When minerals fight back: The relationship between back stress and geometrically necessary dislocation density

Christopher A. Thom¹, David L. Goldsby², Kathryn M. Kumamoto¹ & Lars N. Hansen³

¹ Department of Earth Sciences, University of Oxford
² Department of Earth and Environmental Science, University of Pennsylvania
³ Department of Earth and Environmental Sciences, University of Minnesota

EGU General Assembly 2020
Wednesday, May 6th
Post-seismic stress perturbations can cause transient changes in rheology.

- Post-seismic stress transfer can produce transient changes in viscosity, which are often modeled using a Burgers rheology.

- The functional form of transient creep flow laws is unknown (Masuti et al., 2016), so Burgers elements are often parameterized empirically.

- Our goal is to show evidence for physically motivated parameters.
Long-range interactions among geometrically necessary dislocations (GNDs) suggest a different physical picture than a standard Burgers model.

These slides will present experimental data to support a theoretical prediction of this physical mechanism (Taylor, 1934).
Back stress ($\sigma_{\text{disl}}$) is a function of the Burgers vector ($b$), shear modulus ($G$), and most importantly, GND density ($\rho_{\text{GND}}$)

$$\sigma_{\text{disl}} = \alpha G b \rho_{\text{GND}}^{0.5}$$

(Taylor equation, 1934)

As the spacing between dislocations decreases (i.e. the density of GNDs increases), the elastic distortion of the crystal lattice caused by GNDs begins to interact.
Stress reduction experiments on olivine single crystals demonstrate the importance of long-range dislocation interactions in transient rheology

- Experiments performed at 1250-1300°C and 1 atm.
- Initial elastic strain after a stress reduction is proportional to the magnitude of the stress drop (black dots, see right).
- When the stress is reduced by less than 50%, the sample rapidly reaches a new steady-state creep rate, but larger stress reductions result in a prolonged anelastic recovery before achieving a new creep rate.
- Back stress can therefore be estimated at ~100 MPa.
- High-angular resolution EBSD maps show geometrically necessary dislocation (GND) density of ~7 x 10^{10} m^{-2}.

Wallis et al. 2017; Hansen et al. in prep.
Cyclical deformation experiments on stacked olivine samples in the D-DIA apparatus also measure dislocation-induced back stress.

Room temperature experiments on olivine reveal grain size sensitive yield stress (i.e. “smaller is stronger”), significant strain hardening, and a Bauschinger effect (see blue circles; the yield stress is reduced in extension due to long-range dislocation interactions among GNDs accrued during hardening).

Single crystal in experiment San382 had a back stress of 1.8 GPa and GND density of $4.5 \times 10^{13} \text{ m}^{-2}$. 

Hansen et al. 2019

Wallis et al., in revision at EPSL
Nanoindentation creates extremely large GND density, making it a useful technique to test a wider range of microstructures.

Predicted densities of GNDs from geometric arguments agree with HR-EBSD measurements in geologic materials.

By varying the maximum indentation depth, we can control the GND density.

\[
\rho_{GND} = \frac{3 \tan^2 \theta}{2bh_{\text{max}}}
\]

Pharr et al., 2010

Kumamoto et al. 2017

20 μm

10 μm

Kumamoto et al. 2017
Standard nanoindentation creep test uses contact stiffness and the known elastic modulus to determine the contact stress.

Contact stress = \( \frac{P}{A} \)

- **E**: elastic modulus (known)
- **S**: contact stiffness (measured)
- **A**: contact area (calculated)
- **P**: applied load (known)

\[ A = \frac{\pi S^2}{4E^2} \]

During nanoindentation creep, the tip sinks into the sample, resulting in an increase of stiffness over time.

After Thom et al., 2018; Thom & Goldsby, 2019
In these experiments, after a creep section of 60 s, we drop the load by a prescribed amount and hold for 3600 s at the new applied load, akin to a stress reduction test.

Three behaviors are predicted:
1) Creep continues at a reduced rate
2) No creep (back stress equal to applied stress)
3) Reverse/back creep (back stress bigger than applied stress)
Single crystal olivine loaded to 5 mN (~120 nm depth) at room temperature with different magnitude load drops:
After converting contact stiffness to the applied stress (hardness), we can determine the back stress (~13.8 Gpa).
We tested 3 types of single crystals at several maximum applied loads (to vary initial GND density) and determined the back stress.

![Graph showing back stress vs. GND density for olivine, quartz, and plagioclase](image)

- Each solid line is a best fit to the data (olivine = 0.44, quartz = 0.55, plag = 0.46).
- Dashed lines are a forced fit of the Taylor equation (slope = 0.5).

\[ \sigma_{\text{disl}} = \alpha G b \rho_{\text{GND}}^{0.5} \]

(Taylor equation, 1934)

Average slope for 3 materials = 0.48

This is the first evidence of Taylor hardening in a geologic material.
When normalized by the elastic modulus, a single curve can describe the back stress for all 3 materials

\[
\sigma_{\text{disl}} = \alpha G b \rho_{\text{GND}}^{0.5}
\]

(Taylor equation, 1934)

Dashed line slope = 0.5

Solid line (best fit) slope = 0.44
Compilation of all back stress and GND data for olivine single crystals shows remarkable agreement across wide range of conditions.

- Solid and dashed red lines correspond to fits to olivine nanoindentation data presented in slides above.
- Black line is best fit to all data presented in the plot.
- Data spans several crystal orientations, 298-1573 K, several GPa in pressure, and 5 orders of magnitude in GND density, but it can be unified using Taylor hardening!
Take home messages

1. We have developed a novel nanoindentation load drop method to measure the back stress of materials at very high stresses and GND densities.

2. Our results for 3 different geologic materials demonstrate Taylor hardening, and normalization by the elastic modulus results in a universal curve. This suggests that one may be able to predict the back stress in deforming geologic materials from measurements of GND density.

3. Compilation of all olivine data reveals remarkable consistency of this relationship over 5 orders of magnitude in GND density and a wide range of experimental conditions.

4. These data demonstrate that theoretical considerations of Taylor hardening are accurate for geologic materials, and studies of transient creep should utilize the rheology we laid out above.