Centrifugal Instability of a Geostrophic Jet

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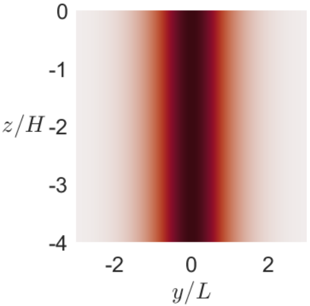
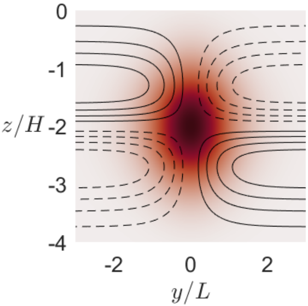
Outline

Types of Jets Considered

Barotropic jet	Baroclinic jet
	

Jets can experience to Baroclinic, Barotropic, Gravitational (GI) and **Centrifugal instabilities (CI)**.

The Work of Carnevale et al.

- Examined unstratified Gaussian barotropic jets.
- Analytically approximated the nonlinear behaviour of the flow:
  - Onset and saturation of CI.
  - Onset and saturation of secondary barotropic instability.
- Provided approximations for the effect of CI.
- Supported these with numerical simulations.

The Work of Ribstein et al.

- Studied the instability of a stratified Bickley jet.
- Linear theory for barotropic jets:
  - Confirmed an ultraviolet catastrophe in the inviscid case.
  - Viscosity arrests the ultraviolet catastrophe.
- Nonlinear simulation for the baroclinic jet:
  - Used WRF, which tends to be diffusive.
  - First a CI instability and then a secondary barotropic instability.

Governing Equations and Nondimentional Paramaters

- Governing system:

$$\frac{D\vec{u}}{Dt} + \vec{f} \times \vec{u} = -\nabla\Phi + b\hat{z} + \nu\nabla^2\vec{u} - \nu\nabla^2\overline{\vec{u}}, \quad (1)$$

$$\nabla \cdot \vec{u} = 0, \quad (2)$$

$$\frac{Db}{Dt} = \kappa\nabla^2 b - \kappa\nabla^2\overline{b}. \quad (3)$$

- BCs: Periodic in x and free slip conditions in y and z.
- Nondimensional parameters:

$$\mathrm{Ro} = \frac{U}{fL}, \quad \mathrm{Re} = \frac{UL}{\nu}, \quad \mathrm{Bu} = \left(\frac{NH}{fL}\right)^2, \quad \delta = \frac{H}{L}, \quad \text{and} \quad \mathrm{Pr} = \frac{\nu}{\kappa}$$

Linear Instability Theory

- Consider a solution $\vec{B}$ to a nonlinear system $\partial_t \vec{B} = \mathcal{N}(\vec{B})$.
- Add a small perturbation and linearize the equations to yield

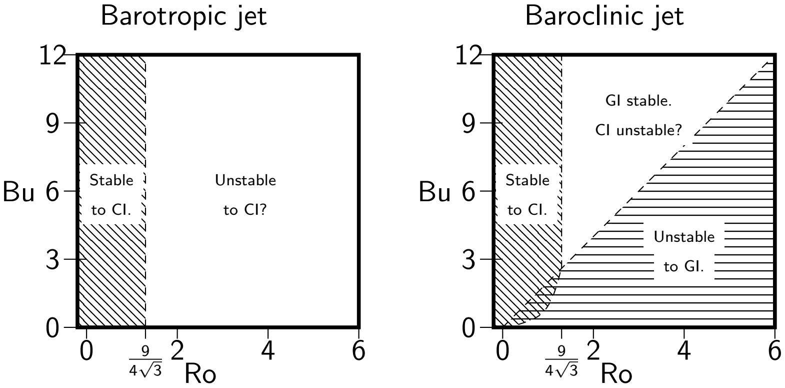
$$\partial_t \vec{b} = \mathcal{L}(\vec{B})\vec{b}.$$

- Use a Fourier decomposition of $\vec{b}$ in time and in x-direction

$$-i\omega\hat{\vec{b}}(\vec{x}) = \mathcal{L}(\vec{k};\vec{B})\hat{\vec{b}}(\vec{x}).$$

- Solve to determine linear stability characteristics.

Paramater Regions of Interest



Can also be unstable to barotropic and baroclinic instabilities.

Mixing Effiecency

Jiao and Dewar found that CI can efficiently mix the flow.
We examine mixing efficiency with flux Richardson number:

$$\mathrm{Ri}_f = \frac{B}{B+\epsilon}.$$

- $B = -\overline{w'b'}$ is the transfer of energy from APE and KE to the BPE.
- $\epsilon = \overline{2\nu\left(e_{ij}e_{ij} - 1/3(e_{ii})^2\right.}$, where $e_{ij} = 0.5(\partial_{x_j}u_i + \partial_{x_i}u_j)$ is the viscous dissipation.
- Can be shown analytically that $Ri_f \in [0,1]$.
- Kelvin-Helmholtz has a typical efficiency of $[0.2, 0.3]$.

Eigenvalue Problems for the LSA Problems

We simplify by making the hydrostatic approximation.

EVP for barotropic jet

- 1D EVP of the form $\omega \begin{bmatrix} \Phi' & u' & iv' \end{bmatrix}^T = A \begin{bmatrix} \Phi' & u' & iv' \end{bmatrix}^T$.
- A depends on k and m, parameters and $\overline{U}(y)$.

EVP for baroclinic jet

- 2D generalized EVP of the form $\omega B \begin{bmatrix} \Phi' & u' & iv' \end{bmatrix}^T = C \begin{bmatrix} \Phi' & u' & iv' \end{bmatrix}^T$.
- B and C depend on k, parmaeters and $\overline{U}(y, z)$.

Eigenvalue Solvers for the LSA Problems

1D EVP for the barotropic jet

- Used a direct EVP solver with a Chebyshev grid.
- Domain must contain the most unstable mode.

2D EVP for the baroclinic jet

- Shift-and-Invert Arnoldi method with linear spacing.
  - Used the barotropic EVP to provide a guess.
  - Want eigenvalues with large growth rates.
- Pick domain to contain region of negative EPV.

Spectral Parallel Incompressible Navier-Stokes (SPINS)

Spins is Spectrally accurate and highly parallelizable.

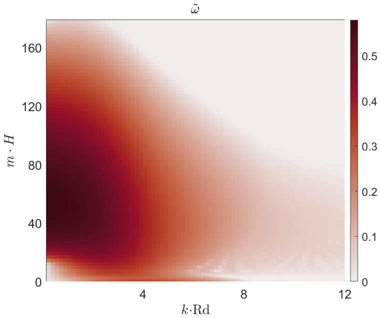
For our purposes we:

- Use periodic BC in the direction of the jet and Free slip BCs in the orthogonal directions.
- Use a Fourier basis and FFTs that scale well using MPI.
- Add a force to balance the dissipation of the jet.

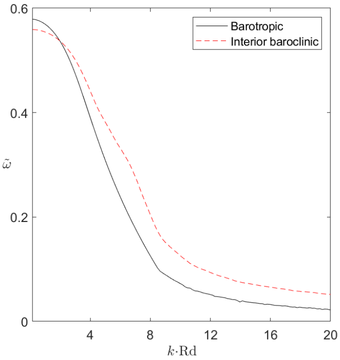
Case 1 - LSA Results I

Nondimensional parameters - (Ro,Re,Bu,δ,Pr)$=(2, 1.1 \times 10^8, 17.26, 0.03, \infty)$.

2D EVP for Barotropic jet

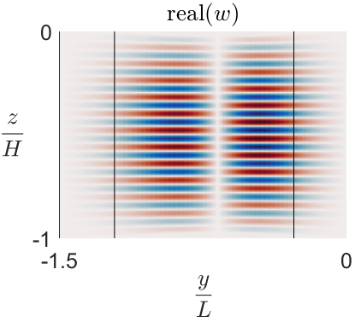


Barotropic vs baroclinic jets

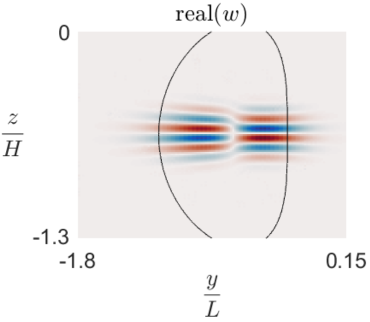


Case 1 - LSA Results II

Fastest growing barotropic mode



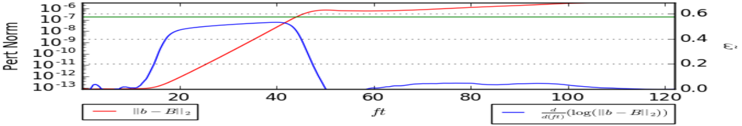
Fastest growing baroclinic mode



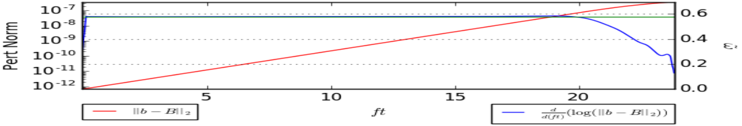
Both instabilities have comparable vertical wavelengths.

Case 1 - Verification of LSA Results I

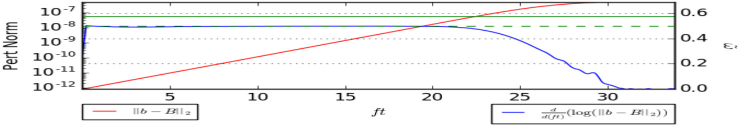
Random initial perturbation



Initial perturbation given by fastest growing mode

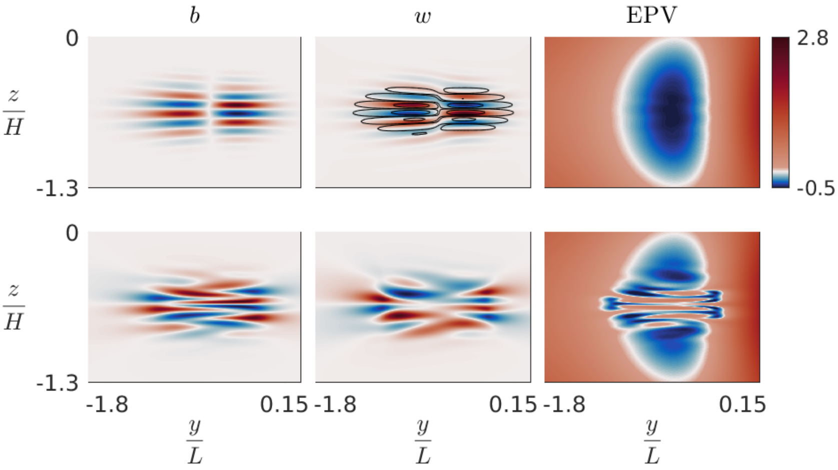


Initial perturbation given by second fastest growing mode



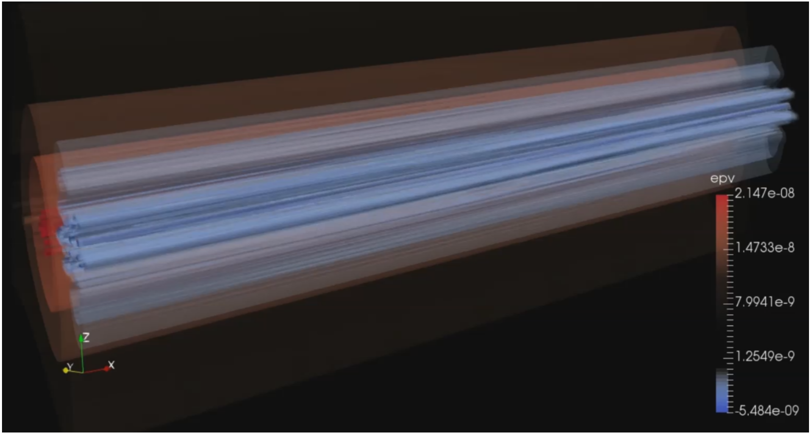
Case 1 - Nonlinear Saturation of CI I

2D 32798 × 4096 results:

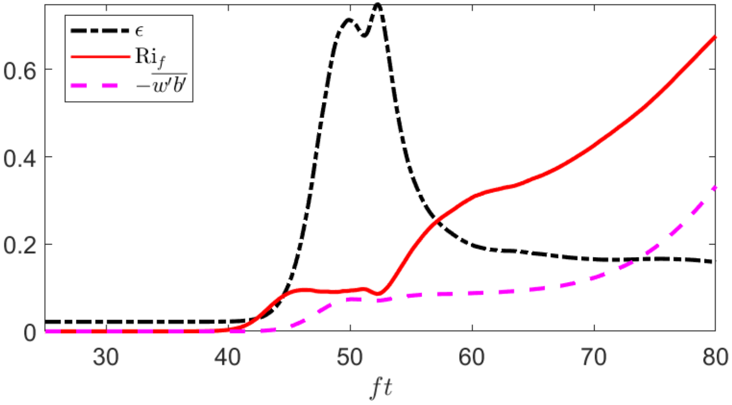


Case 1 - Nonlinear Saturation of CI II

3D $256 \times 512 \times 1024$ EVP field:



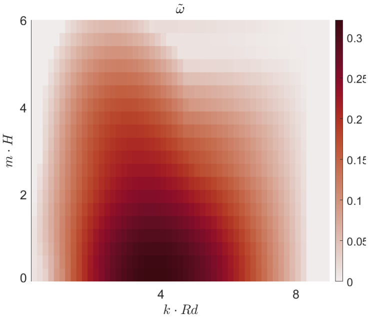
Case 1 - Mixing Efficiency

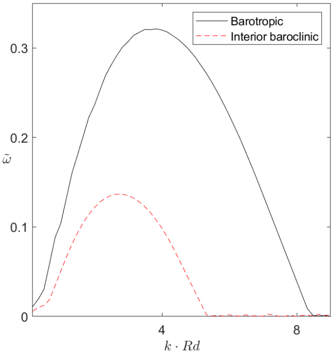


Case 2 - LSA Results I

Nondimensional parameters -
(Ro,Re,Bu,δ,Pr)$=(2, 2.2 \times 10^5, 17.26, 0.03, \infty)$.

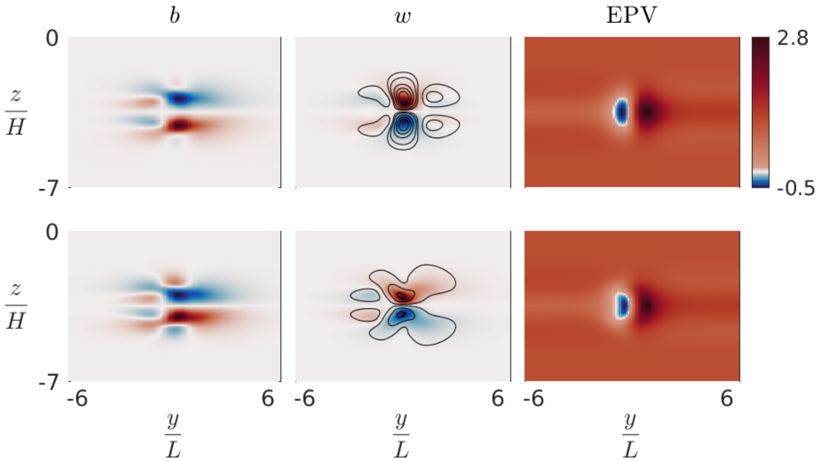
2D EVP for barotropic jet	Barotropic vs baroclinic jets





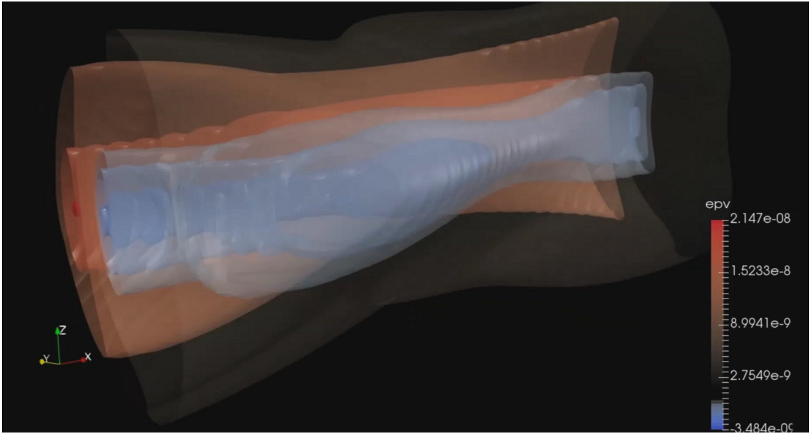
Case 2 - Nonlinear Simulation I

3D 256 × 256 × 1024 results:

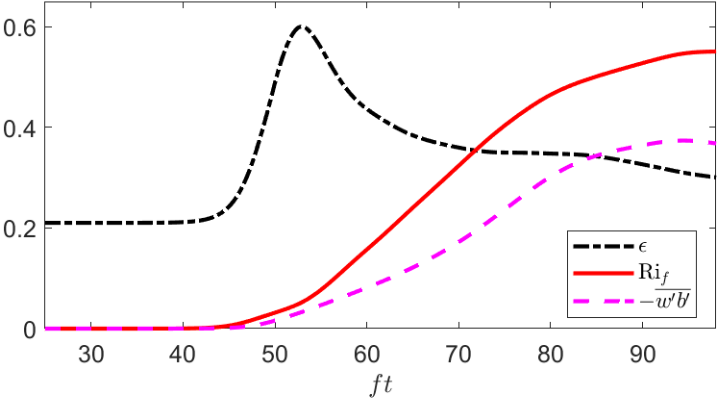


Case 2 - Nonlinear Saturation of CI

3D $256 \times 256 \times 1024$ EVP field:



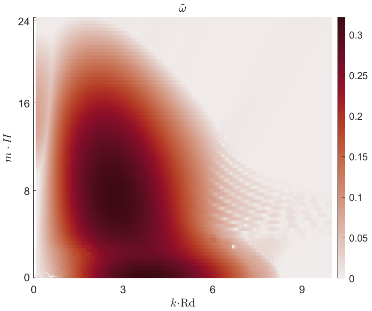
Case 2 - Mixing Efficiency

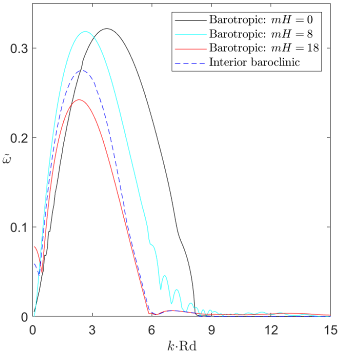


Case 3 - LSA Results I

Nondimensional parameters - (Ro,Re,Bu,δ,Pr)=$(2, 2.2 \times 10^5, 17.26, 0.1, \infty)$.

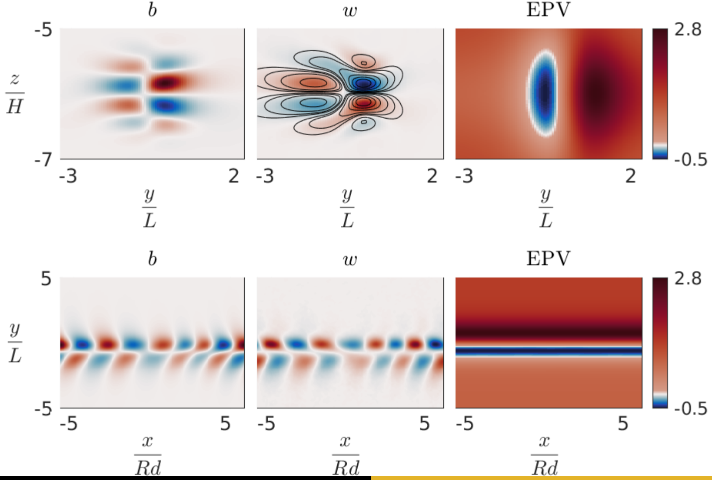
2D EVP for barotropic jet	Barotropic vs baroclinic jets





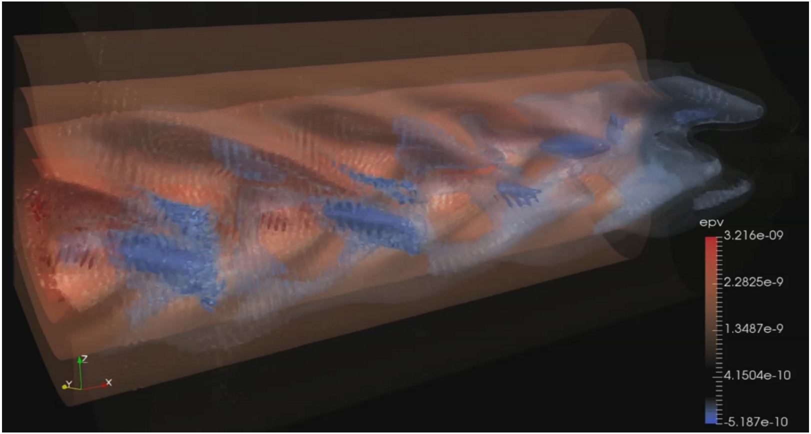
Case 3 - Nonlinear Simulation

3D $256 \times 256 \times 1024$ results:

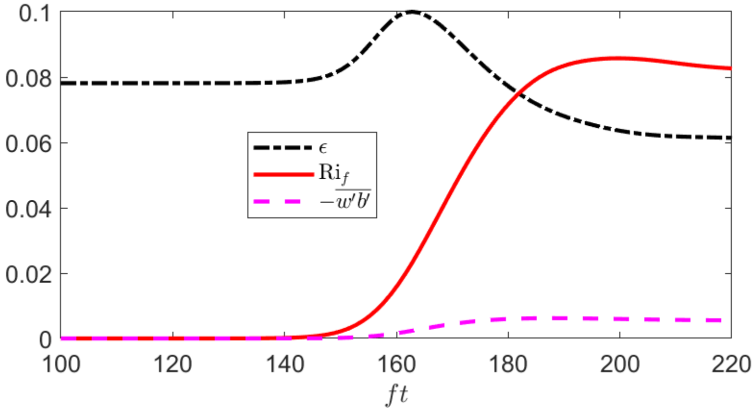


Case 3 - Nonlinear Saturation of CI

3D $256 \times 256 \times 1024$ EVP field:

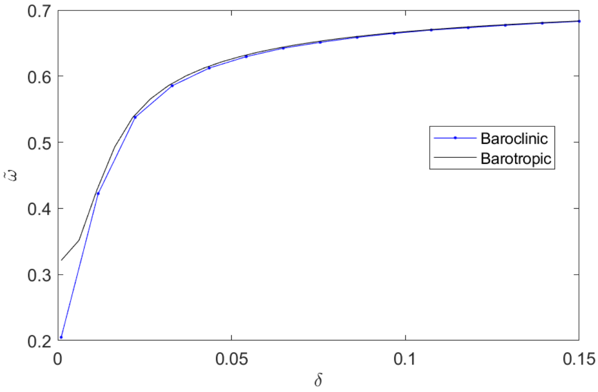


Case 3 - Mixing Efficiency



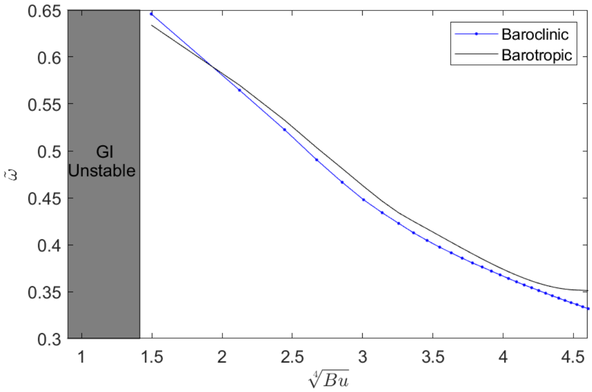
Paramater Study I

δ: The growth rates agree except for small δ.



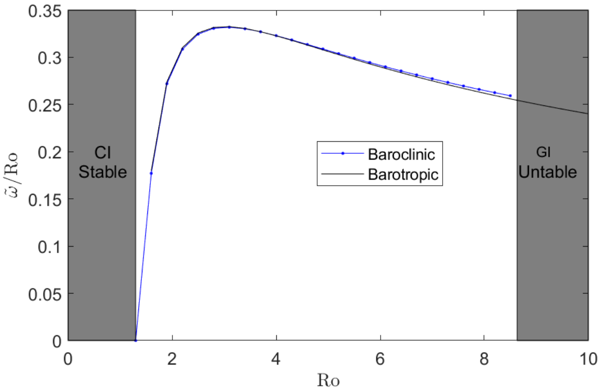
Paramater Study II

Bu: BT growth rates are slightly larger than BC.



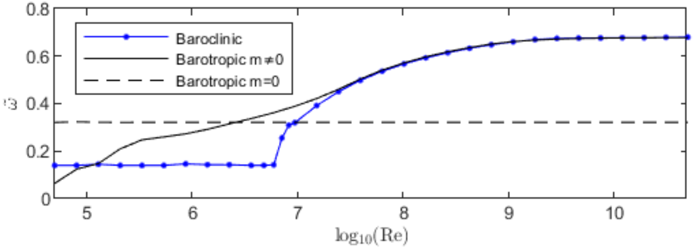
Paramater Study III

Ro: The growth rates agree for all Ro.



Paramater Study IV

Re: Strong dependency on dissipation



Conclusions

- The jets we examined are stable to CI when $\text{Ro} < 9/(4\sqrt{3})$.
- For small Reynolds numbers the stability properties of baroclinic and barotropic jets can differ significantly.
- CI is generally efficient at mixing the water column.
- Depending on the flow parameters, CI may generate a secondary instability.

Future Work

- What are the effects of non-uniform stratification?
- Can we classify the different types of nonlinear saturation based on nondimensional parameters?
- What is the effect of changing the Prandtl number?
- How robust are the parameter study results?

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