Development of a numerical ice-sheet model for simulation of summit migration and dating

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Ice-core topic modeling issues decomposition

Local-scale

- 1d dating model ✔ ——— under review in GMDD
- Higher-order nesting ✔ ——— preliminary tests
- Summit migration ✔ ——— paper in preparation
- Grounding-line migration ——— under development

Large-scale
### Mid Pleistocene Transition (MPT)

41 kyr-world to 100 kyr-world at $\sim$ 1 Ma

Gases trapped in ice are most direct access to the climate variability

Check the following for this topic:

- The Cryosphere Special issue (2013)
- Lisiecki and Raymo (2005)
- Fischer et al. (2013)
- EPICA community members (2004)
  - Dome C (Current oldest core $\sim$ 800 kyr)
- ...
Quest for drilling site(s) around Dome Fuji

Japanese community collaboration

Need to choose the site(s) before drilling
Field works (NIPR/...) + Modeling (JAMSTEC/ILTS/AORI/NIPR)

Radar-echo sounding does good job

- Internal laying visualization
- Layer tracking from Dome Fuji
- Detection of bottom melting at present

Modeling can help....

- Simulation of temperature history
- Simulation of ice flow history
- Age computation (dating)
Age computation with high accuracy

Equation of Age — pure advection equation

\begin{align}
\frac{dA}{dt} = 1 \quad \text{or} \quad \frac{\partial A}{\partial t} + v \cdot \nabla A = 1,
\end{align}

where $A$ is the duration since the deposit.

Studies to show performances of various numerical schemes:

- Mügge et al. (1999), Rybak and Huybrechts (2003) — Comparison of Eulerian and Lagrangian methods
- Greve et al. (2002) — Upwinds, QUICK, TVD-LF
- Tarasov and Peltier (2003), Lhomme et al. (2005); Clarke et al. (2005) — Semi-Lagrangian
- Parrenin et al. (2007) — “Lagrangian thinning and Eulerian age scheme”

Still some possibilities for other schemes, e.g., (R)CIP scheme
CIP schemes (e.g., Yabe et al., 2002)

A variation of semi-Lagrangian method

CIP ≡ Constrained Interpolation Profile scheme

Two advection equations to solve (in 1d case):

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = h, \tag{2}
\]

\[
\frac{\partial g}{\partial t} + u \frac{\partial g}{\partial x} = \hat{h}, \quad x\text{-derivative of (2)}, \tag{3}
\]

where \( g = \frac{\partial f}{\partial x} \), \( h \) and \( \hat{h} \) are non-advection terms.

Interpolation functions \( F \) and \( G \) with constraints:

\[
\begin{aligned}
F_k(x_k) &= f(x_k), & F_k(x_{k+1}) &= f(x_{k+1}), \\
F'_k(x_k) &= g(x_k), & F'_k(x_{k+1}) &= g(x_{k+1}), & F'(x) &= \frac{dF}{dx}.
\end{aligned} \tag{4}
\]

are used to compute upstream values.
RCIP schemes (e.g. Xiao et al., 1996)

RCIP ≡ a Rational function based CIP

Switch interpolation function between:

\[
\begin{align*}
F_k(X) &= C_3 X^3 + C_2 X^2 + C_1 X + C_0 & g_k \leq S_k \leq g_{k+1} \\
F_k(X) &= \frac{C_2 X^2 + C_1 X + C_0}{1 + D_1 X} & \text{otherwise,}
\end{align*}
\]

(5)

where \( X = x - x_k, \ S_k = (f_{k+1} - f_k) / \Delta x_k, \ \Delta x_k = x_{k+1} - x_k. \)

The RCIP shows less diffusive solution as well as Less oscillation at fronts.
Demonstration of RCIP

Highlights of Saito et al. (2020)
Implement RCIP scheme on age computation in an ice-sheet model
- Vertical 1-d age computation under \textit{prescribed} velocity history
- Comparison with first- and second-order upwind schemes
  - Not with various slope limiters
  - Not with even higher-order schemes
  - Not with other semi-Lagrangian
  - Not with Lagrangian
  - These are beyond the scope
- Two variation to compute upstream \textit{departure points} in RCIP scheme — no significant difference
Demonstration of RCIP scheme on dating issue

Equations to solve (1-dimension)

\[
\begin{align*}
\frac{\partial \mathcal{A}}{\partial t} + w \frac{\partial \mathcal{A}}{\partial z} &= 1, \\
\frac{\partial \mathcal{A}'}{\partial t} + w \frac{\partial \mathcal{A}'}{\partial z} &= -\frac{\partial w}{\partial z} \mathcal{A}',
\end{align*}
\]

\(\mathcal{A}:\) age, \(\mathcal{A}' = \frac{\partial \mathcal{A}}{\partial z}\), \(w: \) velocity

Prescribed boundary condition and velocity profiles

\[
(7) \quad w(\zeta, t) = -\left[\left( M_s(t) + M_b(t) - \frac{\partial H}{\partial t} \right) \tilde{w}(\zeta) - M_b(t) \right], \quad \zeta = z/H,
\]

\[
(8) \quad \tilde{w}(\zeta) = 1 - \frac{p + 2}{p + 1} (1 - \zeta) + \frac{1}{p + 1} (1 - \zeta)^{p+2},
\]

\(M_s, M_b: \) surface/basal mass input; \(H: \) thickness; \(p: \) a parameter
Example configuration

- $\rho = 3$ for $w$
- $M_b = 0$, $H = 3000$ m constant
- Square-wave type $M_s$ evolution (green-line)
Example result (square-wave)

Simulated age vs depth:

\[ a_H, a_L = 3, \ 1.5 \text{ cm yr}^{-1} \]

Benchmark

\[ M_s(t) = a_H \]
\[ M_s(t) = a_L \]

RCIP+corr
RCIP
UP-2
UP-1

\[ Ms(t) = a_H \]
\[ Ms(t) = a_L \]
Example result (cosine-wave)

Simulated age vs depth:

\[ a_H, a_L = 3, \ 1.5 \text{ cm yr}^{-1} \]

Benchmark

- \( M_s(t) = a_H \)
- \( M_s(t) = a_L \)

RCIP + corr
- RCIP
- UP-2
- UP-1

\[ a_H, a_L = 3, \ 0.75 \text{ cm yr}^{-1} \]
Example result (square-wave)

Annual layer thickness $\lambda \sim 1/\mathcal{A}'$; Even spacing of 129 levels

- Depth-$\lambda$ relation should follow steady velocity cases (two gray lines) if normalized shape of vertical velocity is the same

Benchmark
- RCIP+corr
- RCIP
- UP-2
- UP-1

$a_H, a_L = 3, 1.5 \text{ cm yr}^{-1}$

$a_H, a_L = 3, 0.75 \text{ cm yr}^{-1}$

SAITO Fuyuki et al.  ICIES for Ice-core topics  May 6 2020  12/34
**Example result (cosine-wave)**

Annual layer thickness $\lambda \sim 1/\mathcal{A}'$; Even spacing of 129 levels

![Graph](image)

- Oscillation may show under smooth mass balance history

**Benchmark**

- $M_s(t) = a_H$
- $M_s(t) = a_L$

- RCIP + corr
- RCIP
- UP-2
- UP-1

$M_s(t) = a_H$

$a_H, a_L = 3, 1.5 \text{ cm yr}^{-1}$

$a_H, a_L = 3, 0.75 \text{ cm yr}^{-1}$
Example result (high resolution)

- Non-uniform smooth discretization of 513 levels
- Square-wave 50 kyr - 50 kyr

Phases are well preserved by RCIP
Brief summary on RCIP implementation

- Computed annual layer thickness is less diffusive by RCIP, as expected
- Phase of changes in annual layer thickness is well preserved by RCIP, as expected

→ under review in GMDD

Development steps

1. Dating with high accuracy under **prescribed** flow history

⇒ How to compute flow history at ice divides?
Ice flow computation at divide

Steady flow structure is expected for a drilling site

**Summit or ice-divide flow**

- Different flow structure from the ‘shear’ region
- Divergent flow, no (little) horizontal velocity
  Horizontal scale $\sim O(\text{thickness})$ (Hindmarsh, 1996)

Need a different flow-regime modeling near divide, with higher-resolution

$\Rightarrow$ **Nesting model** development
Ice flow and approximation

Glen’s flow law:
\[ \dot{\epsilon} = EA(T)\sigma^{n-1}\sigma \]

\[ \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \]

- \( E \): enhancement factor, \( E = E(\text{impurity, \cdots}) \)
- \( A \): rate factor, \( A = A(T) \)
- \( \dot{\epsilon} \): strain rate, \( \frac{\partial u_{ij}}{\partial x_j} \)
- \( \sigma \): (deviatoric) stress
- \( n \): flow-law exponent, typically 3.

**Shallow-Ice Approximation (SIA)**

Use \( \sigma_{xz}, \sigma_{yz} \)

\[ u = u_B - 2 (\rho g)^n \left( \frac{\partial s}{\partial x} \right)^n H^{n+1} \int d\zeta EA(1 - \zeta)^n \]

- Good approx. over the (most) grounded part

**Higher-order Approximation (HOA)**

\[ \text{HOA} = \text{SIA} + \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \]

- Near summit — \( \sigma_{xz}, \sigma_{yz} \sim 0 \)
- Near margin, ice-stream, floating part
High resolution flow computation around Dome

A nesting model is effective

- Large-scale SIA modeling for most part
- Small-scale HOA modeling around the target

Both types of models are in hands....

SIA model: \textit{ICIES} (Saito and Abe-Ouchi, 2010)
HOA model: \textit{ICIES}-HOA (Saito et al., 2003)

Development of a HOA/SIA nesting model

- **Goal**: a high resolution HOA model nesting on prescribed region around the target in low resolution SIA model
- Intermediate: embedding same resolution HOA model on prescribed region
HOA preliminary results

- **ICIES** with 3D higher-order option (Saito et al., 2003)
- **Embedding** HOA model only over prescribed region (gray square) around Dome Fuji, with the same spatial resolution as SIA
- **Trial**: 25kyr experiment starting from SIA result

$t = 25$ kyr topography

$w$ at $DF(hoa)$: black=HOA; red=SIA

Need to investigate further....
Brief summary on HOA embedding

- Higher-order ice flow computation is being implemented
- Just tested for simple cases
  → To develop further

Development steps

1. Dating with high accuracy under **prescribed** flow history
2. HOA flow velocity over **prescribed** summit region

⇒ How to define ice divides?
  — Divide can migrate according to changes in environments
What controls the summit position?

Ice-sheet shape $\leftrightarrow$ Large-scale & local-scale effects
$\leftrightarrow$ Climate + Extent + Bedrock

- Weertman (1973)
  Effect of the spatial pattern of accumulation rate
  $<$ Effect of the ice sheet span for the summit position

- Abe-Ouchi et al. (1994)
  The highest point of the ice sheet is not always at the same position; it migrates towards the center for thickening ice.

- Hindmarsh (1996)
  While flow near ice divides cannot be calculated by the shallow-ice approximation ($\equiv$ shear-dominant flow), their position can be to the order of the ice-sheet thickness.
Experiment configuration: Extent

Sensitivity experiments to **prescribed** ice-sheet area

- advance/retreat in specific/whole area
- AND bedrock elevation > $-800$ m
- AND grounding line advance $\{40 \cdots 600\}$ km from P
- AND ‘holes’ are filled up manually
Experiment configuration: Climate

**Control**: SeaRISE (Nowicki et al., 2013) boundary conditions
- Present-day accumulation (Athern et al. 2006) with modification around Dome Fuji ($\approx 2.75 \text{ cm/yr}$) (Satow et al., 1999)
- Surf. temperature: function of lon lat elev (Fortuin and Oerlemans, 1990)

**LGM**: LGM-like condition
- Constant perturbation from the Control
- Sea level: $-128.25 \text{ m}$
- Background temperature: $-7.81 \text{ K}$
Results: Dome Fuji Position sensitivities

C: Control;  L: LGM climate;  W: W-600 extent;  LW: L+W
Brief summary on Dome Fuji migration

- Potential Dome Fuji migration is examined
- Dome Fuji migration during glacial cycles $\lesssim 50$ km
  → Paper in preparation

Development steps

1. Dating with high accuracy under **prescribed** flow history
2. HOA flow velocity over **prescribed** summit region
3. Summit migration by **prescribed** ice-sheet extent/climate

⇒ How to simulate changes in ice-sheet extent?
Simulation of grounding line migration

Grounding line migration simulation using $I_{CIES}$ with MISMIP configuration (Pattyn et al., 2012)

Under development.....
Summary

A numerical ice-sheet model for ice-core topics has been developed / being developed / will be developed.

Development steps and status

1. (GMDD) RCIP dating with high accuracy
2. (testing) HOA nesting
3. (writing) Summit migration
4. (pending) Grounding line migration by ice-ocean interaction
5. (waiting) Ice-ocean interaction under ice shelves

Another important issue: application for ice-sheet dynamics


References II


References V


References VI


