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Development of a numerical ice-sheet model for simulation of summit migration and dating

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Ice-core topic modeling issues decomposition

Local-scale

- 1d dating model —— under review in GMDD
- Higher-order nesting —— preliminary tests
- Summit migration —— paper in preparation
- Grounding-line migration —— under development

Large-scale

Quest for “Oldest-Ice” ice core

Mid Pleistocene Transition (MPT)

41 kyr-world to 100 kyr-world at ∼ 1 Ma
Gases trapped in ice are most direct access to the climate variability

Check the following for this topic:

- The Cryosphere Special issue (2013)
- Lisiecki and Raymo (2005)
- Fischer et al. (2013)
- EPICA community members (2004)
  — Dome C (Current oldest core∼ 800 kyr)
- . . .

Quest for drilling site(s) around Dome Fuji

Japanese community collaboration

Need to choose the site(s) before drilling
Field works (NIPR/...) + Modeling (JAMSTEC/ILTS/AORI/NIPR)

Radar-echo sounding does good job

- Internal laying visualization
- Layer tracking from Dome Fuji
- Detection of bottom melting **at present**

Modeling can help....

- Simulation of temperature history
- Simulation of ice flow history
- Age computation (**dating**)

Age computation with high accuracy

Equation of Age — pure advection equation

$$\text{(1)} \qquad \frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} = 1 \quad \text{or} \quad \frac{\partial \mathcal{A}}{\partial t} + v \cdot \nabla \mathcal{A} = 1 \ ,$$

where $\mathcal{A}$ is the duration since the deposit.

Studies to show performances of various numerical schemes:

- Mügge et al. (1999), Rybak and Huybrechts (2003)
  — Comparison of Eulerian and Lagrangian methods
- Greve et al. (2002) — Upwinds, QUICK, TVD-LF
- Tarasov and Peltier (2003),Lhomme et al. (2005); Clarke et al. (2005) — Semi-Lagrangian
- Parrenin et al. (2007)
  — “Lagrangian thinning and Eulerian age scheme”

Still some possibilities for other schemes, e.g., **(R)CIP scheme**

CIP schemes (e.g., Yabe et al., 2002)

A variation of semi-Lagrangian method

CIP ≡ Constrained Interpolation Profile scheme

Two advection equations to solve (in 1d case):

$$(2)\qquad \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = h\ ,$$

$$(3)\qquad \frac{\partial g}{\partial t} + u\frac{\partial g}{\partial x} = \hat{h}\ , \qquad x\text{-derivative of (2)}\ ,$$

where $g = \frac{\partial f}{\partial x}$, h and $\hat{h}$ are non-advection terms.
Interpolation functions F and G with constraints:

$$(4)\qquad \begin{cases} F_k(x_k) = f(x_k), & F_k(x_{k+1}) = f(x_{k+1}), \\ F'_k(x_k) = g(x_k), & F'_k(x_{k+1}) = g(x_{k+1}), \end{cases} \qquad F'(x) = \frac{\mathrm{d}F}{\mathrm{d}x}\ .$$

are used to compute upstream values.

RCIP schemes (e.g. Xiao et al., 1996)

RCIP ≡ a Rational function based CIP

Switch interpolation function between:

$$
(5) \quad \begin{cases} F_k(X) = C_3X^3 + C_2X^2 + C_1X + C_0 & g_k \lesssim S_k \lesssim g_{k+1} \\ F_k(X) = \dfrac{C_2X^2 + C_1X + C_0}{1 + D_1X} & \text{otherwise,} \end{cases}
$$

where $X = x - x_k$, $S_k = (f_{k+1} - f_k) / \Delta x_k$, $\Delta x_k = x_{k+1} - x_k$.

The RCIP shows **less diffusive** solution as well as **Less oscillation** at fronts.

Demonstration of RCIP

Highlights of Saito et al. (2020)
Implement RCIP scheme on age computation in an ice-sheet model

- Vertical 1-d age computation under **prescribed** velocity history
- Comparison with first- and second-order upwind schemes
    - Not with various slope limiters
    - Not with even higher-order schemes
    - Not with other semi-Lagrangian
    - Not with Lagrangian
    - These are beyond the scope
- Two variation to compute upstream *departure points* in RCIP scheme — no significant difference

Demonstration of RCIP scheme on dating issue

Equations to solve (1-dimension)

(6)
$$\begin{cases} \dfrac{\partial \mathcal{A}}{\partial t} + w\dfrac{\partial \mathcal{A}}{\partial z} = 1 \,, \\ \dfrac{\partial \mathcal{A}'}{\partial t} + w\dfrac{\partial \mathcal{A}'}{\partial z} = -\dfrac{\partial w}{\partial z}\mathcal{A}' \,, \end{cases}$$

$\mathcal{A}$: age, $\mathcal{A}' = \partial\mathcal{A}/\partial z$, w: velocity

Prescribed boundary condition and velocity profiles

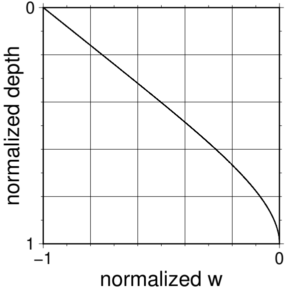
(7)
$$w(\zeta,t) = -\left[\left(M_\mathrm{s}(t) + M_\mathrm{b}(t) - \frac{\partial H}{\partial t}\right)\tilde{w}(\zeta) - M_\mathrm{b}(t)\right] \,, \quad \zeta = z/H \,,$$

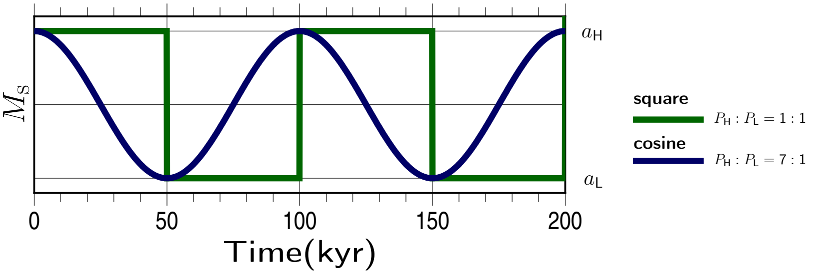
(8)
$$\tilde{w}(\zeta) = 1 - \frac{p+2}{p+1}(1-\zeta) + \frac{1}{p+1}(1-\zeta)^{p+2} \,,$$

M_s,M_b: surface/basal mass input; H: thickness; p: a parameter

Example configuration

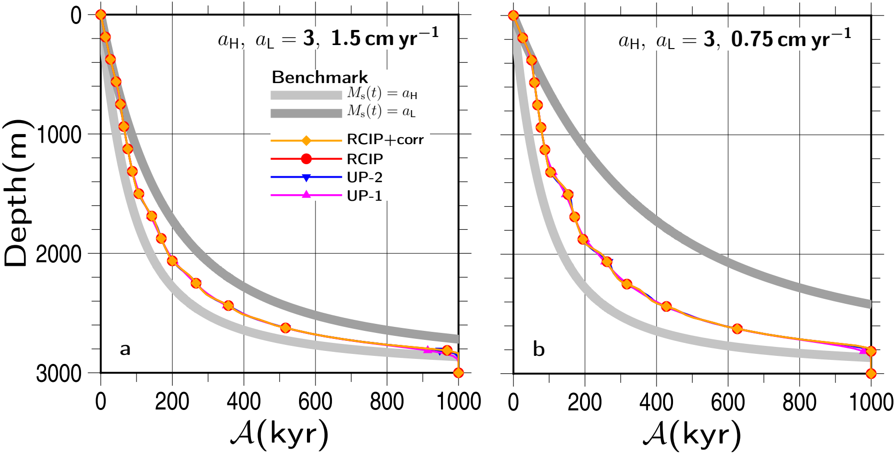
- $p = 3$ for w
- $M_b = 0$, $H = 3000$ m constant
- Square-wave type M_s evolution (green-line)

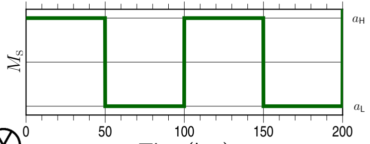




Example result (square-wave)

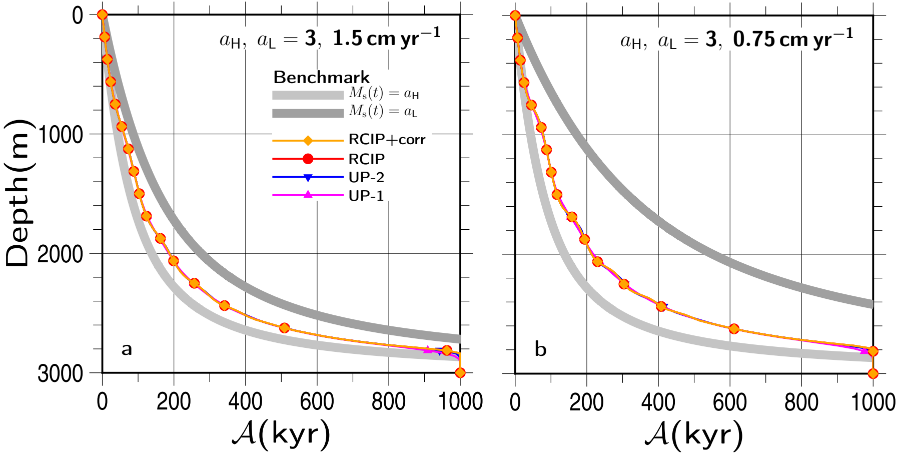
Simulated age vs depth:

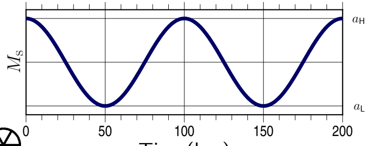




Example result (cosine-wave)

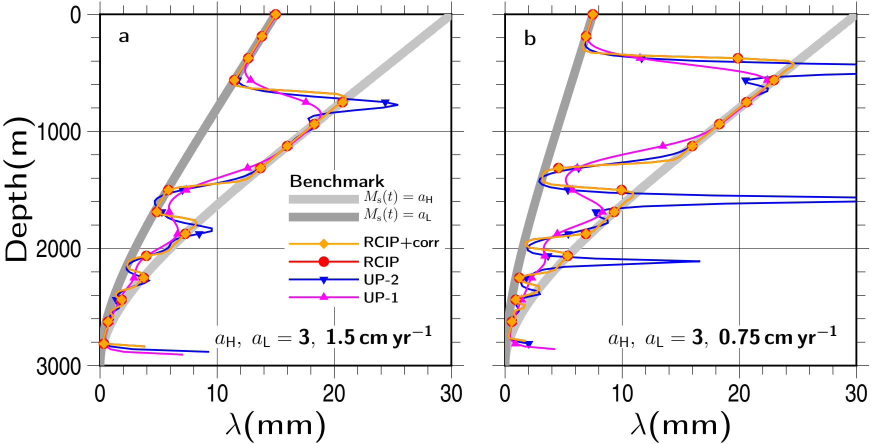
Simulated age vs depth:





Example result (square-wave)

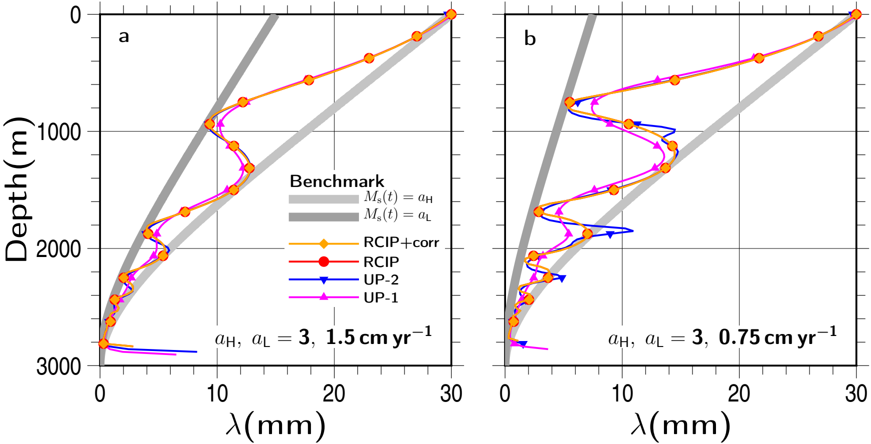
Annual layer thickness $\lambda \sim 1/\mathcal{A}'$; Even spacing of 129 levels



- Depth-λ relation should follow steady velocity cases (two gray lines) if normalized shape of vertical velocity is the same

Example result (cosine-wave)

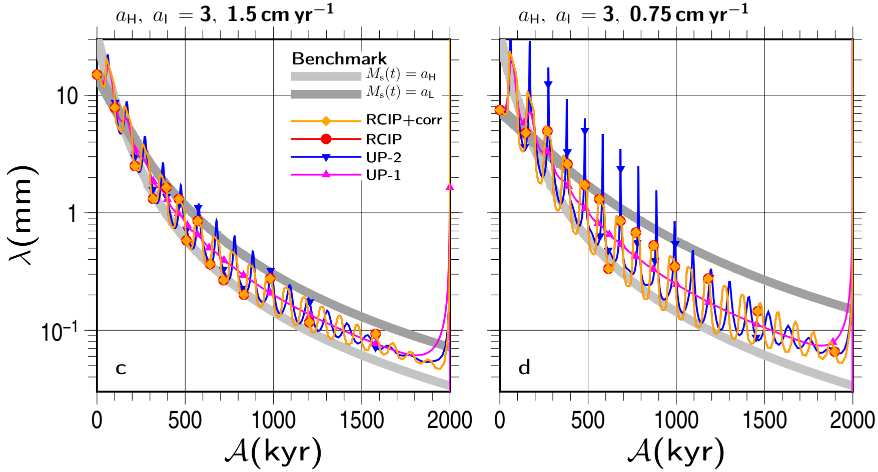
Annual layer thickness $\lambda \sim 1/\mathcal{A}'$; Even spacing of 129 levels



- Oscillation may show under smooth mass balance history

Example result (high resolution)

- Non-uniform smooth discretization of 513 levels
- Square-wave 50 kyr - 50 kyr



Phases are well preserved by RCIP

Brief summary on RCIP implementation

- Computed annual layer thickness is less diffusive by RCIP, as expected
- Phase of changes in annual layer thickness is well preserved by RCIP, as expected

→ under review in GMDD

Development steps

1. Dating with high accuracy under **prescribed** flow history

⇒ How to compute flow history at ice divides?

Ice flow computation at divide

Steady flow structure is expected for a drilling site

Summit or ice-divide flow

- Different flow structure from the ‘shear’ region
- Divergent flow, no (little) horizontal velocity
  Horizontal scale $\sim$ O(thickness)(Hindmarsh, 1996)

Need a different flow-regime modeling near divide, with higher-resolution

$\Rightarrow$ **Nesting model** development

Ice flow and approximation

Glen's flow law:

$$\dot{\boldsymbol{\epsilon}} = EA(T)\sigma^{n-1}\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \sigma_{yz} \\ & & \sigma_{zz} \end{pmatrix}$$

E: enhancement factor, $E = E(\text{impurity}, \cdots)$

A: rate factor, $A = A(T)$

$\dot{\boldsymbol{\epsilon}}$: strain rate, $\dfrac{\partial u_{ij}}{\partial x_j}$

$\boldsymbol{\sigma}$: (deviatoric) stress

n: flow-law exponent, typically 3.

Shallow-Ice Approximation (SIA)

Use σ_{xz}, σ_{yz}

$$u = u_B - 2(\rho g)^n \left(\frac{\partial s}{\partial x}\right)^n H^{n+1} \int d\zeta EA(1-\zeta)^n$$

- Good approx. over the (most) grounded part

Higher-order Approximation (HOA)

HOA = SIA + $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$

- **Near summit** — $\sigma_{xz}, \sigma_{yz} \sim 0$
- Near margin, ice-stream, floating part

High resolution flow computation around Dome

A nesting model is effective

- Large-scale SIA modeling for most part
- Small-scale HOA modeling around the target

Both types of models are in hands....

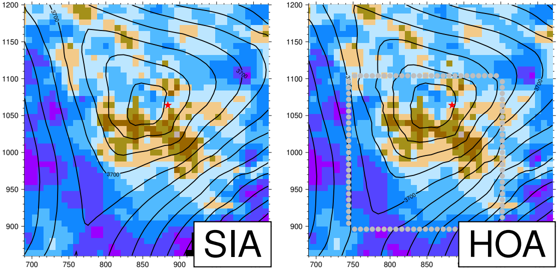
SIA model: I_C*IES* (Saito and Abe-Ouchi, 2010)
HOA model: I_C*IES*-HOA (Saito et al., 2003)

Development of a HOA/SIA nesting model

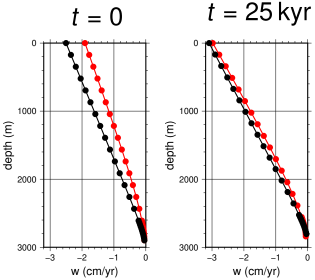
- **Goal**: a high resolution HOA model nesting on prescribed region around the target in low resolution SIA model
- Intermediate: embedding same resolution HOA model on prescribed region

HOA preliminary results

- ***I_CIES*** with 3D higher-order option (Saito et al., 2003)
- **Embedding** HOA model only over prescribed region (gray square) around Dome Fuji, with the same spatial resolution as SIA
- Trial: 25kyr experiment starting from SIA result



t = 25 kyr topography



w at DF(hoa): black=HOA; red=SIA

Need to investigate further....

Brief summary on HOA embedding

- Higher-order ice flow computation is being implemented
- Just tested for simple cases

→ To develop further

Development steps

1. Dating with high accuracy under **prescribed** flow history
2. HOA flow velocity over **prescribed** summit region

⇒ How to define ice divides?

— Divide can migrate according to changes in environments

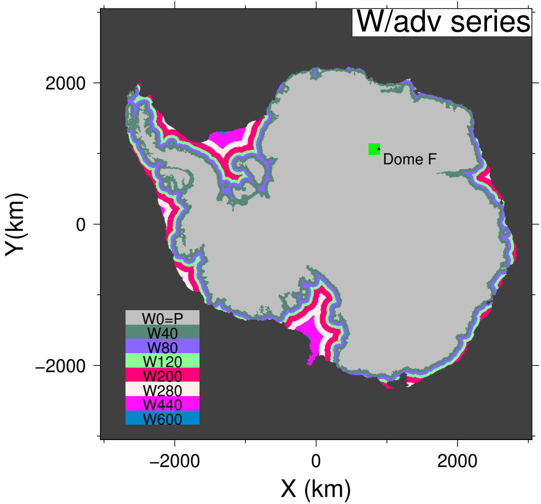
What controls the summit position?

Ice-sheet shape $\Leftarrow$ Large-scale & local-scale effects
$\Leftarrow$ Climate + Extent + Bedrock

- Weertman (1973)
  Effect of the spatial pattern of accumulation rate
  $<$ Effect of the ice sheet span for the summit position
- Abe-Ouchi et al. (1994)
  The highest point of the ice sheet is not always at the same position; it migrates towards the center for thickening ice.
- Hindmarsh (1996)
  While flow near ice divides cannot be calculated by the shallow-ice approximation ($\equiv$ shear-dominant flow), their **position can be** to the order of the ice-sheet thickness.

Experiment configuration: Extent

Sensitivity experiments to **prescribed** ice-sheet area



- advance/retreat in specific/whole area
- AND bedrock elevation > -800m
- AND grounding line advance $\{40 \cdots 600\}$km from P
- AND 'holes' are filled up manually

Experiment configuration: Climate

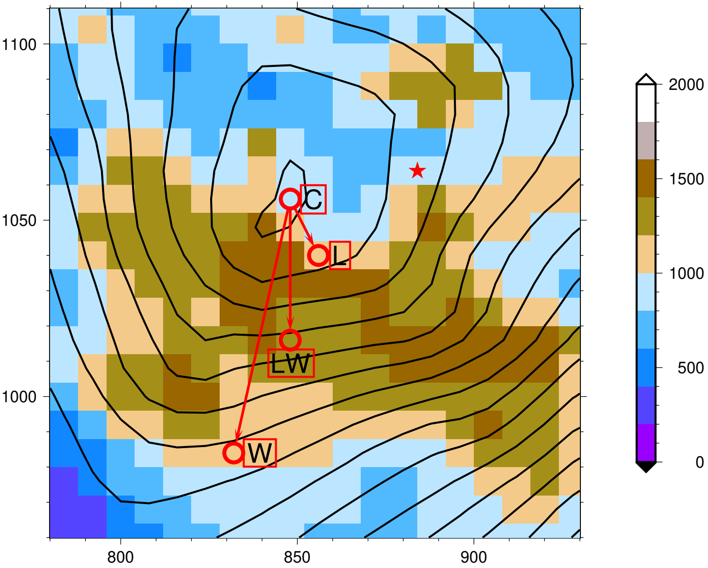
Control: SeaRISE (Nowicki et al., 2013) boundary conditions

- Present-day accumulation (Athern et al. 2006) with modification around Dome Fuji ($\simeq 2.75\,\mathrm{cm/yr}$) (Satow et al., 1999)
- Surf. temperature: function of lon lat elev (Fortuin and Oerlemans, 1990)

LGM: LGM-like condition

- Constant perturbation from the Control
- Sea level: $-128.25\,\mathrm{m}$
- Background temperature: $-7.81\,\mathrm{K}$

Results: Dome Fuji Position sensitivities



C: Control; L: LGM climate; W: W-600 extent; LW: L+W

Brief summary on Dome Fuji migration

- Potential Dome Fuji migration is examined
- Dome Fuji migration during glacial cycles $\lesssim 50\,\mathrm{km}$

$\longrightarrow$ Paper in preparation

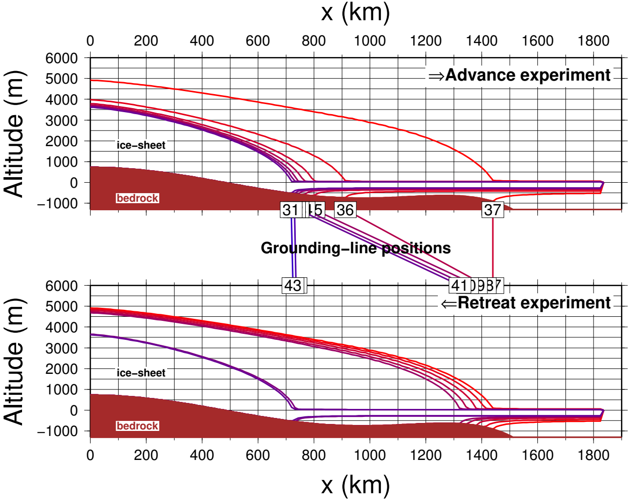
Development steps

1. Dating with high accuracy under **prescribed** flow history
2. HOA flow velocity over **prescribed** summit region
3. Summit migration by **prescribed** ice-sheet extent/climate

$\Rightarrow$ How to simulate changes in ice-sheet extent?

Simulation of grounding line migration

Grounding line migration simulation using $I_C IES$ with MISMIP configuration(Pattyn et al., 2012)



Under development.....

Summary

A numerical ice-sheet model for ice-core topics has been developed / being developed / will be developed

Development steps and status

1. **(GMDD ↗)** RCIP dating with high accuracy
2. **(testing)** HOA nesting
3. **(writing)** Summit migration
4. **(pending)** Grounding line migration by ice-ocean interaction
5. **(waiting)** Ice-ocean interaction under ice shelves

Another important issue: application for ice-sheet dynamics

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