Rainfall-Runoff Hydrograph Prediction Using a Dynamic Wave Based Instantaneous Unit Hydrograph

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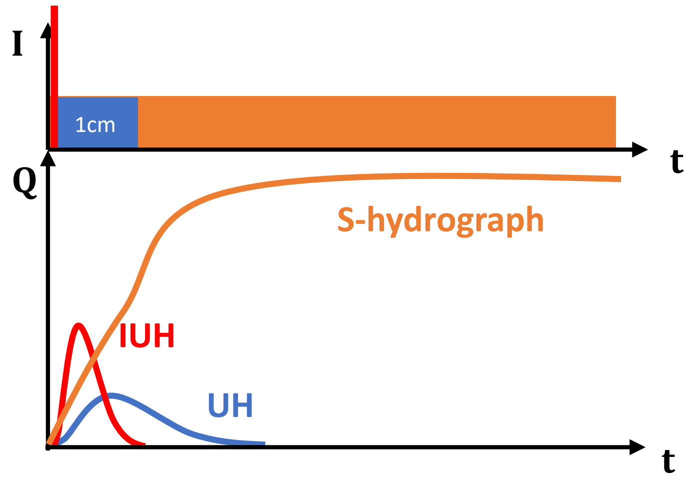
Contents

Concept of Unit hydrograph (UH)

- ✓ Unit hydrograph (UH) is a direct runoff hydrograph resulting from one unit (1 cm or 1 inch) of effective rainfall generated uniformly over the entire watershed at a constant rate for an effective duration.
- ✓ A UH model is a simple linear model that can be used to derive runoff hydrographs using the principles of superposition and proportionality.
- ✓ The UHs for a watershed have traditionally been derived by relating measured rainfall and runoff discharge.
- ✓ If the effective rainfall is of unit amount and its duration is infinitesimally small, the unit hydrograph may be called as an Instantaneous unit hydrograph (IUH).
- ✓ S-hydrograph is a direct runoff hydrograph generated by a continuous effective rainfall occurring at an uniform rate for an indefinite period.
- ✓ S-hydrograph can be obtained by summation of an infinite series of UHs or IUHs.



Linsley, R. K., Kohler, M. A., Paulhus, J. L. H., 1958. Hydrology for Engineers. McGraw-Hill, New York.

Chow, V., 1964. Applied Hydrology. McGraw-Hill, New York.

Concept of Unit hydrograph (UH)

✓ Brief history of UH models

- Sherman (1932) : Introduction of unit hydrograph (UH)

  Sherman, L.K., 1932. Streamflow from Rainfall by Unit-Graph Method. Engineering News-Record. 108, 501-505.

- Snyder (1938), Clark (1945) : Synthetic Unit hydrographs

  Snyder, F. F., 1938. Synthetic Unit Graphs. Eos, Transactions American Geophysics Union. 19, 447-454.

  Clark, C.O., 1945. Storage and the Unit Hydrograph. Transactions of the American Society of Civil Engineers. 110 (1), 1419-1446.

- Nash (1957), Dooge (1959) : Instantaneous unit hydrographs (IUH)

  Nash, J.E., 1957. The Form of the Instantaneous Unit Hydrograph. International Association of Hydrological Sciences. 45, 114-121.

  Dooge, J.C.I., 1959. A general theory of the unit hydrograph. Journal of Geophysical Research. 64 (2), 241-256.

- Rodriguez-Iturbe and Valdes (1979), Gupta et al. (1980) : Geomorphological instantaneous unit hydrographs (GIUH)

  Rodriguez-Iturbe, I., Valdés, J.B., 1979. The geomorphologic structure of hydrologic response. Water Resources Research. 15(6), 1409-1420.

  Gupta, V.K., Waymire, E., Wang, C.T., 1980. A representation of an instantaneous unit hydrograph from geomorphology. Water Resources Research, 16(6), 855-862.

- Lee and Yen (1997) : Kinematic wave based GIUH (KW-GIUH)

  Lee, K. T., Yen, B. C., 1997. Geomorphologic and kinematic-wave-based hydrograph derivation. Journal of Hydraulic Engineering. 123(1), 73– 80.

Dynamic wave based IUH (DIUH) method

- ✓ Dynamic wave models are known as one of the most physically representative and accurate rainfall-runoff modeling methods, thus can consider the nonlinear relationship between rainfall and runoff.
- ✓ However, the dynamic wave models are not appropriate for a practical purpose owing to their numerical instability and high computational cost.
- ✓ Our purpose is to present a hybrid method composed of a dynamic wave approach and instantaneous unit hydrograph (IUH) to predict rainfall-runoff discharge physically, stably, and instantly.

Dynamic wave

(+) - Physical
- Nonlinear

(−) - Unstable
- Long computation time

\+

IUH

(+) - Stable, Reliable
- Simple, Fast

(−) - Conceptual, Not physical
- Linear

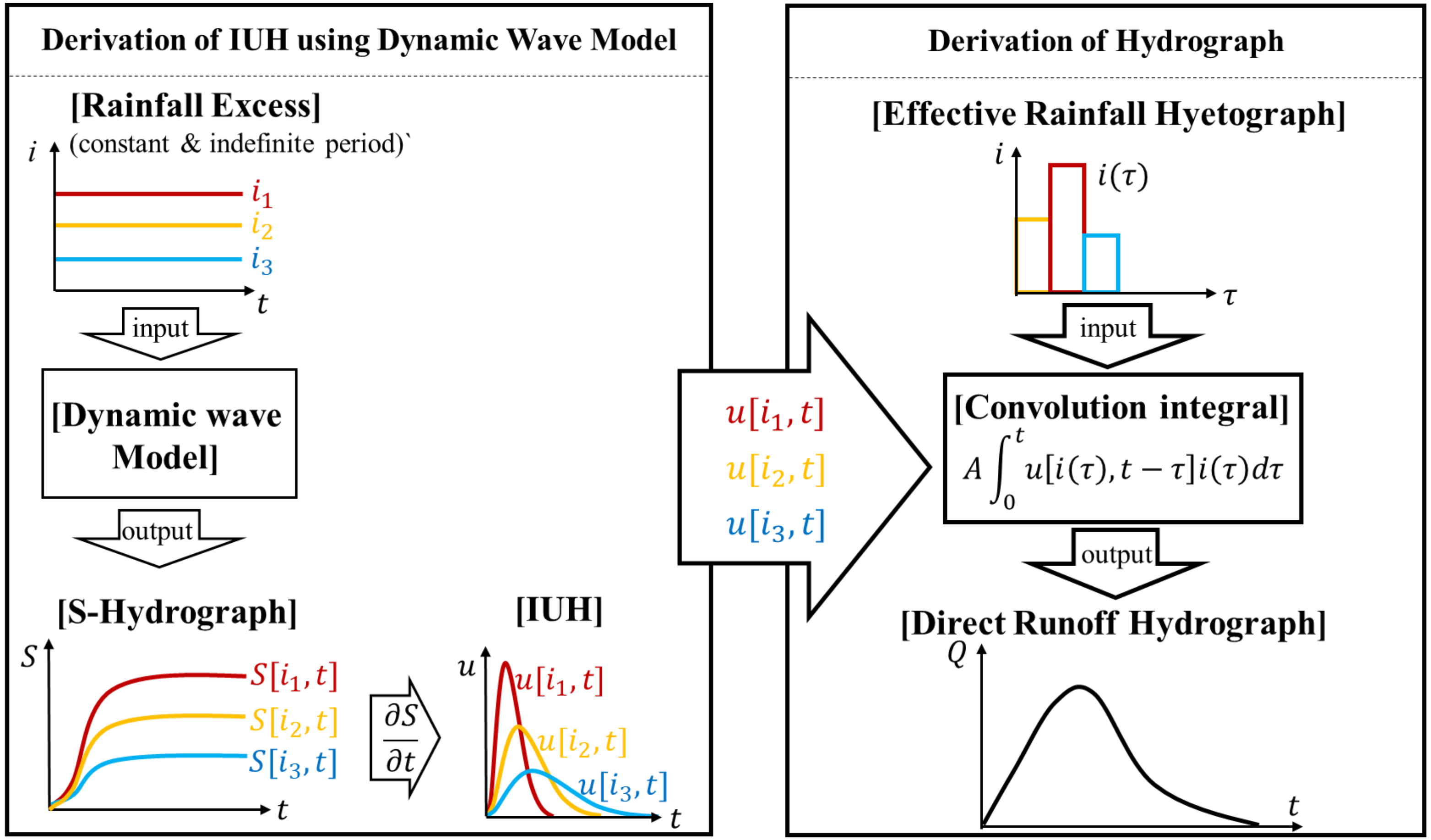
=

Dynamic wave based IUH

- Stable, Reliable
- Simple, Fast
- Physical
- Nonlinear

Dynamic wave based IUH (DIUH) method

Dynamic wave based IUH (DIUH) method



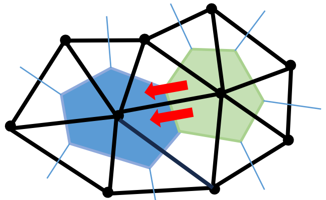
Schematic of dynamic wave based IUH (DIUH) method

Dynamic wave based IUH (DIUH) method

Step 1 : S-hydrograph Generation

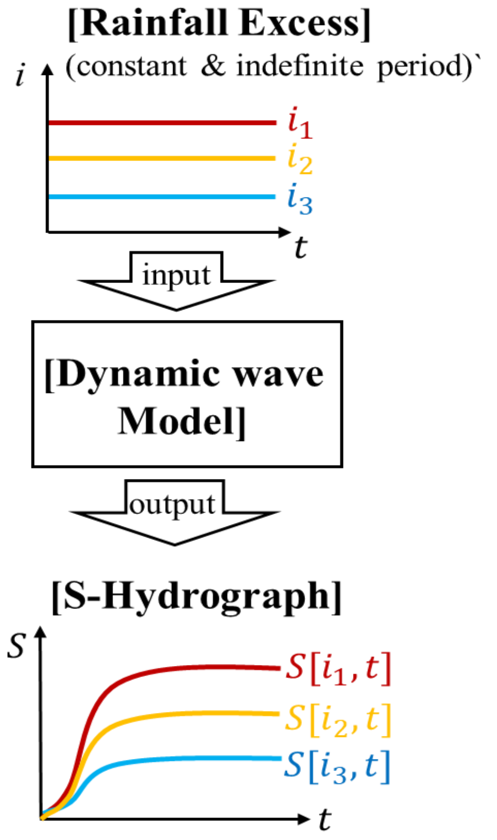
- ✓ S-hydrographs corresponding to each of the various rainfall intensities are produced by simulating uniform rainfalls in a watershed using a dynamic wave model.
- ✓ The dynamic wave model we use is a fully coupled hydrologic and hydrodynamic model, tRIBS-FEaST developed by Kim et al. (2012, 2013).
- ✓ The governing equations of FEaST for fluid motion are the shallow water equations.

$$\frac{\partial h}{\partial t}+\frac{\partial (Uh)}{\partial x}+\frac{\partial (Vh)}{\partial y}=S_r\,,\quad \frac{\partial (Uh)}{\partial t}+\frac{\partial (U^2h+\frac{1}{2}gh^2)}{\partial x}+\frac{\partial (UVh)}{\partial y}=-gh\frac{\partial z_b}{\partial x}-C_DU\sqrt{U^2+V^2}\,,$$



$$\frac{\partial (Vh)}{\partial t}+\frac{\partial (UVh)}{\partial x}+\frac{\partial (V^2h+\frac{1}{2}gh^2)}{\partial y}=-gh\frac{\partial z_b}{\partial y}-C_DV\sqrt{U^2+V^2}$$

- ✓ To solve the governing equations, the finite volume method on an unstructured triangular grid and Roe's approximate Riemann solver (Roe, 1981) is used.



Kim, J., Warnock, A., Ivanov, V.Y., Katopodes, N.D., 2012. Coupled modeling of hydrologic and hydrodynamic processes including overland and channel flow. Advances in Water Resources. 37, 104–126.

Kim, J., Ivanov, V.Y., Katopodes, N.D., 2013. Modeling erosion and sedimentation coupled with hydrological and overland flow processes at the watershed scale. Water Resources Research. 49, 5134-5154.

Roe, P.L., 1981. Approximate Riemann solvers, parameter vectors, and difference schemes. Journal of Computational Physics. 43, 357-372.

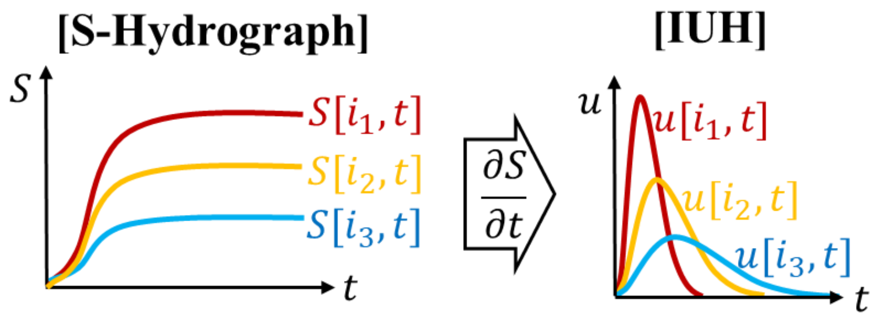
Dynamic wave based IUH (DIUH) method

Step 2 : Derivation of Dynamic wave based IUH

Ding, J. Y., 2011. A measure of watershed nonlinearity: interpreting a variable instantaneous unit hydrograph model on two vastly different sized watersheds. Hydrology and Earth System Sciences. 15(1), 405–423.

✓ Mathematically, the IUH is the first derivative of an S-hydrograph normalized by the rainfall excess intensity and the area of a watershed.

$$u(t) = \frac{1}{i_o A}\frac{dS(t)}{dt}$$



✓ Because $S(t)$ is computed numerically with the dynamic wave model, we calculate $u(t)$ using a discretized equation.

$$u(t) = \frac{S(t + 0.5\Delta t) - S(t - 0.5\Delta t)}{i_o A \Delta t} + O(\Delta t^2)$$

✓ Since we produced S-hydrographs corresponding to each of the various rainfall intensities, the DIUH we derived is a function of both the watershed and rainfall properties.

Dynamic wave based IUH (DIUH) method

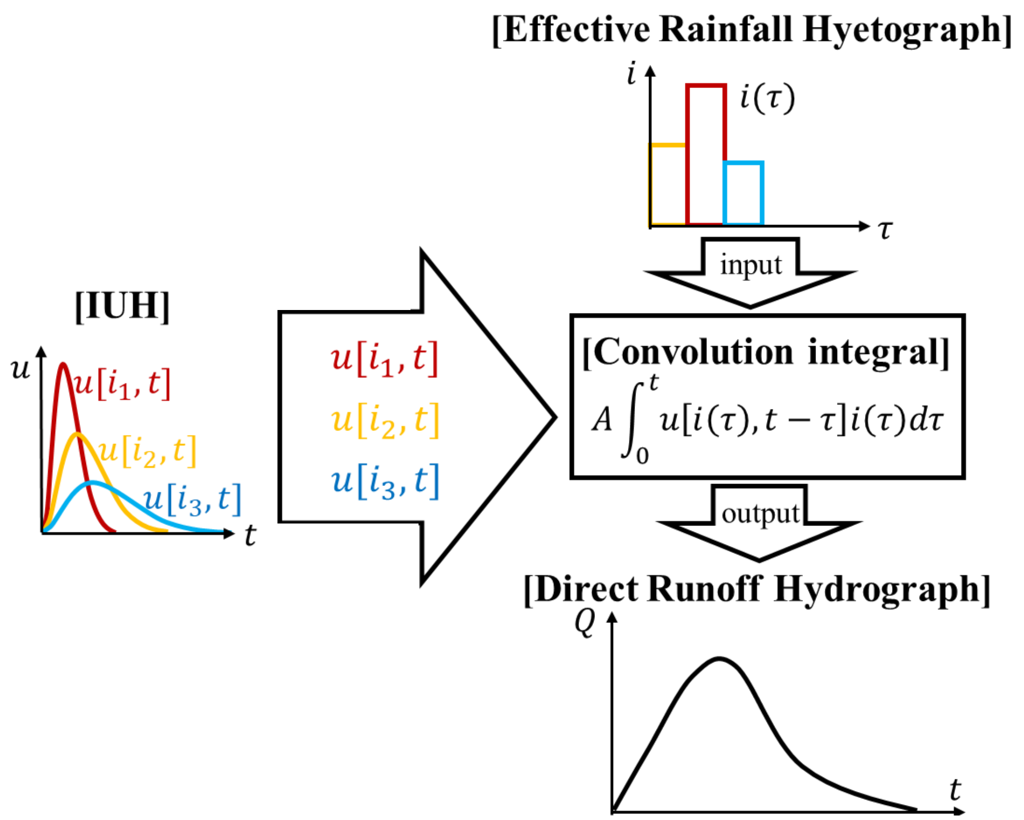
Step 3 : Hydrograph Derivation

- ✓ To derive the direct surface runoff hydrograph of a watershed, we use an alternative form of convolution integral proposed by Amorocho (1967).

$$Q(t) = A\int_0^t u[i(\tau), t-\tau]i(\tau)d\tau$$

- ✓ Because our method produces only discrete values of the DIUH, we numerically integrate the convolution integral equation.

$$Q(t) = A\sum_{\tau=0}^{t} u[i(\tau), t-\tau]i(\tau)\Delta\tau$$

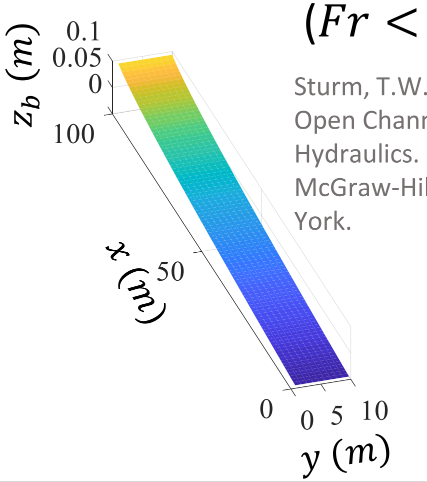
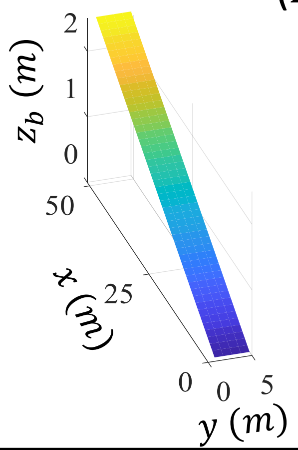
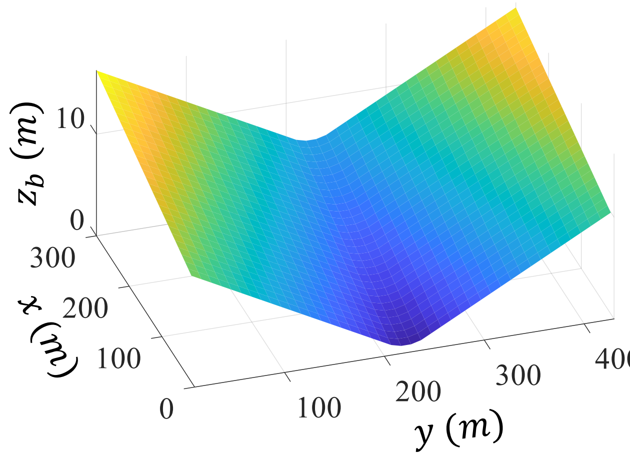


- ✓ The DIUH in the equation, $u[i(\tau), t-\tau]$, is a function of both time and rainfall intensity for a watershed, thus nonlinear relationship between rainfall and runoff can be considered.

Amorocho, J., 1967. The nonlinear prediction problem in the study of the runoff cycle. Water Resources Research. 3, 861–874.

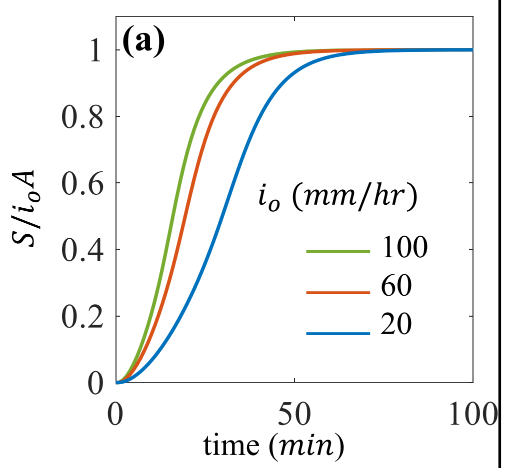
Test in ideal basins

✓ To test the performance of the proposed DIUH method, three impervious ideal basins are adopted.

Basin	**(a) Mild-sloped basin** $(Fr < 1)$  Sturm, T.W., 2001. Open Channel Hydraulics. McGraw-Hill, New York.	**(b) Steep-sloped basin** $(Fr > 1)$ 	**(c) V-shaped basin** 
Size	$100\ m \times 10\ m$	$50\ m \times 5\ m$	Two Planes: $200\ m \times 300\ m$ Channel: $20\ m \times 300\ m$
Bottom slope	$S_o \propto A_d^{-\theta}$ A_d : drainage area along the main channel $\theta = 0.6$: concavity index Flint, J.J., 1974. Stream gradient as a function of order, magnitude, and discharge. Water Resources Research. 10, 969– 973.	0.05	Two planes : 0.05 channel : 0.02
Manning's roughness	$n = 0.04\ s/m^{1/3}$	$n = 0.015\ s/m^{1/3}$	$n = 0.015\ s/m^{1/3}$
Nodes and grids	1,224 nodes and 2,222 grids	364 nodes and 610 grids	1,408 nodes and 2,664 grids

Test in ideal basins

- ✓ Figures show S-hydrographs and DIUHs of the three ideal basins. (DIUHs for any i_o values can be derived.)
- ✓ The peak and time to the peak of the DIUHs are proportional and inversely proportional to the rainfall intensity, respectively.
- ✓ Therefore, DIUH is a function of both the watershed and rainfall properties.

<table>
<tr><th></th><th>(a) Mild-sloped basin</th><th>(b) Steep-sloped basin</th><th>(c) V-shaped basin</th></tr>
<tr><td>S-hydrographs
(i_o=rainfall excess intensity)</td>
<td>
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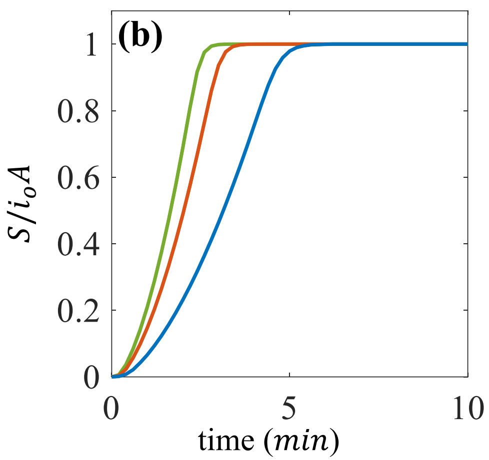
<td>
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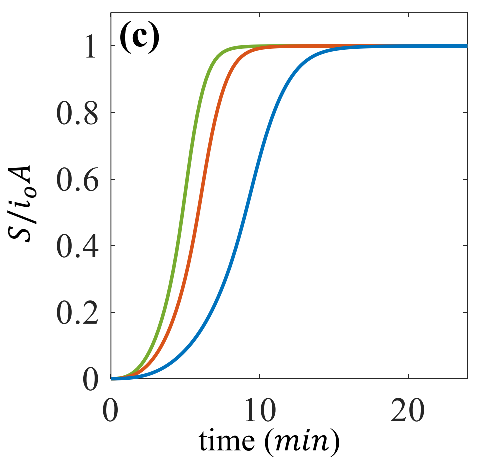
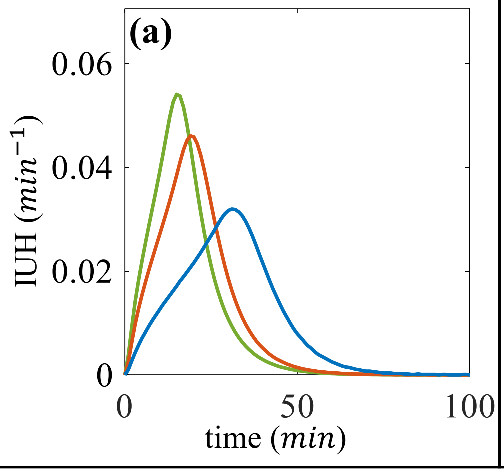
<td>
</td></tr>
<tr><td>DIUHs
(i_o=rainfall excess intensity)</td>
<td>
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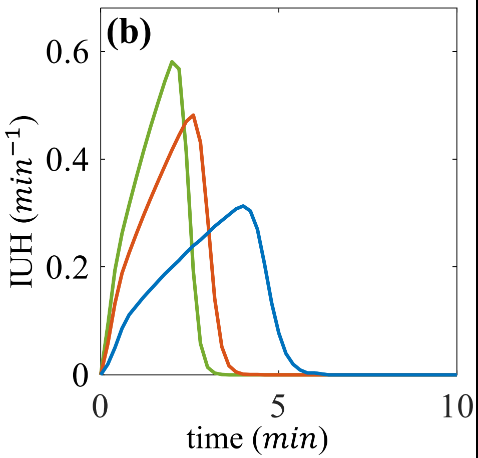
<td>
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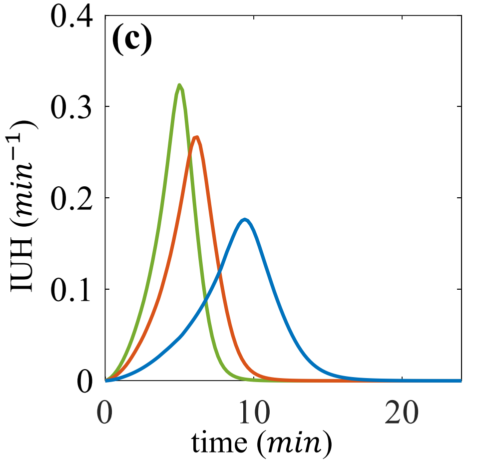
<td>
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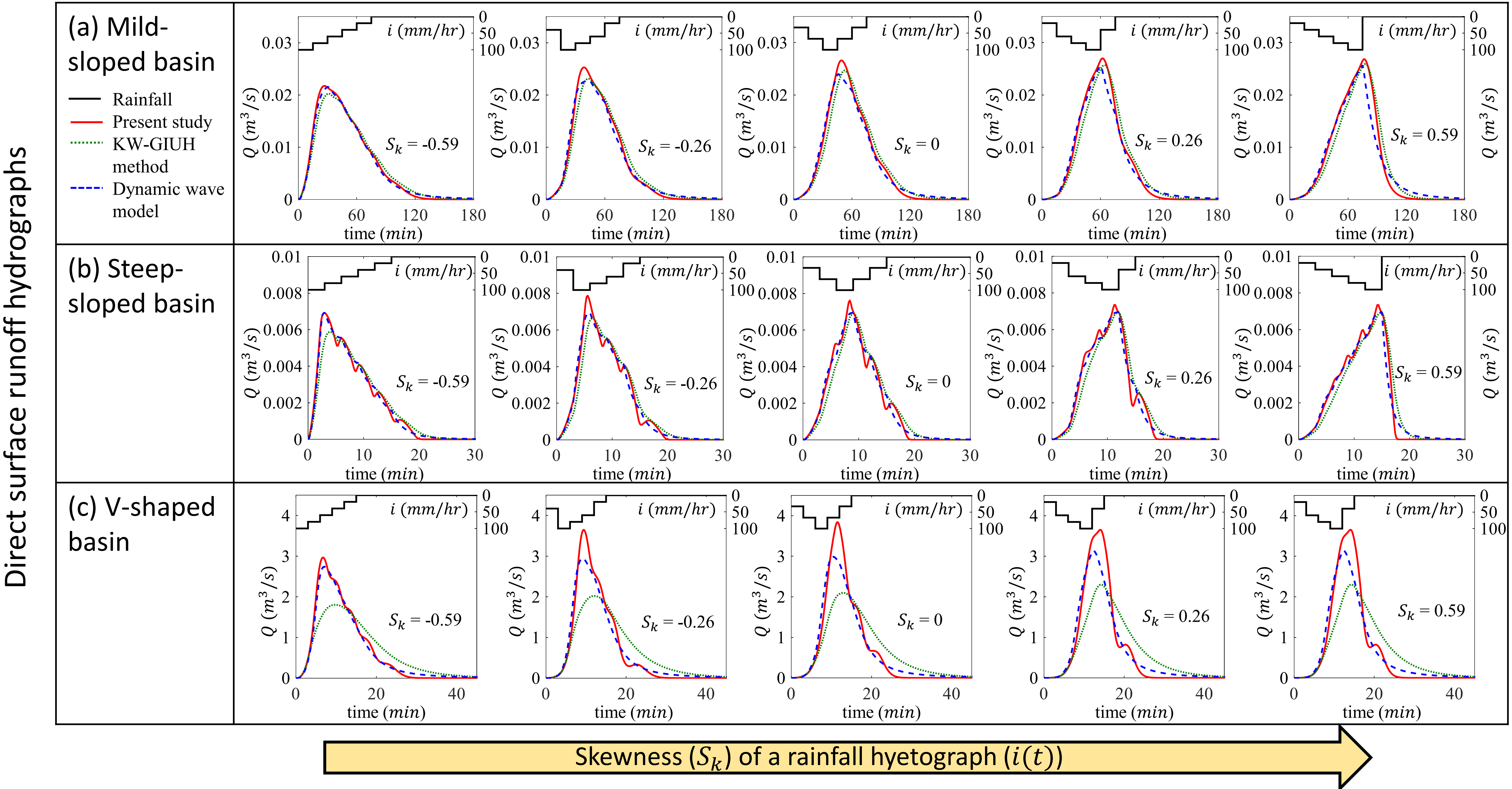
Test in ideal basins



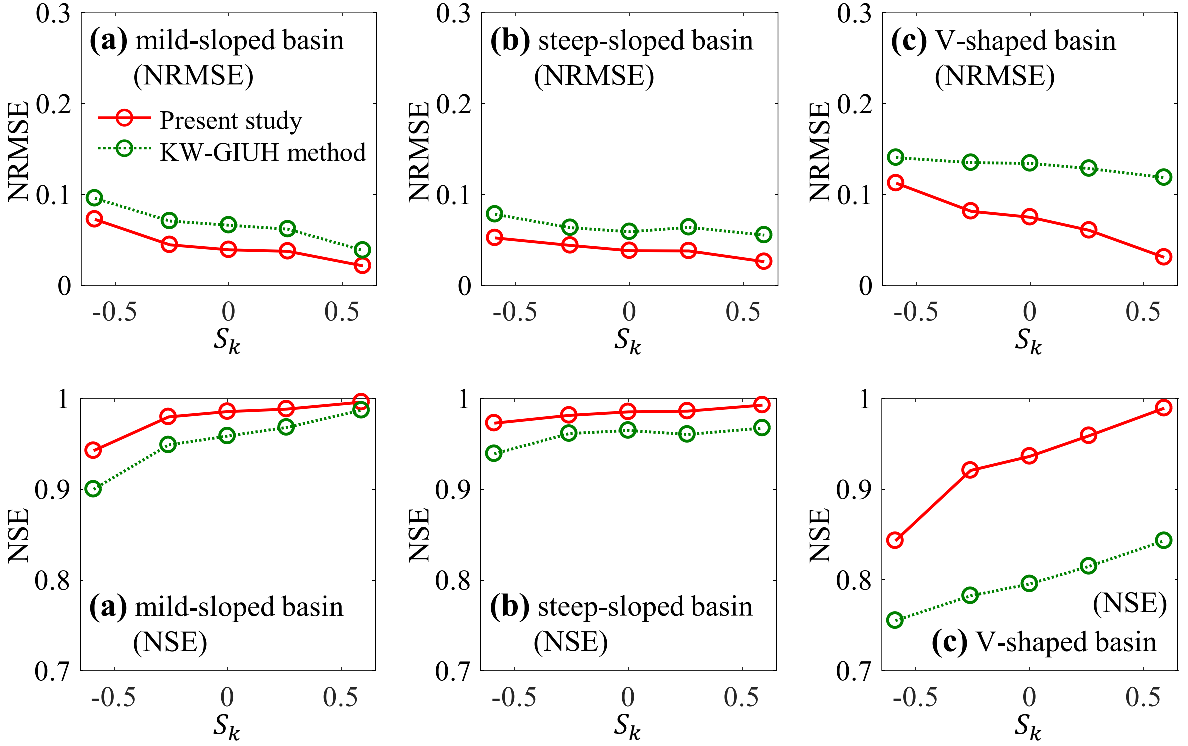
Test in ideal basins

- ✓ We produced the hydrographs by numerically integrating the derived DIUHs, as shown on the previous page.
- ✓ For the temporally varying rainfall scenarios in a basins, the average of the rainfall intensities was 60 mm/h. The kurtosis of the rainfall hyetographs are the same, but the skewness (S_k) ranges from -0.59 to 0.59.
- ✓ The hydrographs obtained with the DIUH method show reasonable agreement with the criterion discharges computed by the dynamic wave model, tRIBS-FEaST.
- ✓ All of the NSE for the predicted results are in the 'very good ($NSE > 0.8$)' range.

Nash, J.E., Sutcliffe, J. V., 1970. River flow forecasting through conceptual models part I - A discussion of principles. Journal of Hydrology. 10, 282-290.

Moriasi, D.N., Gitau, M.W., Pai, N., Daggupati, P., 2015. Hydrologic and water quality models: Performance measures and evaluation criteria. Transactions of the American Society of Agricultural and Biological Engineers. 58(6), 1763-1785.

- ✓ The accuracy of the DIUH method increases as the S_k of the rainfall distribution increases.
- ✓ We also compared the results with the hydrographs produced using the kinematic wave based GIUH (KW-GIUH) proposed by Lee and Yen (1997).



Lee, K. T., Yen, B. C., 1997. Geomorphologic and kinematic-wave-based hydrograph derivation. Journal of Hydraulic Engineering. 123(1), 73– 80.

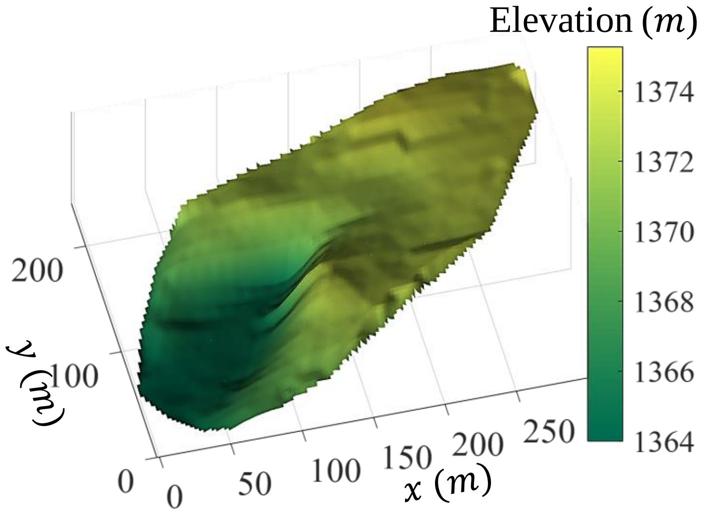
Application to a real watershed

✓ To demonstrate the capability of the proposed DIUH method for runoff prediction in a real watershed, we applied the proposed method to a small-size experimental watershed, the Lucky Hills 103 Watershed.

Lucky Hills 103 Watershed	
Location	Walnut Gulch Experimental Watershed, southeastern Arizona, USA
Area	36,800 m^2
Elevation	ranges from 1,364 to 1,375 m above sea level
Average slope	approximately 0.03
Nodes and grids	481 nodes and 928 grids

Arizona, USA

Walnut Gulch Experimental Watershed

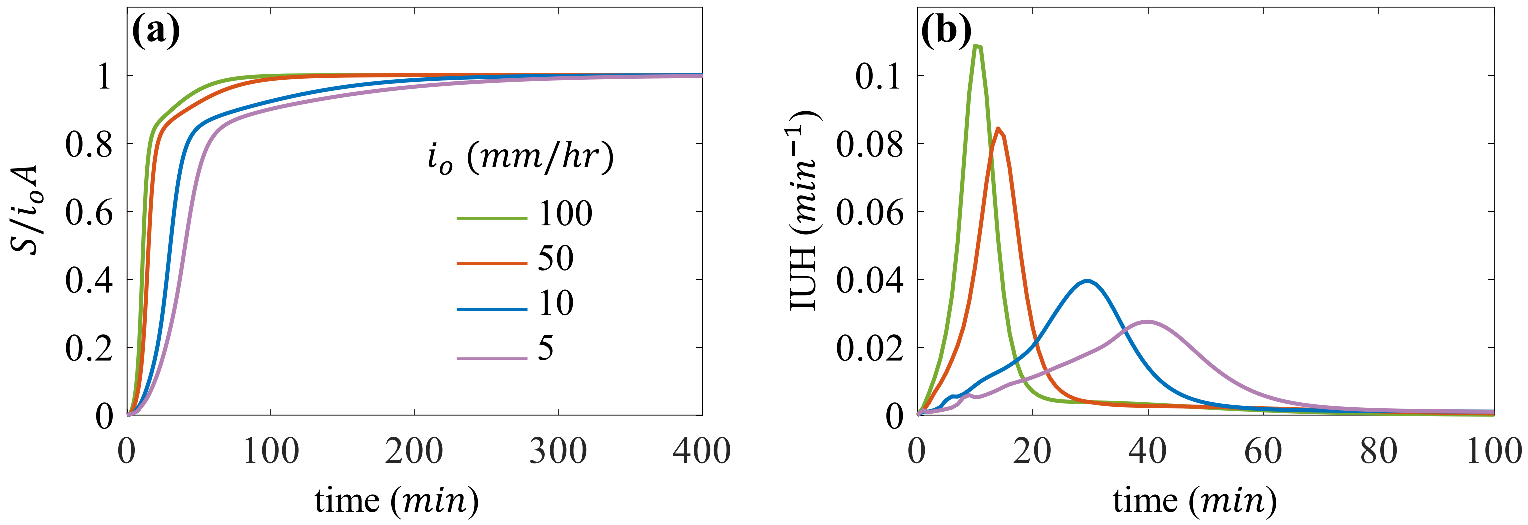


Lucky Hills 103 Watershed

Kim, J., Ivanov, V.Y., Katopodes, N.D., 2013. Modeling erosion and sedimentation coupled with hydrological and overland flow processes at the watershed scale. Water Resources Research. 49, 5134-5154.

Application to a real watershed

✓ Figures show (a) S-hydrographs and (b) DIUHs for the Lucky Hills 103 Watershed.

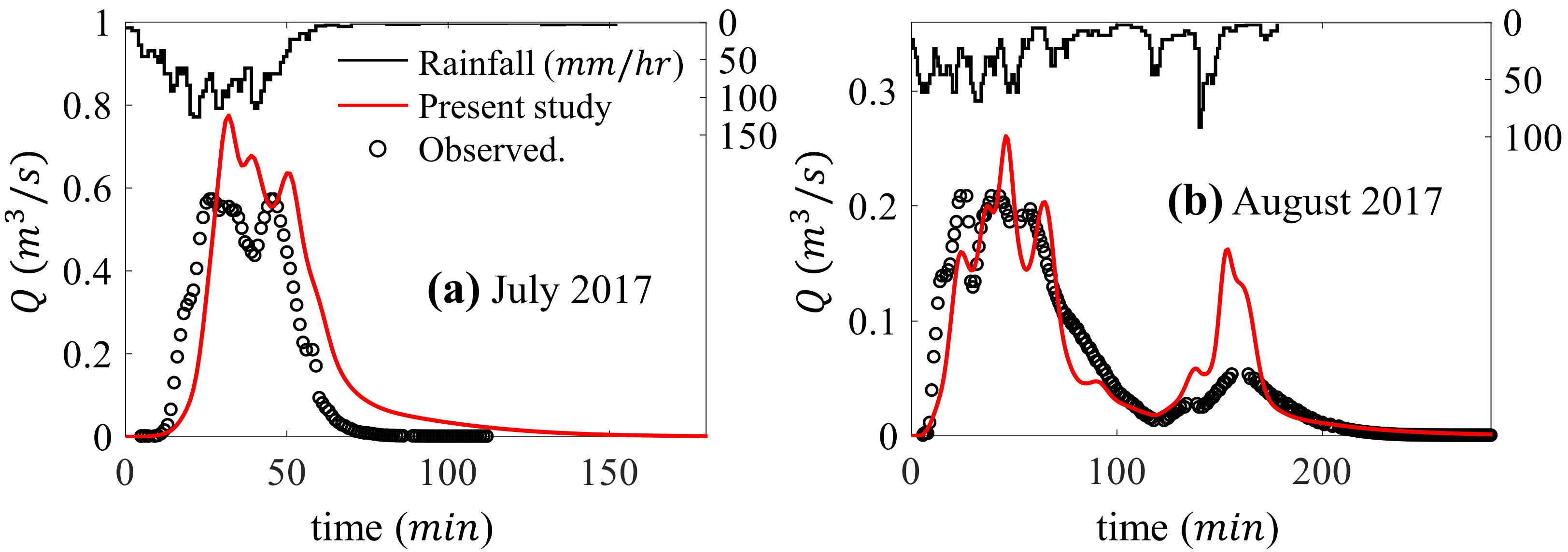


✓ DIUHs in the Lucky Hills 103 watershed showed consistent characteristics with those in ideal basins.

- The peak and time to the peak of the DIUHs are proportional and inversely proportional to the rainfall intensity, respectively.
- DIUH is a function of both the watershed and rainfall properties.

Application to a real watershed

✓ Figures show the measured and predicted runoff discharges of the two rainfall-runoff events in the Lucky Hills 103 Watershed.



✓ To consider the rainfall loss, the infiltration was estimated using the Green–Ampt model (Green and Ampt, 1911)

Green, W.H., Ampt, G.A., 1911. Studies on Soil Physics. Journal of Agricultural Science. 4(1), 1–24.

✓ The DIUH method resulted in NRMSE=0.232 and NSE=0.648 for event(a) and NRMSE=0.141 and NSE=0.788 for event(b).

Nash, J.E., Sutcliffe, J. V., 1970. River flow forecasting through conceptual models part I - A discussion of principles. Journal of Hydrology. 10, 282-290.

✓ Therefore, the proposed method can result in 'good' performance (NSE>0.6) for the real watershed.

Moriasi, D.N., Gitau, M.W., Pai, N., Daggupati, P., 2015. Hydrologic and water quality models: Performance measures and evaluation criteria. Transactions of the American Society of Agricultural and Biological Engineers. 58(6), 1763-1785.

Conclusion

- ✓ To predict rainfall-runoff discharge physically, stably, and instantly, a method predicting hydrograph, based on the IUH approach combined with a dynamic wave model, is presented.
- ✓ The presented method is capable of generating IUHs without measured runoff data only if the bottom friction drag is known.
- ✓ The DIUH is a function of both the watershed characteristics and rainfall intensity, thus the nonlinear relationship between rainfall and runoff can be partially considered in the hydrograph derivation.
- ✓ The peak and time to peak of the DIUHs were proportional and inversely proportional to the rainfall intensity, respectively.
- ✓ The test results for several rainfall-runoff events on ideal and real watersheds show reasonable accuracy, thus we can conclude that the proposed DIUH method can provide good performance for both simple and complex basins.
- ✓ The proposed method still needs further improvement in practical applications.
  - The method should be coupled with more physical and accurate infiltration and interception models.
  - We need to resolve the inaccuracy observed in the recession limb of the hydrographs.
  - Nonlinear methods considering the relationship among the water depth, flow velocity, and celerity should be adopted to improve the prediction accuracy.

Acknowledgement

✓ This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning (2017R1E1A1A01074399).

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Thank you