Linking surface enthalpy fluxes to the forces driving the secondary circulation: Towards a causal theory of Tropical Cyclone intensification

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Axisymmetric vortex evolution

$$\frac{Dv}{Dt} + \left(f + \frac{v}{r}\right)u = D_v \quad \Rightarrow \quad \frac{DM}{Dt} = r\,D_v$$

'Primary' circulation

$$M = r\,v + \frac{fr^2}{2}$$

Angular Momentum

$$\frac{Du}{Dt} - \left(f + \frac{v}{r}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial r} = D_u$$

$$\frac{Dw}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -g + D_w$$

'Secondary' circulation

$$\frac{D\eta}{Dt} = \frac{\dot{Q}}{T}$$

Thermodynamics

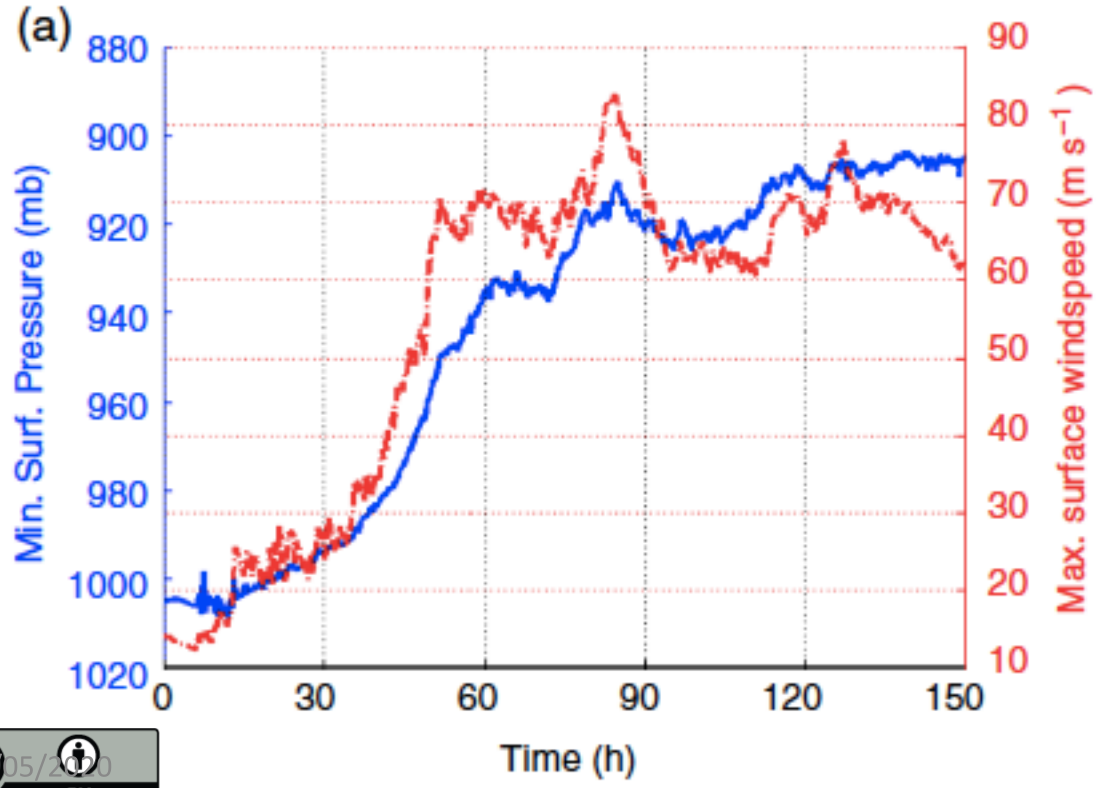
$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0$$

Continuity

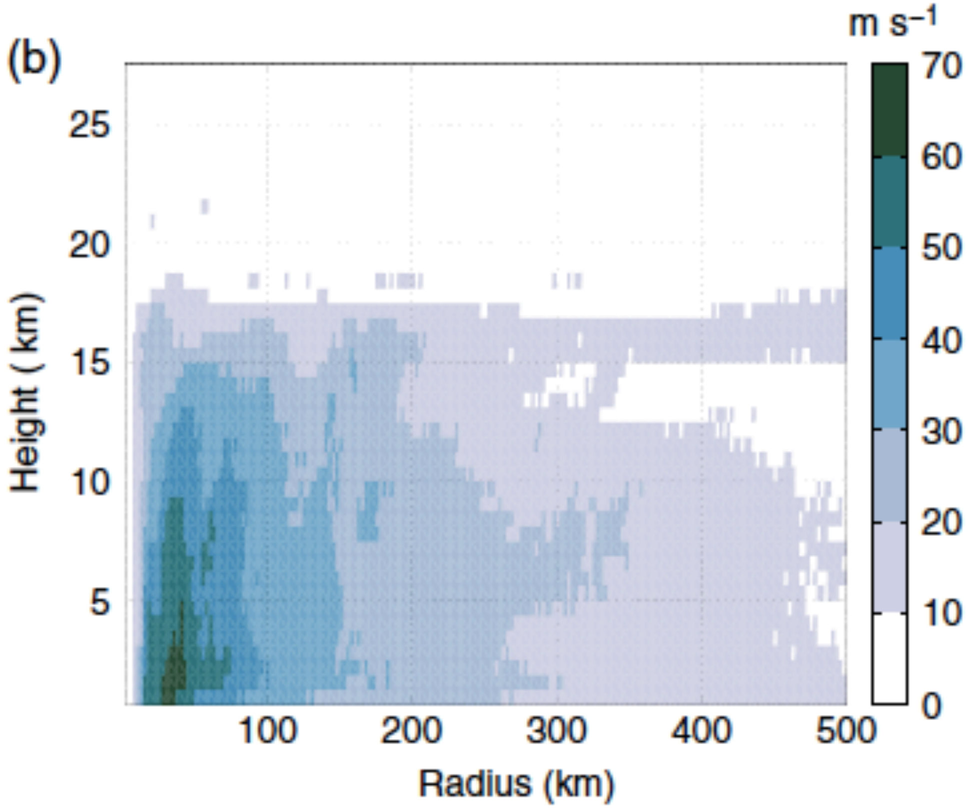
Idealized Tropical Cyclone Simulation
Rotunno and Emanuel (1987) axisymmetric hurricane model

Wong, Tailleux and Gray (QJ, 2015)

Minimum Surface Pressure
Maximum Surface Wind



Azimuthal Wind



Energetics approach:

- Some fraction of surface fluxes generate available potential energy, rest goes into background potential energy
- Part of APE generated gets into kinetic energy, rest goes into APE storage
- Kinetic energy generated eventually dissipates

$$\frac{d\,APE}{dt} = G - C_{APE \to KE}$$

$$\frac{d\,KE}{dt} = C_{APE \to KE} - D_k$$

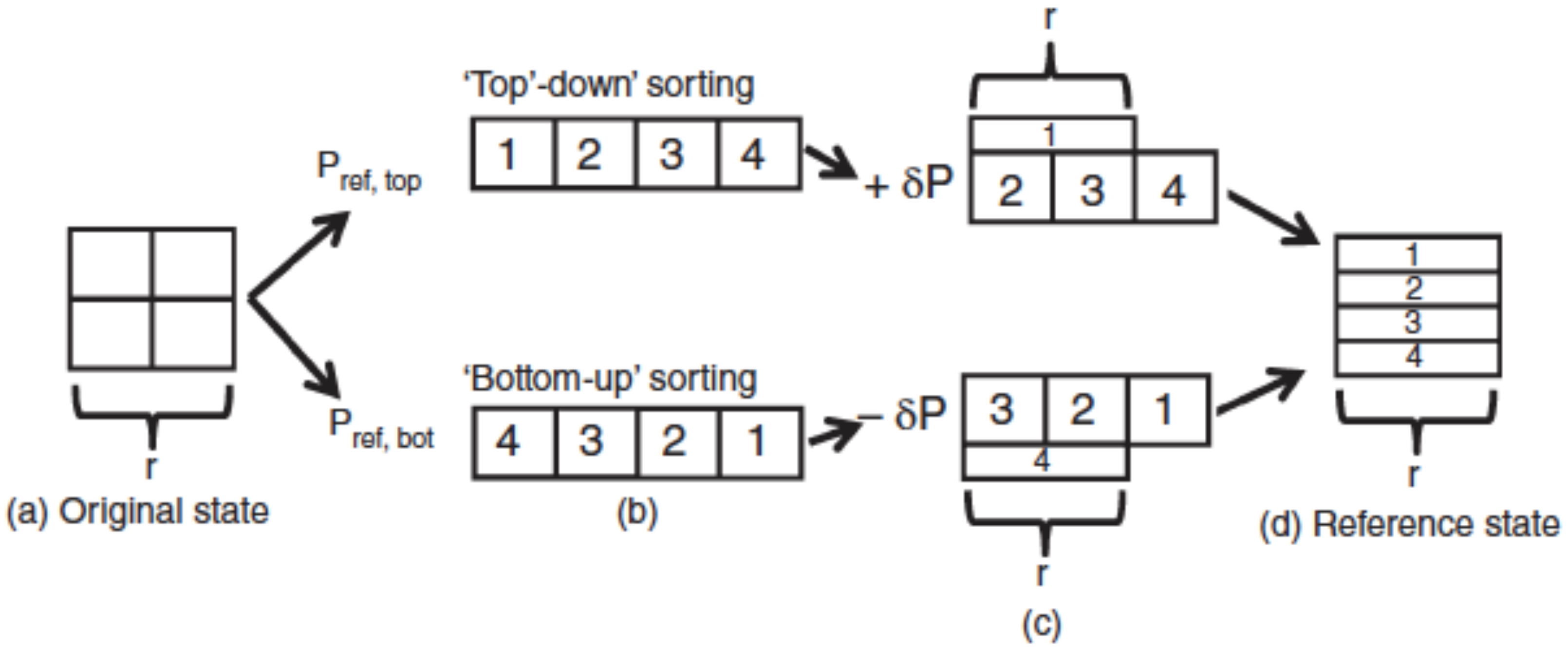
Challenge:

Reference state impacts on APE and APE generation, but not the APE/KE conversion.

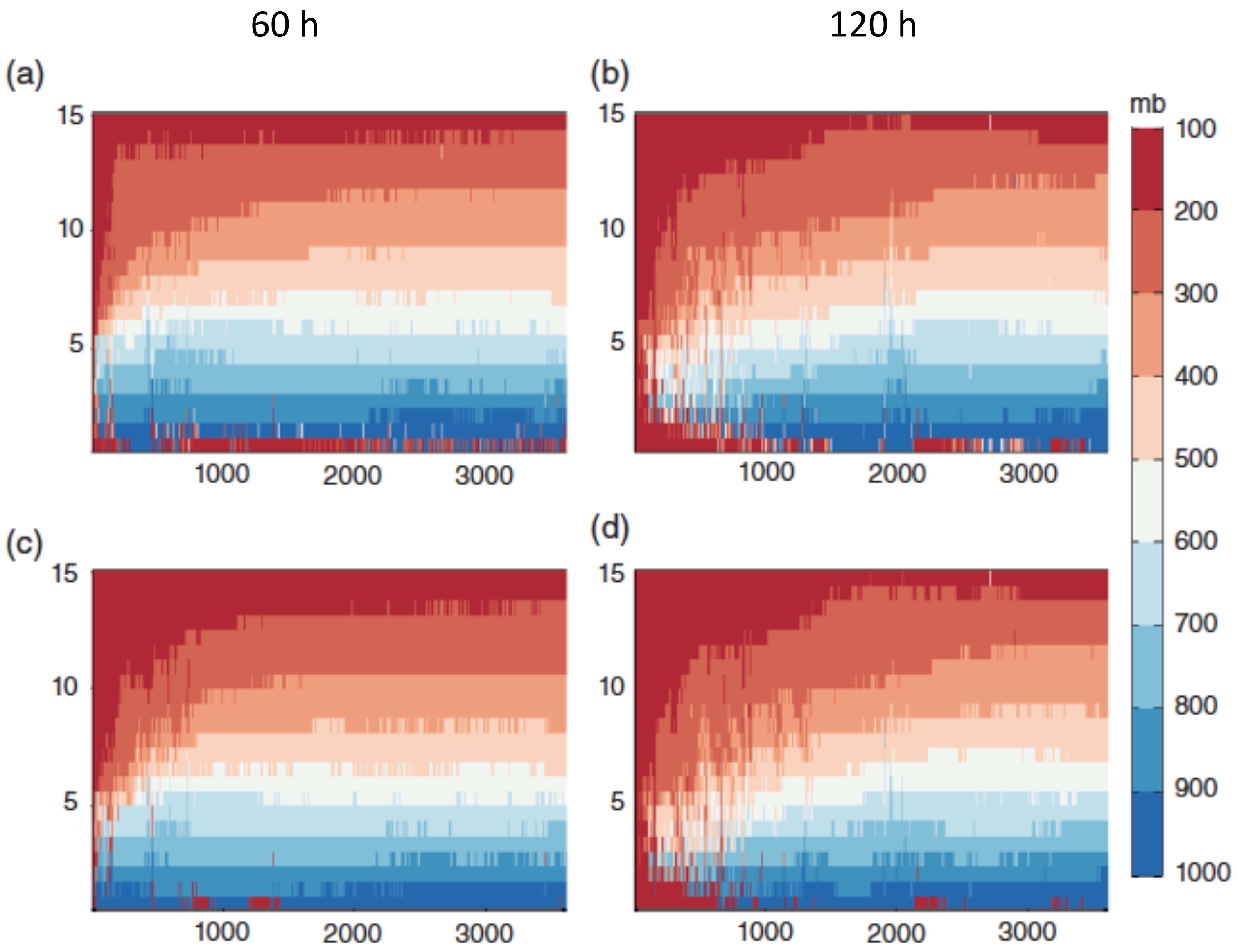
What choice of reference state yields APE generation rate equal APE to KE conversion?

Top down vs bottom-up sorting algorithms (Wong et al., 2016)

See also Harris and Tailleux (2018) for inter-comparison of algorithms for computing moist APE



Reference position



Top Down

Bottom up

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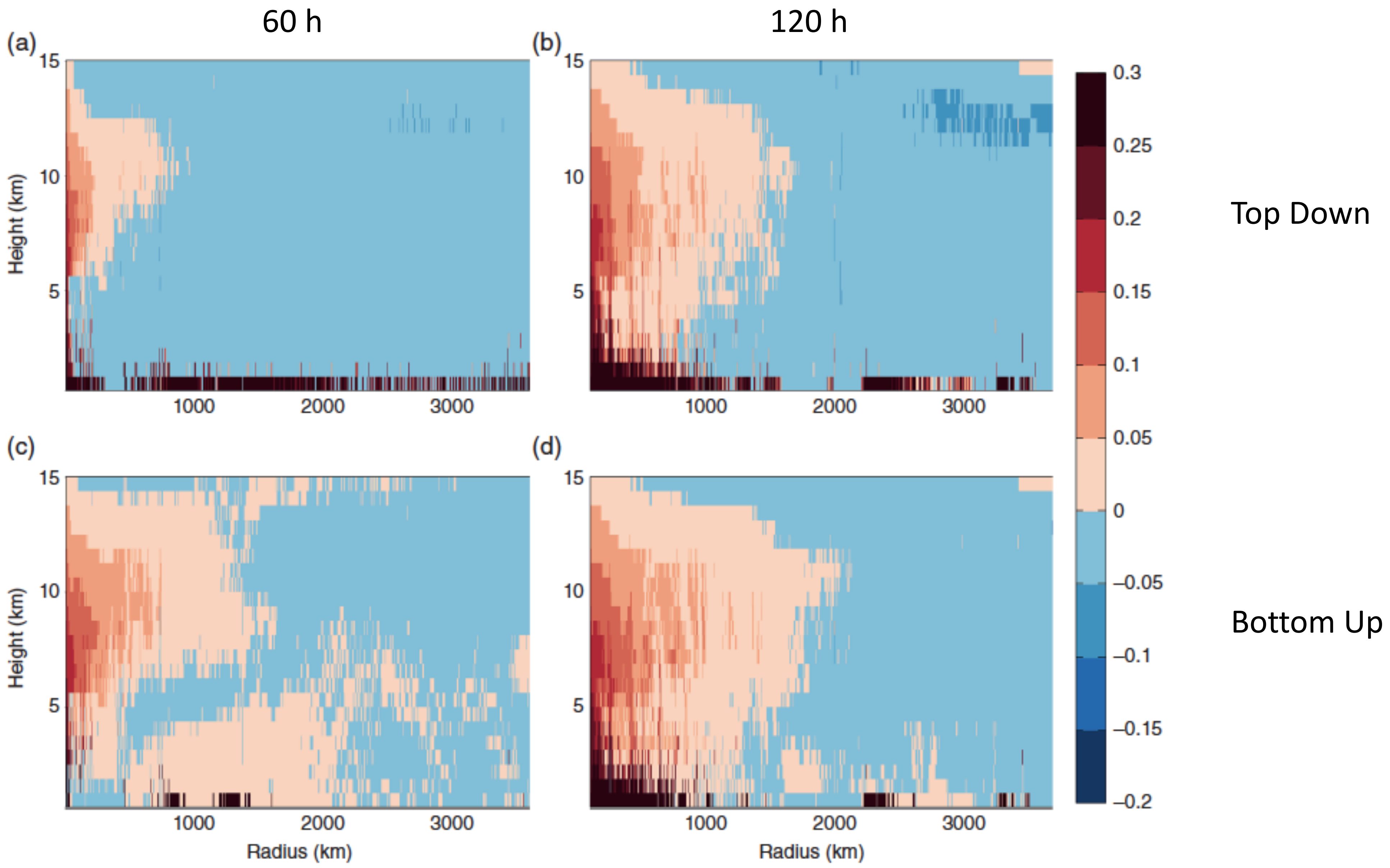
APE production efficiency factor



ure 6. APE production efficiency $(T_1 - T_{ref})/T_1$ computed using (a, b) the top-down and (c, d) the bottom-up sorting method at (a, c) 60 h and (b, d) 120 h.

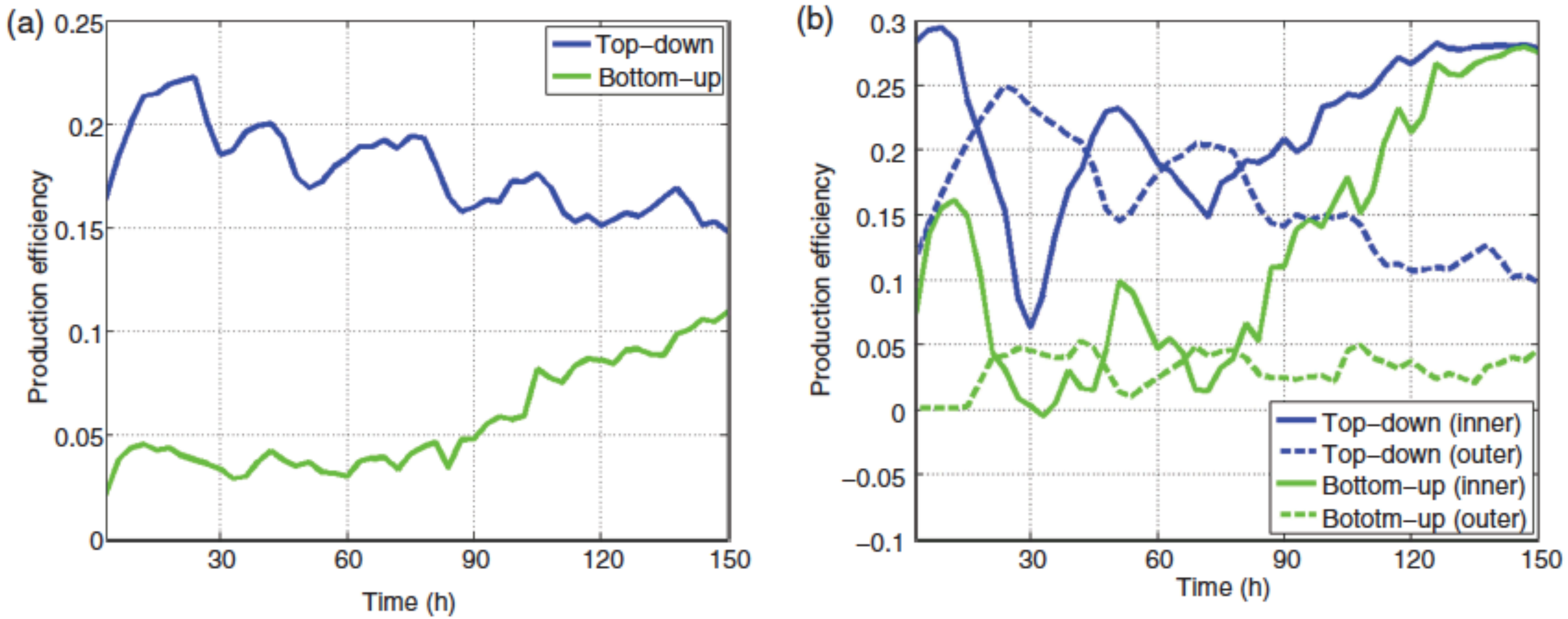


Figure 7. Time series of the area-averaged top-down (blue) and bottom-up (green) APE production efficiency in the surface layer over (a) the whole domain, and (b) the inner region (radius ≤1000 km, solid lines) and outer region (radius >1000 km, dashed lines).

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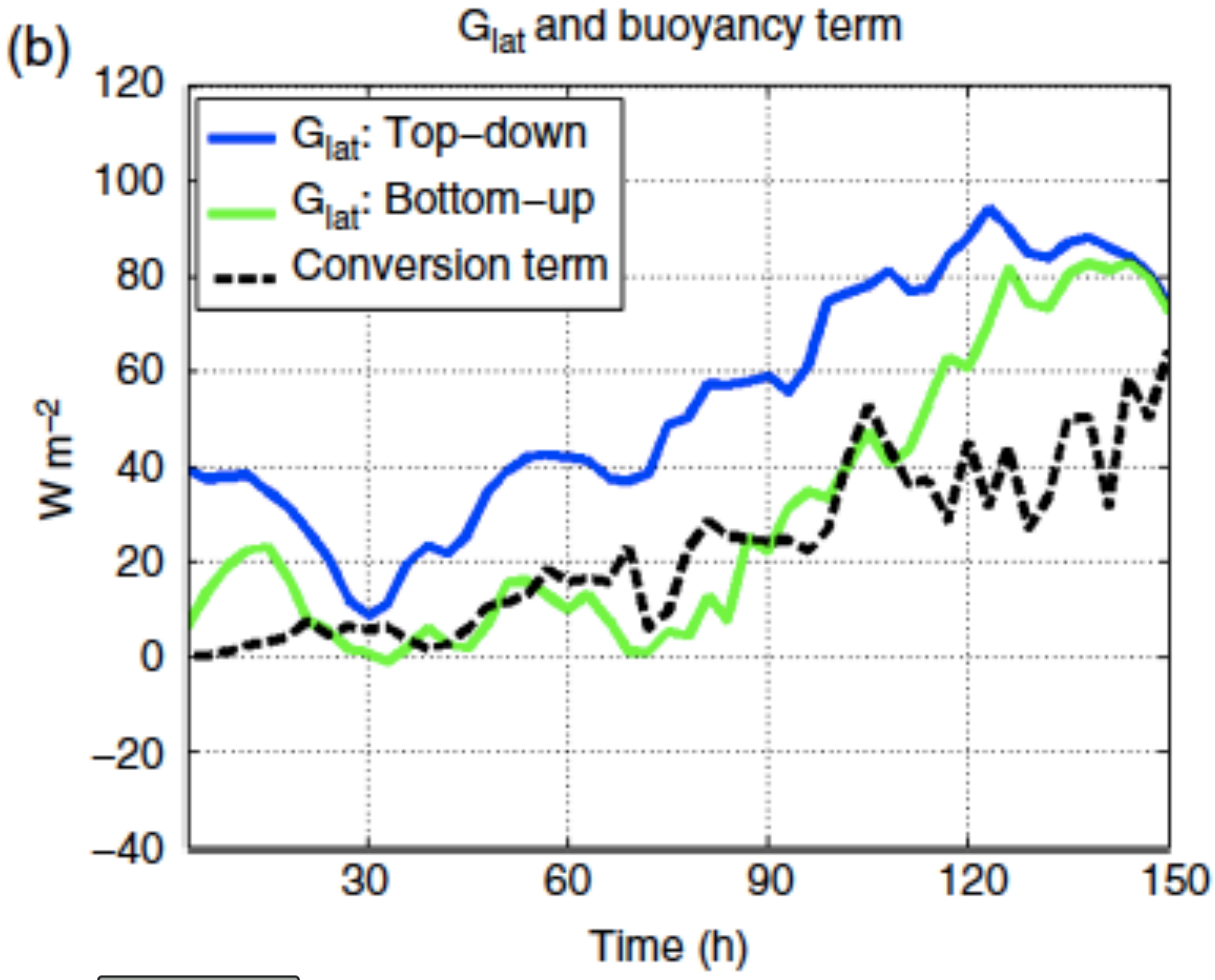
APE PRODUCTION VERSUS DIAGNOSED ENERGY GENERATION



Wong, Tailleux and Gray (QJ, 2015)

$$\frac{\mathrm{d}\,APE}{\mathrm{d}t} = G - C_{APE\rightarrow KE},$$

$$\frac{\mathrm{d}\,KE}{\mathrm{d}t} = C_{APE\rightarrow KE} - D,$$

What about reference state in gradient-wind and hydrostatic balance?

Construct reference state $p_0(r, z, t)$ and $\rho_0(r, z, t)$ so that

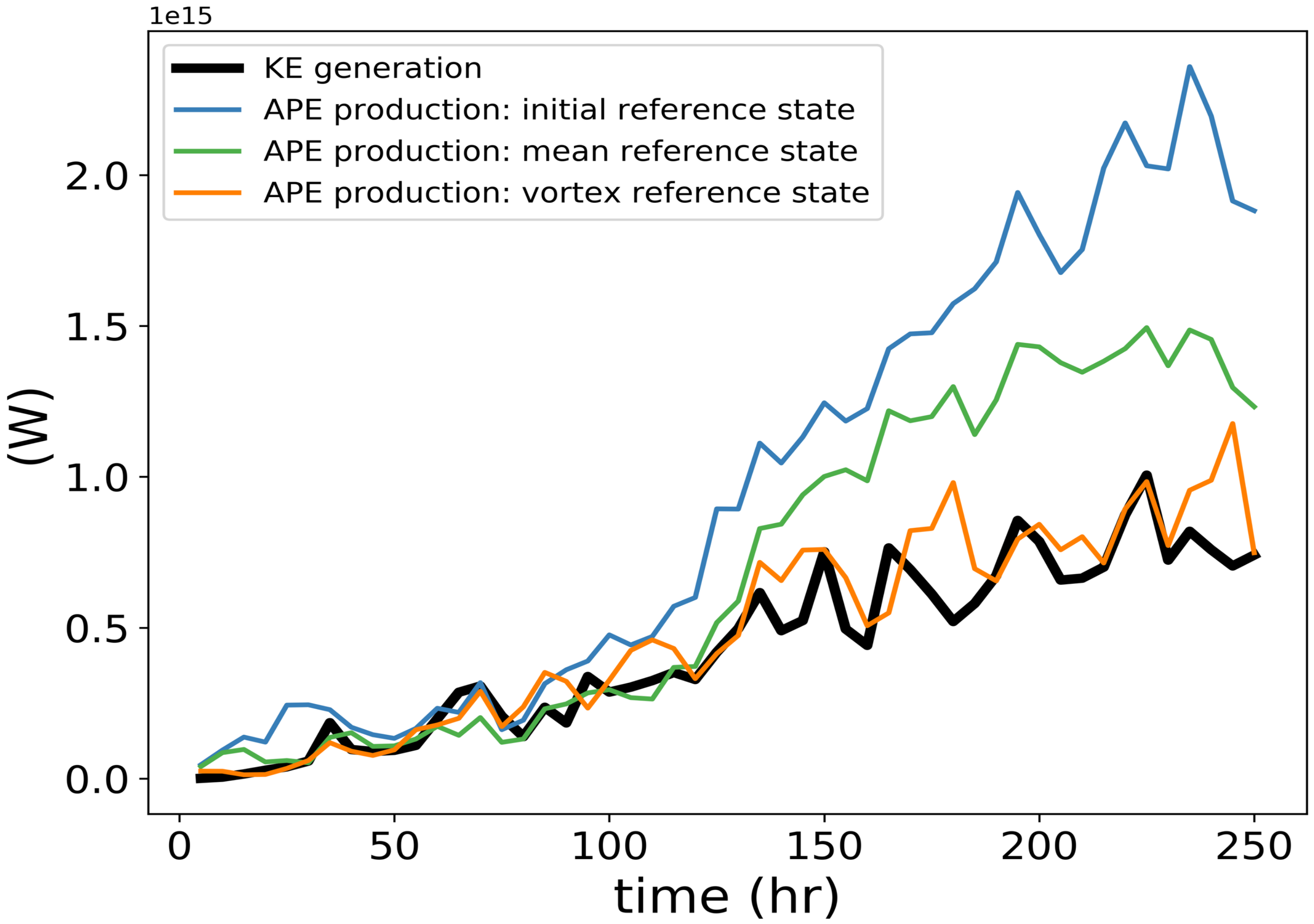
$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial r} = \left(f + \frac{v}{r}\right)v$$

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial z} = -g$$

Method: Iterative procedure based on successively integrating these equations radially and vertically

Nolan and Montgomery (2002)

Impact of reference state on APE production rate



Bethan Harris
PhD work

Comparison between APE production rate predicted using different choices of reference state and diagnose kinetic energy generation

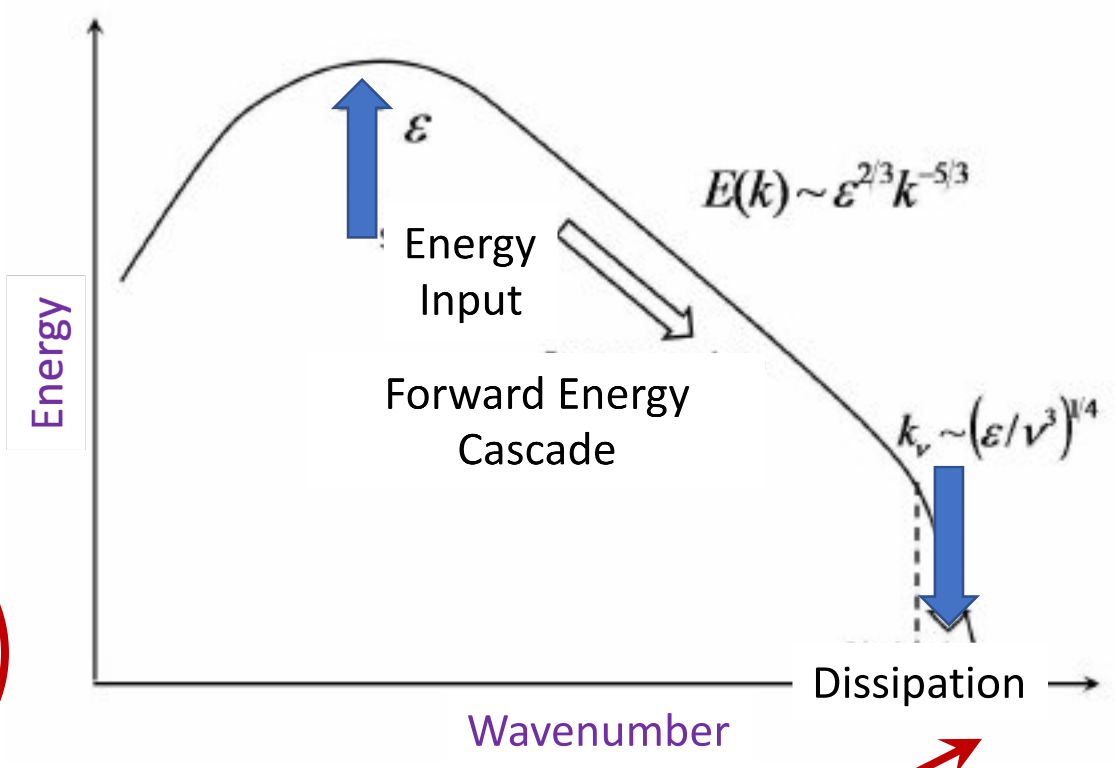
Thermodynamic study of the oceanic and atmospheric heat engines (in a steady-state)

Total Energy Budget

$$Q_{in} - Q_{out} + W_{mech} = 0$$

Mechanical Energy Budget

$$\int_V p \frac{Dv}{Dt}\, dm + W_{mech} = \int_V \varepsilon\, dm$$



Aim of thermodynamic heat engine theories

$$\int_V p \frac{Dv}{Dt}\, dm + W_{mech} = \int_V \epsilon_K\, dm$$

$$\int_V p \frac{Dv}{Dt}\, dm = P_A - D_A$$

P_A = Thermodynamic Production

D_A = Non-Viscous Dissipation

Entropy budget approach

$$\frac{Q_{in}}{T_{in}} - \frac{Q_{out}}{T_{out}} + \dot{\Sigma}_{irr} + \frac{1}{T_\epsilon}\int_V \epsilon_K dm = 0$$

Gouy-Stodola Theorem

$$\int_V \epsilon_K \, dm = \frac{T_\epsilon}{T_{in}}\left(\frac{T_{in} - T_{out}}{T_{out}}\right) Q_{out} + \frac{T_\epsilon}{T_{in}} W_{mech} - T_\epsilon \, \dot{\Sigma}_{irr}$$

Carnot Power

Mechanical Forcing

Lost Work Non-viscous

APE budget approach

$$\int_V \epsilon_K \, dm = G_{APE} + W_{mech} - D_{APE}$$

- G_{APE}: APE production by surface buoyancy fluxes
- W_{mech}: Mechanical Energy input by Wind
- D_{APE}: APE dissipation by non-viscous mixing processes

APE production by surface buoyancy fluxes (freshwater fluxes neglected)

$$G_{APE} \approx \int_S \left(\frac{T - T_R}{T}\right) Q_{net} \, dS = \Upsilon_{out} Q_{out} - \Upsilon_{in} Q_{in}$$

Summary of APE versu Entropy view of thermodynamic forcing and dissipation

	PRODUCTION BY BUOYANCY FLUXES	NON-VISCOUS DISSIPATION
ENTROPY APPROACH	$\frac{T_\epsilon}{T_{in}}\left(\frac{T_{in}-T_{out}}{T_{out}}\right) Q_{out}$	$T_\varepsilon\, \dot{\Sigma}_{irr}^{nonviscous}$
APE APPROACH	$\Upsilon_{ape}\, Q_{out}$	D_{ape}

Local theory of Available Potential Energy

(Andrews 1981; Tailleux 2018; Novak and Tailleux 2018)

$$B \;= \Pi + B_r$$

$$\int_V \Pi\, \rho dV = APE_{Lorenz}$$

$$B = \Phi(z) + h(\sigma, S, p) + \frac{p_R(z) - p}{\rho}$$

$$\Pi = B \; - B_r$$

$$B_r = \Phi(z_r) + h\big(\sigma, S, p_R(z_R)\big)$$

Potential Energy of fluid + Environment

Available Potential Energy density
= Work

Background Potential Energy density
= Heat

Local definition of available energy

$$\Pi = \Phi(z) - \Phi(z_R) + h(\sigma, S, p) - h(\sigma, S, p_R) + \frac{p_R(z) - p}{\rho}$$

$$\approx \frac{(p - p_R)^2}{2\,(\rho\, c)^2} + \frac{N_R^2 (z - z_R)^2}{2}$$

Positive definite. Sum of compressible work + work against buoyancy forces to construct actual state from reference state by means of adiabatic and isohaline transformation. Can be further decomposed into mean/eddy components.

Evolution equation for Available Energy density

$$\rho \frac{D\Pi}{Dt} = \frac{\rho(T - T_*)}{T} \dot{Q} + C(\Pi, E_k) + \cdots$$

Thermodynamic Efficiency

Kinetic Energy Conversion

APE accounting for momentum constraints

- Shepherd (1993): A unified theory of available potential energy. AO
- Codoban and Shepherd (2003): Energetics of a symmetric circulation including momentum constraints. JAS
- Codoban and Shepherd (2006): On the available energy of an axisymmetric vortex. Met. Zeit.
- Andrews (2006): On the available energy density for axisymmetric motions of a compressible stratified fluid. JFM
- Tailleux and Harris (2020): The generalized buoyancy/inertial forces and available energy of axisymmetric compressible stratified vortex motions. JFM, in review. https://arxiv.org/abs/1911.10333

Available energetics of axisymmetric motions relative to a non-resting balanced vortex state

Balanced/hydrostatic reference state

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial r} = \left(f + \frac{v_0}{r}\right) v_0$$

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial z} = -g$$

Reference position solution of

$$\eta_0(r_*, z_*) = \eta$$

$$M_0(r_*, z_*) = M$$

Available energetics of axisymmetric motions (Tailleux and Harris, 2020)

$$A = \Pi_1 + \Pi_k + \Pi_e$$

Available Acoustic Energy (AEE): Compress/Expand from $p_0(r,z)$ to p

$$\Pi_1 = h(\eta, p) - h\big(\eta, p_0(r,z)\big) + \frac{p_0(r,z) - p}{\rho} \approx \frac{\big(p - p_0(r,z)\big)^2}{\rho^2 c_s^2}$$

Centrifugal Potential Energy (proportional to azimuthal kinetic energy anomaly):

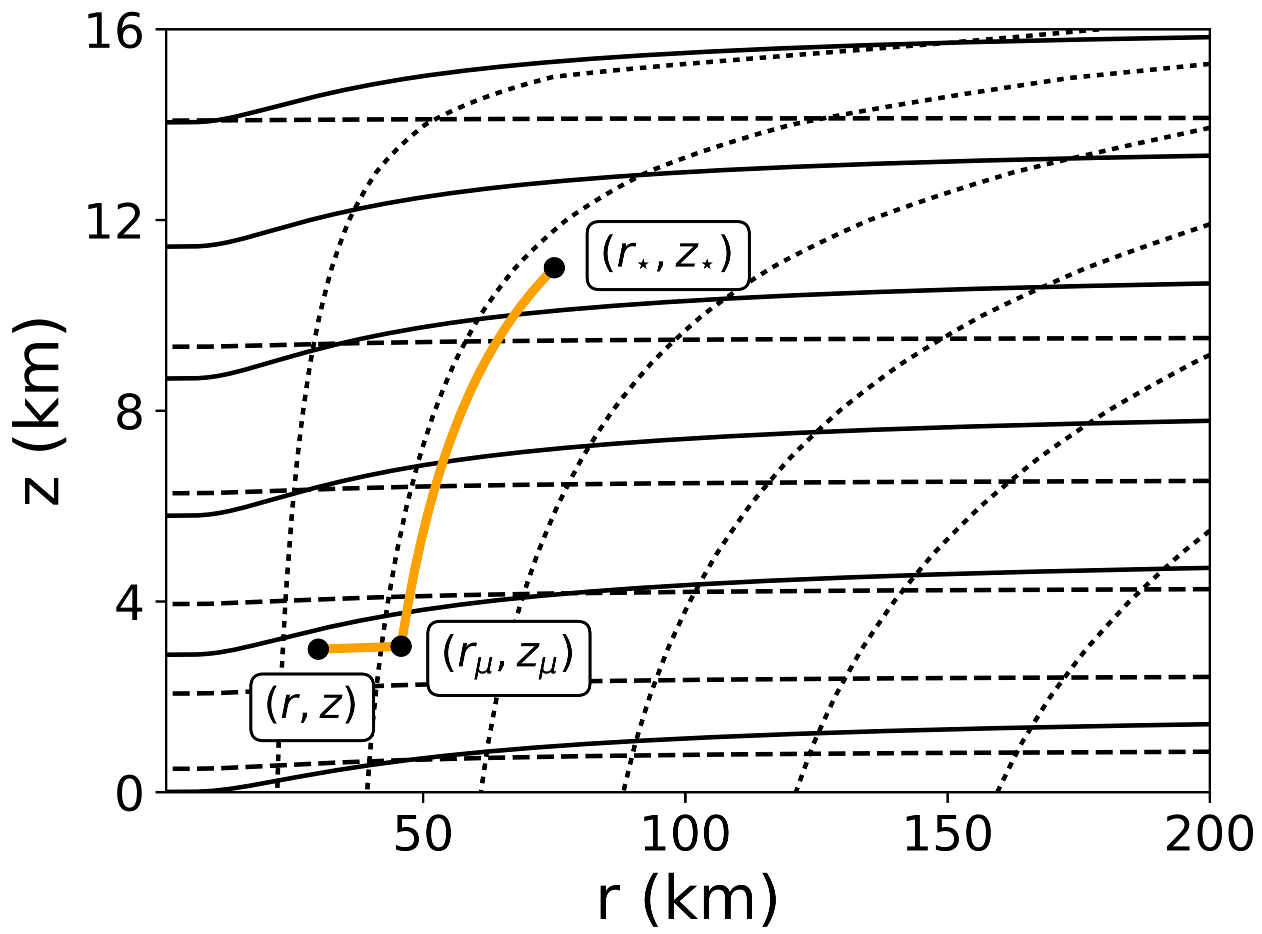
$$\Pi_k = -\frac{\partial \tilde{\chi}}{\partial \mu}\frac{(\mu - \mu_m)^2}{2} \propto \frac{(v - v_m)^2}{2}$$

Slantwise available potential energy (equivalent to SCAPE in a moist atmosphere)

$$\Pi_e = \int_{p_*}^{p_m} \big(v(\eta, p') - v_m(\mu, p')\big)\, dp'$$

Integral of local buoyancy along surface of constant angular momentum

Physical interpretation of available energy relative to vortex reference state (Tailleux and Harris, 2020)



Centrifugal potential energy: Work needed to move the fluid parcel from intermediate position (r_μ, z_μ) isobarically to actual position (r, z)

Slantwise available potential energy: Work needed to move fluid parcel from reference position (r_*, z_*) in balanced vortex state along surface of constant angular momentum up to intermediate position $\left(r_\mu, z_\mu\right)$

Isobaric surfaces: - - -

Dry entropy surfaces: _____

Angular momentum surfaces: . . . .

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Available energy and the forces driving the secondary circulation $\overrightarrow{u_s} = (u, w)$

$$\frac{D\overrightarrow{\boldsymbol{u}_s}}{Dt} \approx -\nabla\Pi_k - \nabla\Pi_e + Dissipation + \ \ldots$$

$$\frac{\mu - \mu_m}{r^3}\,\hat{\boldsymbol{r}}\ \ (purely\ radial)$$

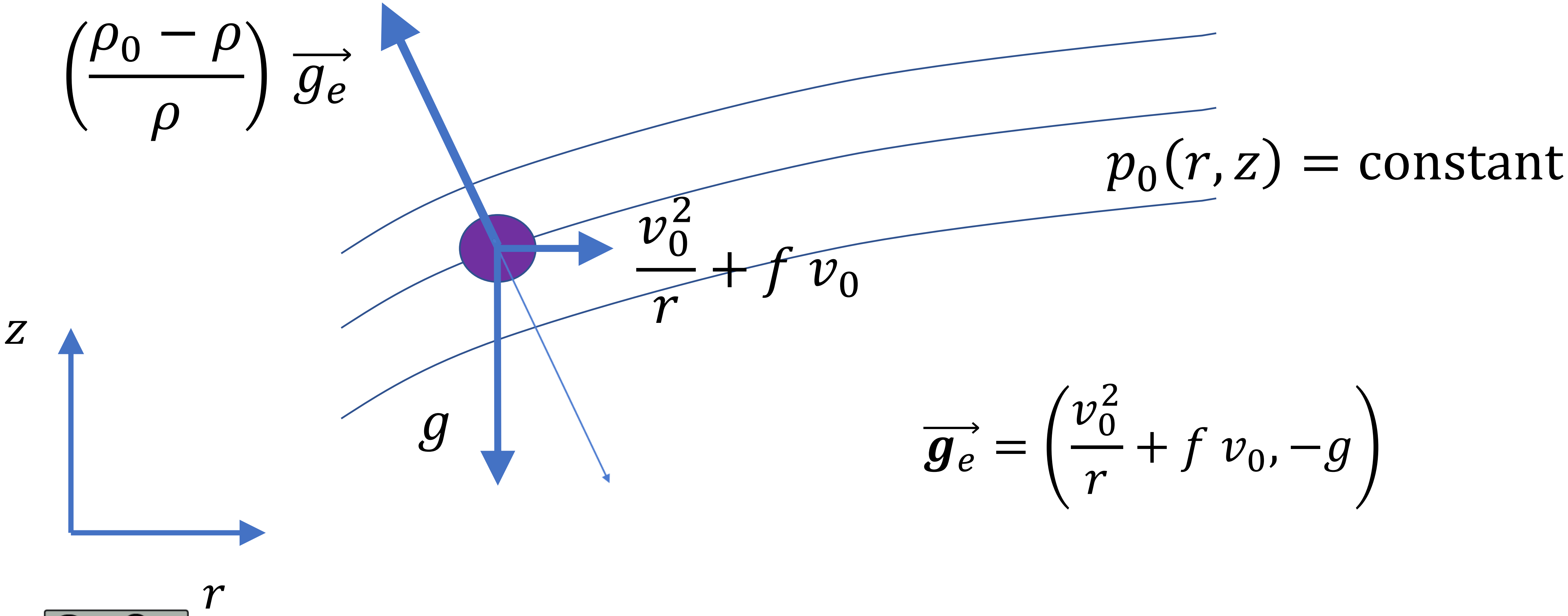
$$-(\nu_h - \nu_0)\nabla \mathrm{p}_0$$

Generalised buoyancy force of Smith et al. (2006). Inward and upward for positive buoyancy

Gradient of available energy defines the generalized buoyancy/inertial force driving the adiabatic component of the secondary circulation!

Generalised buoyancy force: inward and upward for positive buoyancy anomaly

Smith, Montgomery and Zhu (DAO, 2005)



Evolution equation for Available Energy density

$$\rho \frac{DA}{Dt} = \frac{\rho(T - T_*)}{T}\dot{Q} + \rho M \left(\frac{1}{r^2} - \frac{1}{r_*^2}\right)\frac{DM}{Dt} + C(A, E_k) + \cdots$$

Thermodynamic Efficiency

'Mechanical' Efficiency

Kinetic Energy Conversion

Sustained positive diabatic heating required to generate positive buoyancy anomaly needed to sustain the generalized buoyancy force driving the adiabatic secondary circulation. Link with thermodynamic `heat engine' view of tropical cyclones

Equations for the 'diabatic' secondary circulation

$$u_* = \frac{Dr_*}{Dt} = \frac{1}{J_0}\left[\frac{\partial \eta_0}{\partial z}\frac{DM}{Dt} - \frac{\partial M_0}{\partial z}\frac{\dot{Q}}{T}\right]$$

$$w * = \frac{Dz_*}{Dt} = \frac{1}{J_0}\left[-\frac{\partial \eta_0}{\partial r}\frac{DM}{Dt} + \frac{\partial M_0}{\partial r}\frac{\dot{Q}}{T}\right]$$

Secondary circulation sum of diabatic and adiabatic components

$$u = u_* + \delta u$$
$$w = w_* + \delta w$$

If reference vortex state evolves only slowly, diabatic secondary circulation depends only on local sinks/sources of entropy and angular momentum. Much simpler than Eliassen-Sawyer equations.

Summary and conclusions

- Theory of available potential energy can be generalized to account for momentum constraints => Available energy for perturbations to an axisymmetric reference vortex in gradient wind balance
- Available energy is the sum of available acoustic energy, centrifugal potential energy and slantwise available potential energy
- Gradient of the centrifugal potential energy and slantwise potential energy defines the generalized buoyancy/inertial force driving the adiabatic secondary circulation, whose kinetic energy is transferred to that of the primary circulation
- Maintenance of such a generalized buoyancy/inertial force required sustained positive diabatic heating to sustain positive buoyancy anomaly

Summary and conclusions (cont'd)

- Available Energy defined relative to a non-resting state is a special case of 'eddy' APE
- Available energy production defined relative to a non-resting reference state is a very accurate predictor of kinetic energy creation
- Assumption of axisymmetry forbids exchanges between 'eddy' APE and 'mean' APE. Axysymmetric TC evolution lacks a potentially crucial intensification mechanism compared to asymmetric TC evolution, consistent with Persing et al. (2013)
- APE production of generalized APE includes both thermodynamic and mechanical production terms
- Introduction of reference position allows a rigorous decomposition of the total circulation into adiabatic and diabatic components.

References

- Wong, K. C., R. Tailleux and S. L. Gray,, 2016: The computation of reference state and APE production by diabatic processes. QJRMS, 142, 2646—2657.
- Harris, B. and R. Tailleux, 2018: Assessment of algorithms for computing moist available energy. QJRMS, 144, 1501—1510.
- Tailleux, R. and B. Harris (2020): The generalized buoyancy/inertial forces and available energy of axisymmetric compressible stratified vortex motions. Journal of Fluid Mechanics, in review. Available at: https://arxiv.org/abs/1911.10333