Linking surface enthalpy fluxes to the forces driving the secondary circulation: Towards a causal theory of Tropical Cyclone intensification

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EGU Online – 6 May 2020
Axisymmetric vortex evolution

\[
\frac{Dv}{Dt} + \left( f + \frac{v}{r} \right) u = D_v
\]

\[
\frac{Du}{Dt} - \left( f + \frac{v}{r} \right) v + \frac{1}{\rho} \frac{\partial p}{\partial r} = D_u
\]

\[
\frac{Dw}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial z} = -g + D_w
\]

\[
\frac{D\eta}{Dt} = \frac{\dot{Q}}{T}
\]

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{DM}{Dt} = r \ D_v
\]

'Primary' circulation

\[
M = r \ v + \frac{f r^2}{2}
\]

Angular Momentum

'Secondary' circulation

Thermodynamics

Continuity
Idealized Tropical Cyclone Simulation
Rotunno and Emanuel (1987) axisymmetric hurricane model

Minimum Surface Pressure
Maximum Surface Wind

Azimuthal Wind

Wong, Tailleux and Gray (QJ, 2015)
Energetics approach:

- Some fraction of surface fluxes generate available potential energy, rest goes into background potential energy
- Part of APE generated gets into kinetic energy, rest goes into APE storage
- Kinetic energy generated eventually dissipates

\[
\frac{d\text{APE}}{dt} = G - C_{\text{APE}\rightarrow\text{KE}}
\]

\[
\frac{d\text{KE}}{dt} = C_{\text{APE}\rightarrow\text{KE}} - D_k
\]

Challenge:
Reference state impacts on APE and APE generation, but not the APE/KE conversion.

What choice of reference state yields APE generation rate equal APE to KE conversion?
Top down vs bottom-up sorting algorithms (Wong et al., 2016)

See also Harris and Tailleux (2018) for inter-comparison of algorithms for computing moist APE
APE production efficiency factor

Figure 6. APE production efficiency \( \frac{T_i - T_{ref}}{T_i} \) computed using (a, b) the top-down and (c, d) the bottom-up sorting method at (a, c) 60 h and (b, d) 120 h.
Figure 7. Time series of the area-averaged top-down (blue) and bottom-up (green) APE production efficiency in the surface layer over (a) the whole domain, and (b) the inner region (radius ≤1000 km, solid lines) and outer region (radius >1000 km, dashed lines).
APE PRODUCTION VERSUS DIAGNOSED ENERGY GENERATION

Wong, Tailleux and Gray (QJ, 2015)

\[
\frac{dA_{PE}}{dt} = G - C_{A_{PE}\rightarrow KE},
\]

\[
\frac{dKE}{dt} = C_{A_{PE}\rightarrow KE} - D,
\]
What about reference state in gradient-wind and hydrostatic balance?

Construct reference state \( p_0(r, z, t) \) and \( \rho_0(r, z, t) \) so that

\[
\frac{1}{\rho_0} \frac{\partial p_0}{\partial r} = \left( f + \frac{v}{r} \right) v
\]

\[
\frac{1}{\rho_0} \frac{\partial p_0}{\partial z} = -g
\]

Method: Iterative procedure based on successively integrating these equations radially and vertically

Nolan and Montgomery (2002)
Impact of reference state on APE production rate

Comparison between APE production rate predicted using different choices of reference state and diagnose kinetic energy generation

Bethan Harris
PhD work
Thermodynamic study of the oceanic and atmospheric heat engines (in a steady-state)

**Total Energy Budget**

\[ Q_{in} - Q_{out} + W_{mech} = 0 \]

**Mechanical Energy Budget**

\[ \int_V p \frac{Du}{Dt} \, dm + W_{mech} = \int_V \varepsilon \, dm \]
Aim of thermodynamic heat engine theories

\[ \int_V p \frac{Dv}{Dt} \, dm + W_{mech} = \int_V \epsilon_K \, dm \]

\[ \int_V p \frac{Dv}{Dt} \, dm = P_A - D_A \]

- \( P_A \) = Thermodynamic Production
- \( D_A \) = Non-Viscous Dissipation
Entropy budget approach

\[
\frac{Q_{in}}{T_{in}} - \frac{Q_{out}}{T_{out}} + \dot{\Sigma}_{irr} + \frac{1}{T_\epsilon} \int_V \epsilon_K dm = 0
\]

Gouy-Stodola Theorem

\[
\int_V \epsilon_K dm = \frac{T_\epsilon}{T_{in}} \left( \frac{T_{in} - T_{out}}{T_{out}} \right) Q_{out} + \frac{T_\epsilon}{T_{in}} W_{mech} - T_\epsilon \dot{\Sigma}_{irr}
\]
APE budget approach

\[ \int_V \epsilon_K \, dm = G_{\text{APE}} + W_{\text{mech}} - D_{\text{APE}} \]

APE production by surface buoyancy fluxes

Mechanical Energy input by Wind

APE dissipation by non-viscous mixing processes

APE production by surface buoyancy fluxes (freshwater fluxes neglected)

\[ G_{\text{APE}} \approx \int_S \left( \frac{T - T_R}{T} \right) Q_{\text{net}} \, dS = \gamma_{\text{out}} Q_{\text{out}} - \gamma_{\text{in}} Q_{\text{in}} \]
### Summary of APE versus Entropy view of thermodynamic forcing and dissipation

<table>
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<tr>
<th></th>
<th>PRODUCTION BY BUOYANCY FLUXES</th>
<th>NON-VISCOUS DISSIPATION</th>
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<tr>
<td>ENTROPY APPROACH</td>
<td>$T_\epsilon \left( \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{out}}} \right) Q_{\text{out}}$</td>
<td>$T_\epsilon \dot{\Sigma}_{\text{nonviscous}}$</td>
</tr>
<tr>
<td>APE APPROACH</td>
<td>$\gamma_{\text{ape}} Q_{\text{out}}$</td>
<td>$D_{\text{ape}}$</td>
</tr>
</tbody>
</table>
Local theory of Available Potential Energy
(Andrews 1981; Tailleux 2018; Novak and Tailleux 2018)

\[ B = \Pi + B_r \]

\[ \Pi = B - B_r \]

\[ B_r = \Phi(z_r) + h(\sigma, S, p_R(z_r)) \]

\[ \int_V \Pi \rho dV = APE_{Lorenz} \]

Potential Energy of fluid + Environment

Available Potential Energy density
= Work

Background Potential Energy density
= Heat
Local definition of available energy

\[ \Pi = \Phi(z) - \Phi(z_R) + h(\sigma, S, p) - h(\sigma, S, p_R) + \frac{p_R(z) - p}{\rho} \]

\[ \approx \frac{(p - p_R)^2}{2 \rho c^2} + \frac{N_R^2 (z - z_R)^2}{2} \]

Positive definite. Sum of compressible work + work against buoyancy forces to construct actual state from reference state by means of adiabatic and isohaline transformation. Can be further decomposed into mean/eddy components.
Evolution equation for Available Energy density

\[ \rho \frac{D \Pi}{Dt} = \frac{\rho(T - T_*)}{T} \dot{Q} + C(\Pi, E_k) + \ldots \]

Thermodynamic Efficiency

Kinetic Energy Conversion
APE accounting for momentum constraints

- Shepherd (1993): A unified theory of available potential energy. AO
- Codoban and Shepherd (2003): Energetics of a symmetric circulation including momentum constraints. JAS
Available energetics of axisymmetric motions relative to a non-resting balanced vortex state

Balanced/hydrostatic reference state

Reference position solution of

\[
\begin{align*}
\frac{1}{\rho_0} \frac{\partial p_0}{\partial r} &= \left( f + \frac{v_0}{r} \right) v_0 \\
\frac{1}{\rho_0} \frac{\partial p_0}{\partial z} &= -g
\end{align*}
\]

\[
\eta_0(r_*, z_*) = \eta \\
M_0(r_*, z_*) = M
\]
Available energetics of axisymmetric motions (Tailleux and Harris, 2020)

\[ A = \Pi_1 + \Pi_k + \Pi_e \]

Available Acoustic Energy (AEE): Compress/Expand from \( p_0(r, z) \) to \( p \)

\[ \Pi_1 = h(\eta, p) - h(\eta, p_0(r, z)) + \frac{p_0(r, z) - p}{\rho} \approx \frac{(p - p_0(r, z))^2}{\rho^2 c_s^2} \]

Centrifugal Potential Energy (proportional to azimuthal kinetic energy anomaly):

\[ \Pi_k = -\frac{\partial \tilde{\chi}}{\partial \mu} \frac{(\mu - \mu_m)^2}{2} \alpha \frac{(v - v_m)^2}{2} \]

Slantwise available potential energy (equivalent to SCAPE in a moist atmosphere)

\[ \Pi = \int_{p_*}^{p_m} (v(\eta, p') - v_m(\mu, p')) dp' \]

Integral of local buoyancy along surface of constant angular momentum
Physical interpretation of available energy relative to vortex reference state (Tailleux and Harris, 2020)

Centrifugal potential energy: Work needed to move the fluid parcel from intermediate position \((r_\mu, z_\mu)\) isobarically to actual position \((r, z)\)

Slantwise available potential energy: Work needed to move fluid parcel from reference position \((r_*, z_*)\) in balanced vortex state along surface of constant angular momentum up to intermediate position \((r_\mu, z_\mu)\)

Isobaric surfaces: - - -
Dry entropy surfaces: _____
Angular momentum surfaces: . . . .
Available energy and the forces driving the secondary circulation $\vec{u}_s = (u, w)$

$$\frac{D\vec{u}_s}{Dt} \approx -\nabla \Pi_k - \nabla \Pi_e + \text{Dissipation} + \ldots$$

$$\frac{\mu - \mu_m}{r^3} \hat{r} \text{ (purely radial)}$$

$$- (\nu_h - \nu_0) \nabla p_0$$

Generalised buoyancy force of Smith et al. (2006). Inward and upward for positive buoyancy

Gradient of available energy defines the generalized buoyancy/inertial force driving the adiabatic component of the secondary circulation!
Generalised buoyancy force: inward and upward for positive buoyancy anomaly  

Smith, Montgomery and Zhu (DAO, 2005)

\[
\left( \frac{\rho_0 - \rho}{\rho} \right) \overrightarrow{g_e} = p_0(r, z) = \text{constant}
\]

\[
\overrightarrow{g_e} = \left( \frac{v_0^2}{r} + f v_0, -g \right)
\]
Evolution equation for Available Energy density

\[
\rho \frac{DA}{Dt} = \frac{\rho(T - T_*)}{T} \dot{Q} + \rho M \left( \frac{1}{r^2} - \frac{1}{r_*^2} \right) \frac{DM}{Dt} + C(A, E_k) + \ldots
\]

Thermodynamic Efficiency

‘Mechanical’ Efficiency

Kinetic Energy Conversion

Sustained positive diabatic heating required to generate positive buoyancy anomaly needed to sustain the generalized buoyancy force driving the adiabatic secondary circulation. Link with thermodynamic ‘heat engine’ view of tropical cyclones.
Equations for the ‘diabatic’ secondary circulation

\[ u_* = \frac{D r_*}{D t} = \frac{1}{J_0} \left[ \frac{\partial \eta_0}{\partial z} \frac{D M}{D t} - \frac{\partial M_0}{\partial z} \dot{Q} \right] \]

\[ w_* = \frac{D z_*}{D t} = \frac{1}{J_0} \left[ -\frac{\partial \eta_0}{\partial r} \frac{D M}{D t} + \frac{\partial M_0}{\partial r} \dot{Q} \right] \]

Secondary circulation sum of diabatic and adiabatic components

\[ u = u_* + \delta u \]

\[ w = w_* + \delta w \]

If reference vortex state evolves only slowly, diabatic secondary circulation depends only on local sinks/sources of entropy and angular momentum. Much simpler than Eliassen-Sawyer equations.
Summary and conclusions

• Theory of available potential energy can be generalized to account for momentum constraints => Available energy for perturbations to an axisymmetric reference vortex in gradient wind balance
• Available energy is the sum of available acoustic energy, centrifugal potential energy and slantwise available potential energy
• Gradient of the centrifugal potential energy and slantwise potential energy defines the generalized buoyancy/inertial force driving the adiabatic secondary circulation, whose kinetic energy is transferred to that of the primary circulation
• Maintenance of such a generalized buoyancy/inertial force required sustained positive diabatic heating to sustain positive buoyancy anomaly
Summary and conclusions (cont’d)

• Available Energy defined relative to a non-resting state is a special case of ‘eddy’ APE

• Available energy production defined relative to a non-resting reference state is a very accurate predictor of kinetic energy creation

• Assumption of axisymmetry forbids exchanges between ‘eddy’ APE and ‘mean’ APE. Axysymmetric TC evolution lacks a potentially crucial intensification mechanism compared to asymmetric TC evolution, consistent with Persing et al. (2013)

• APE production of generalized APE includes both thermodynamic and mechanical production terms

• Introduction of reference position allows a rigorous decomposition of the total circulation into adiabatic and diabatic components.
References


• Harris, B. and R. Tailleux, 2018: Assessment of algorithms for computing moist available energy. QJRMS, 144, 1501—1510.