lange^{1,*}, Michael Hauhs², Katharina Funk², Sebastian Sippel^{1,3}
and Henning Meesenburg⁴

⁴ Northwest German Forest Research Institute, Göttingen, Germany

In the Harz mountains (Lower Saxony, Germany), forest monitoring of the Norway spruce stands (*Picea abies* Karst.) in the Bramke valley has been ongoing since the 18th century. Just after WWII, hydrological records started, while meteorology, deposition, hydrochemistry, soil hydrology and other ecosystem variables were measured since the 1960ies. Three small catchments – Lange (LB), Steile (SB) and Dicke (DB) Bramke (Fig. 1) – can be used for a paired catchment approach to quantify the impact of forest management on catchment-scale hydrology. The expected water balance under climate change and growing stands has been modelled with process models (Sutmöller and Meesenburg 2018), indicating drought stress and increased vulnerability of these forests.

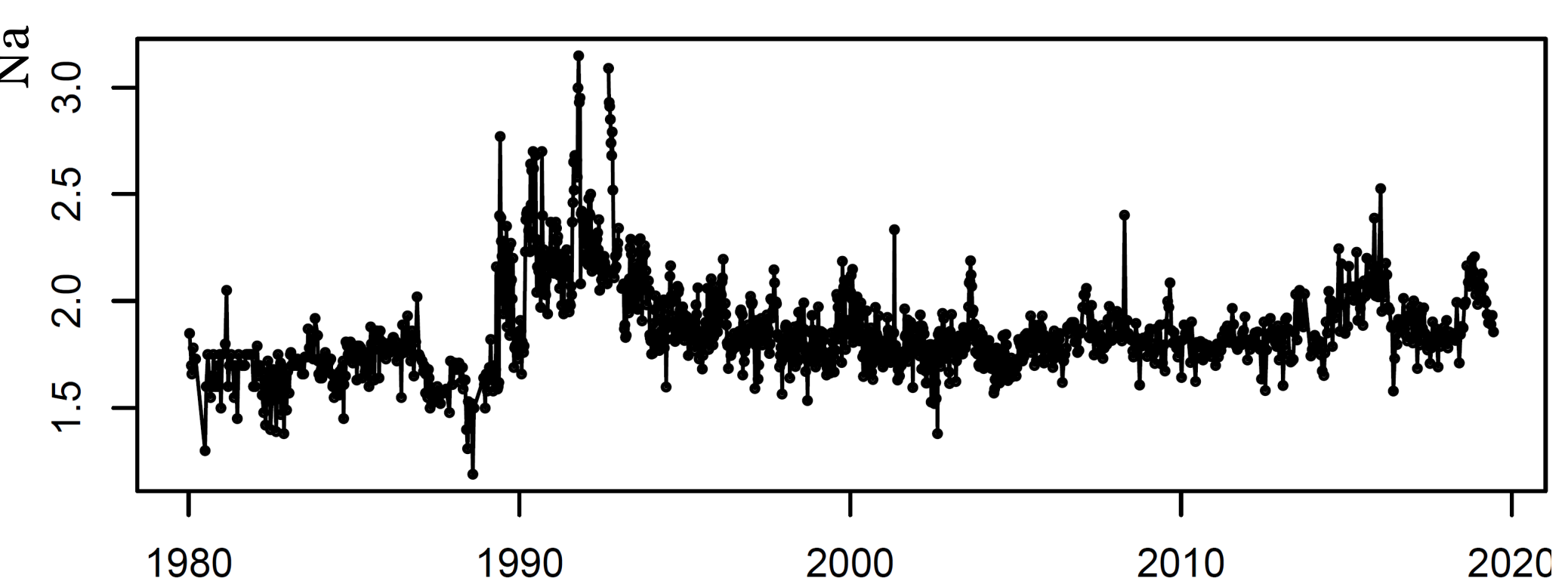
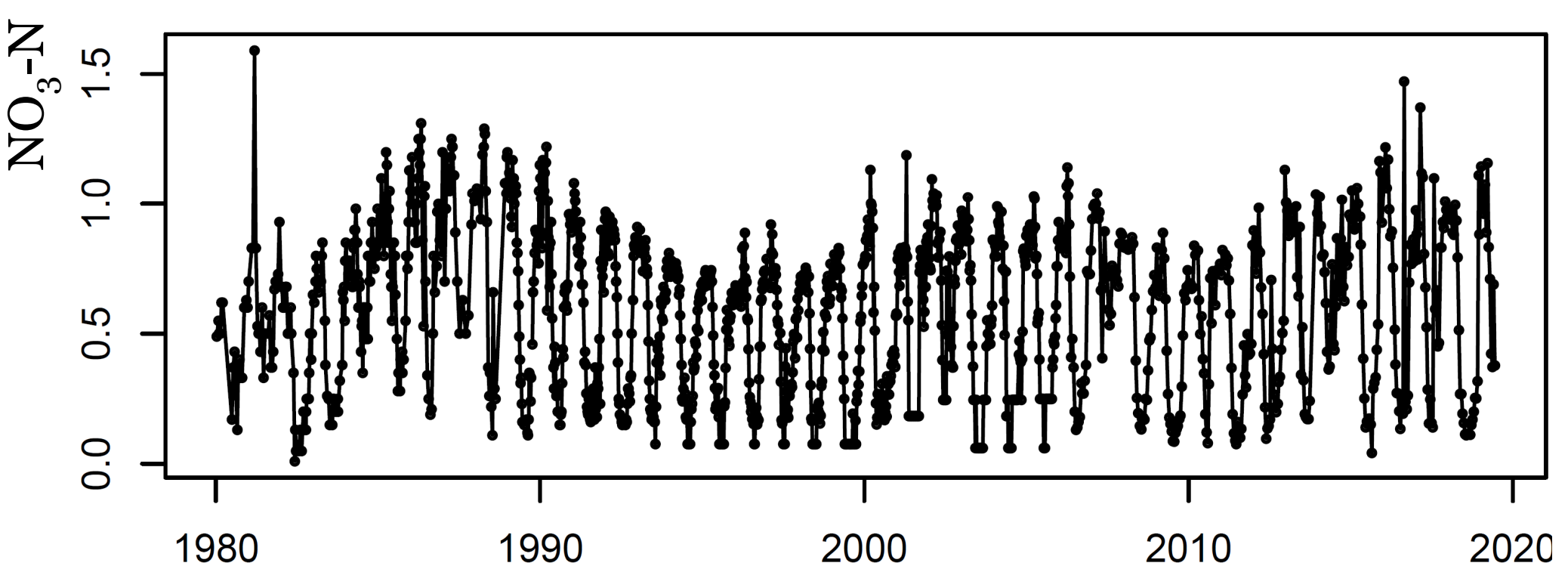
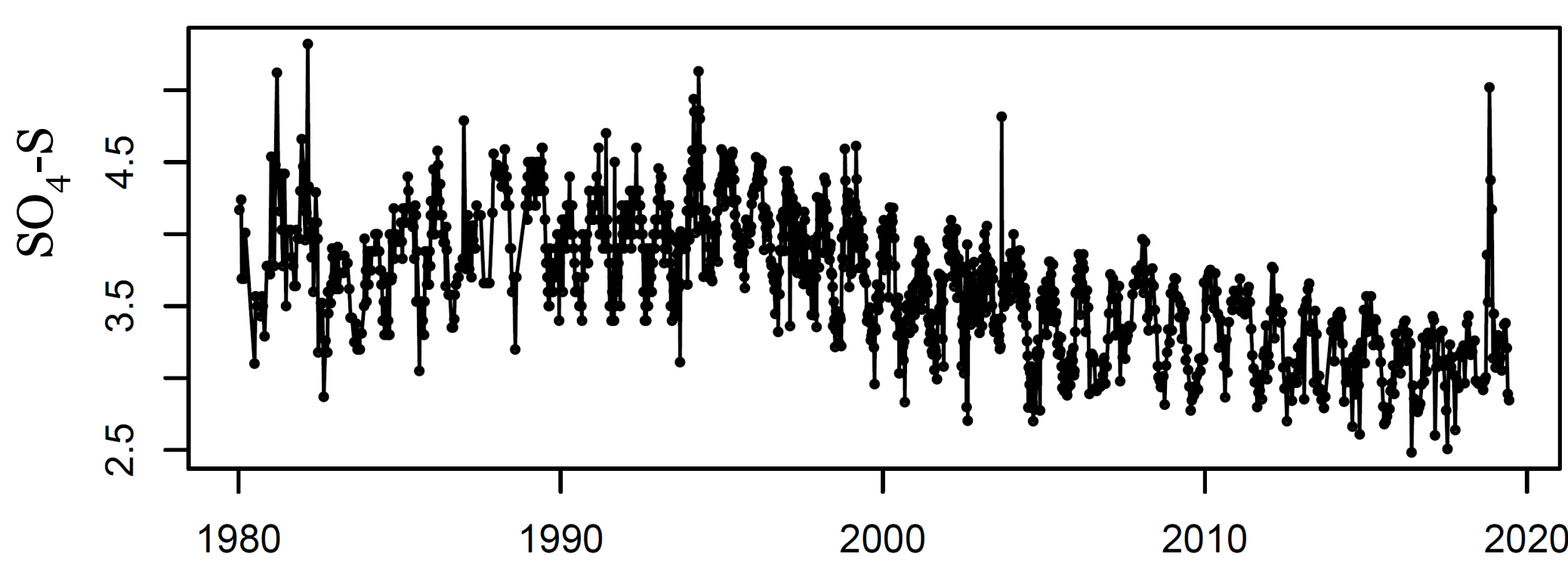
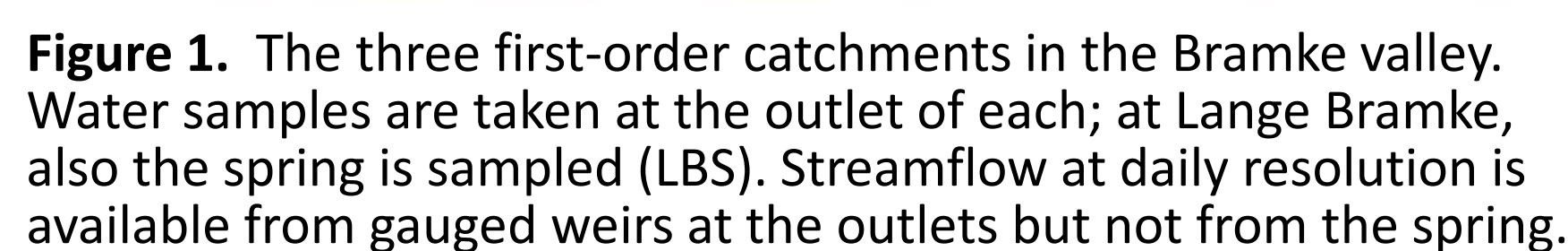


Figure 2. Some examples of the Bramke time series: hydrochemical concentrations at LBW 1980-2019. Note the strong decline of sulfate, the peak in nitrate in the mid-80ies, and the sharp increase in Na in the early 1990ies, also seen in Ca and Cl. Units are mg/l.

The recently introduced 2D diagrams of Tarnopolski (2016) display normalized squared successive differences vs. turning points. White noise is located at (1,1), parametric noise processes are 1D-curves.

The permutation entropy, complexity (Rosso et al. 2010) and Fisher information (Sippel et al. 2016) are based on ordinal patterns generated from the time series with word length $D = 4$. Recent extensions are (i) the q -entropy and q -complexity (Ribeiro et al. 2017) based on the q -logarithm $\log_q(x) = (x^{1-q} - 1)/(1 - q)$ (Tsallis 2009; $0 \leq q < \infty$ and $\log_1(x) = \log(x)$), and (ii) the (Renyi) α -entropy and α -complexity (Jauregui et al. 2018). They separate stochastic from deterministic-chaotic processes: the former show closed loops in the q -entropy vs. q -complexity plane, deterministic ones do not. They also differ qualitatively in their curvature in the α plots. The methods classify time series and enable a detailed data—model comparison.

The Tarnopolski diagram (Fig. 3) shows that Bramke hydrochemistry does not belong to a simple correlated stochastic process. The spread between catchments is as large as between the variables. The Entropy-Fisher Information (FI) plot for 8 different ions (Fig. 4; both concentrations and fluxes) shows that the time series are high-entropic, differ between catchments, and have systematically higher FI than a standard reference process, the fractional Brownian motion. Here, trends were removed nonlinearly using Singular System Analysis (Golyandina and Korobeynikov 2014). The q-entropy and complexity for NO_3 (Fig. 5) show closed loops, typical for stochastic processes with a fixed Hurst exponent. The difference between the catchments is pronounced.

The α -entropy and α -complexity (Fig. 6) for hydrometeorological data and catchment runoff reveals that the output from the systems is qualitatively different from the input since the curvature is different.

The comprehensive data from the Bramke valley show complex dynamic behaviour, not easily attributable to simple stochastic processes. There is a main difference between input and output variables, and although the output time series belong to the same process class, they differ markedly between catchments. In a next step, we will relate these differences to historic forest management and disturbance regimes, and also investigate the stability of the patterns against changing temporal resolution. The framework also provides a very strong testbed for the validation of process-based models.

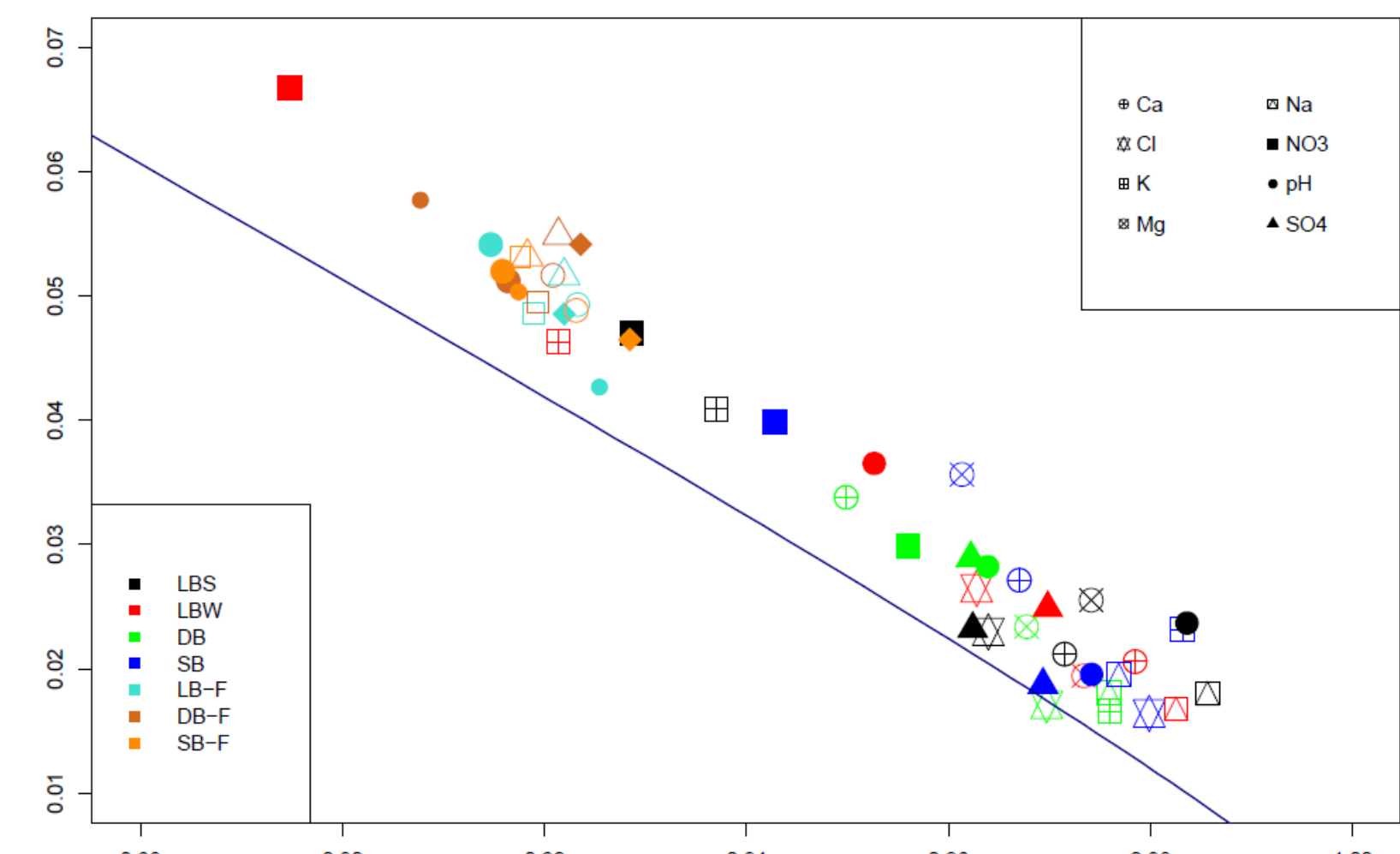
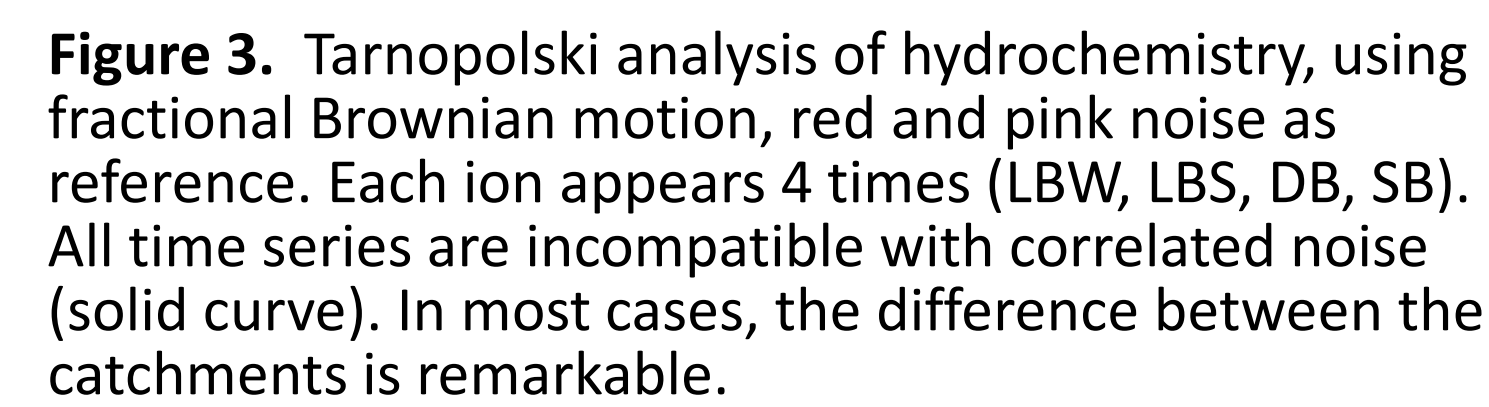


Figure 4. Permutation Entropy and Fisher Information for eight different ions from the Bramke. Colours indicate catchments. The solid line is for the fractional Brownian motion.

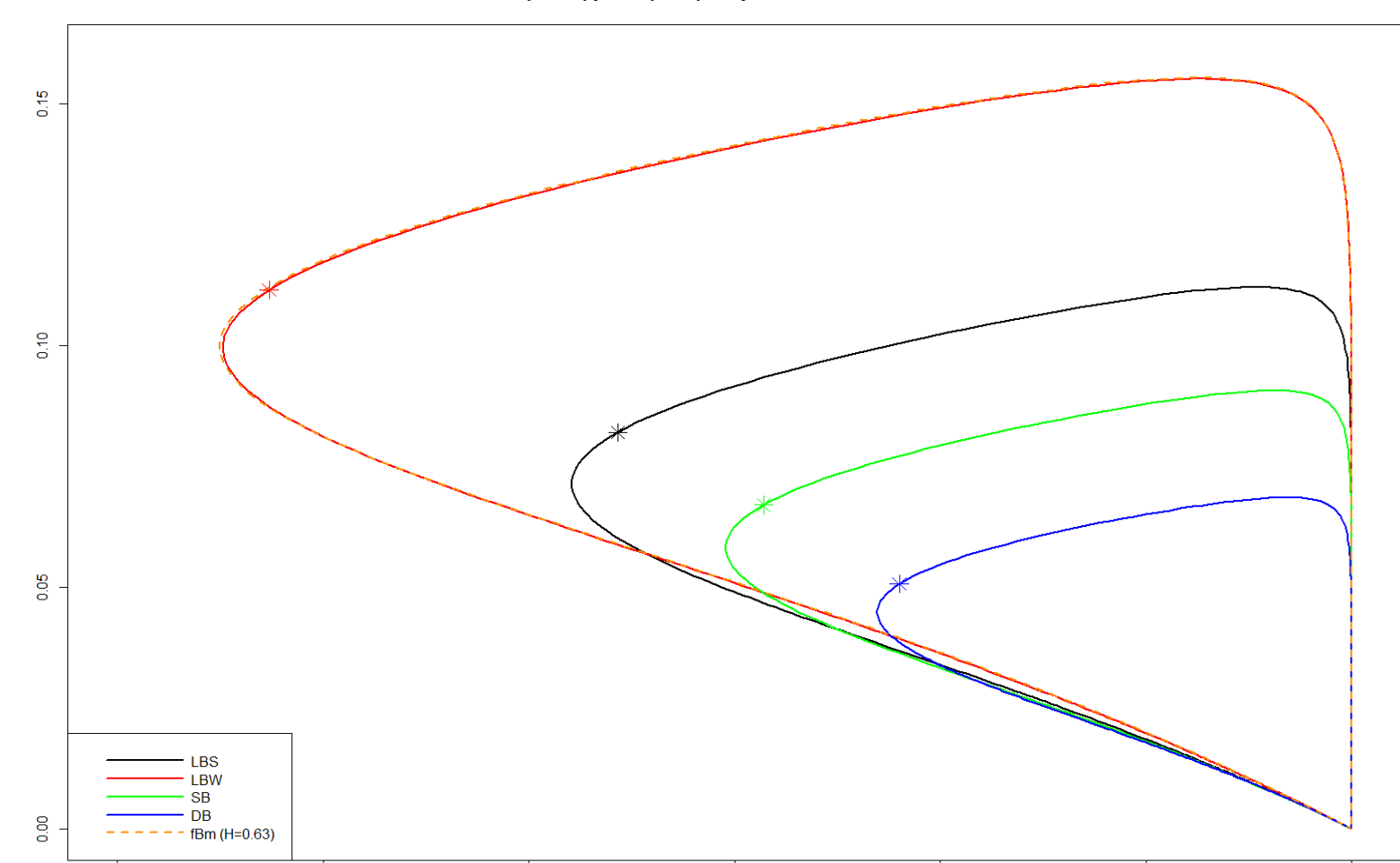


Figure 5. q-entropy and complexity for NO_3 from the Bramke catchments. q runs from 10^{-3} to 10^2 . Closed loops indicate long-range stochastic processes. The dashed line is for fractional Brownian motion with a Hurst exponent $H=0.63$. The conventional values ($q=1$) are marked.

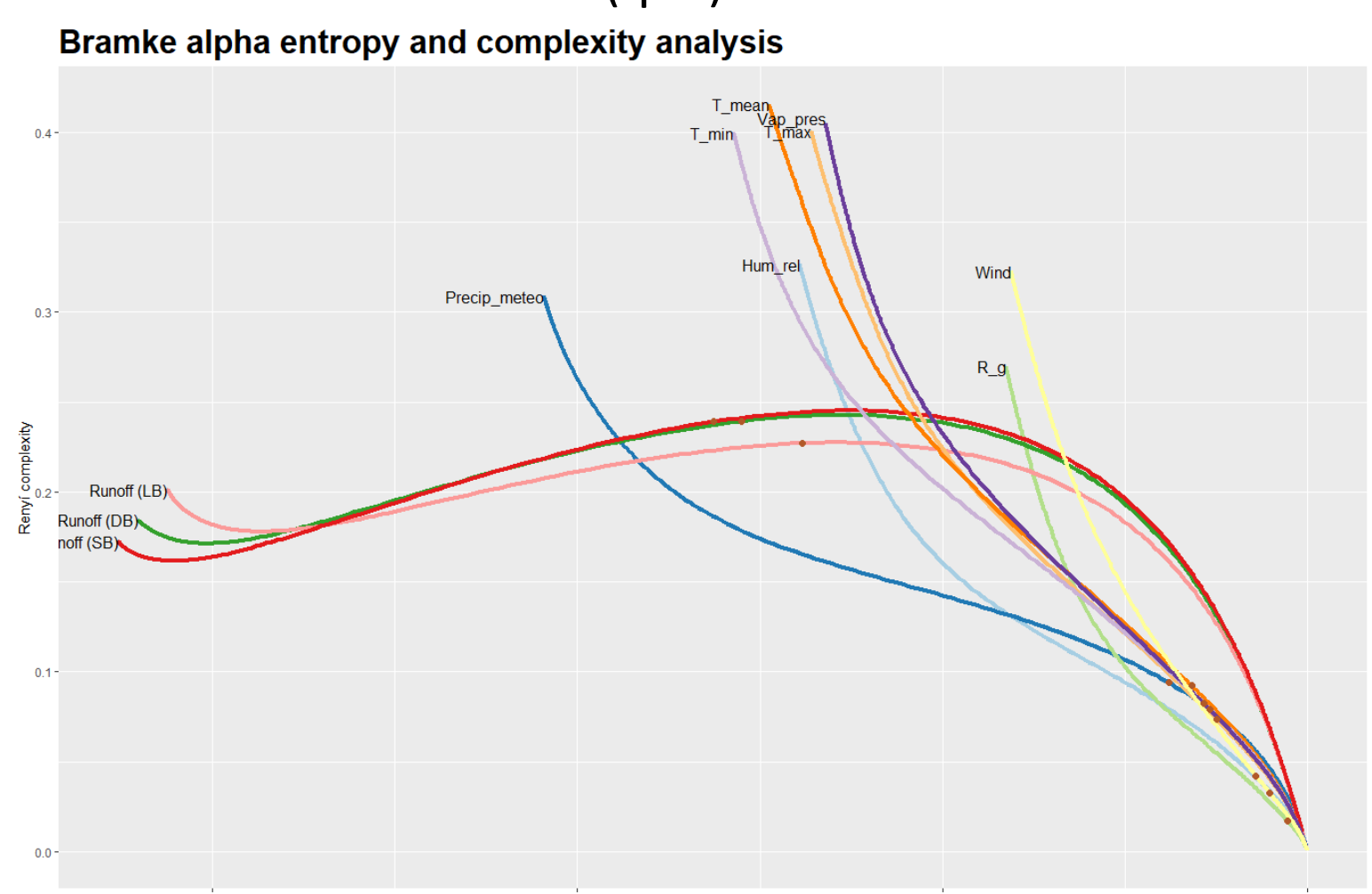


Figure 6. α -entropy and α -complexity climatological variables and runoff flow rates. The curvature indicates the process class. The conventional values ($\alpha = 1$) are marked.

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