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Dynamic emergence of domino effects in systems of interacting tipping elements in ecology and climate

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Introduction and Motivation

- **Tipping elements** identified as (sub-)systems that undergo transition into qualitatively different state when a critical threshold is crossed [1]
- Complex **interactions** exist between tipping elements [2]

Key research questions

- What are the consequences of tipping element interactions for the tipping behavior?
- Is it possible that the tipping of one element triggers a critical transition in a coupled tipping element?

Overview

Explore **qualitative long-term behavior of interacting systems** using a **simple model** (based on [3],[4]), with focus on conditions that favour **tipping cascades**

1. A Simple **Model** of Interacting Tipping Elements
2. **Classification of Emerging Tipping Behavior** in a simple Master-Slave System
3. **Application** to a Real-World Example: Greenland ice sheet-AMOC

A Simple Model of Interacting Tipping Elements

Continuous dynamical system $\dot{x}_i(\mathrm{t}) = f_i(x_1, \ldots, x_n)$ in n dimensions, where each component $x_i \in \mathbb{R}$ corresponds to a tipping element X_i

- **Dynamics** of each tipping element

$$f_i(x_1, \ldots, x_n) = a_i x_i - b_i {x_i}^3 + c_i + C_i\big(x_1(t), \ldots, x_n(\mathrm{t})\big)$$

c_i: control parameter

$a_i, b_i = 1$ in the following

qualitatively represents long-term behavior of many real-world tipping elements

- **Linear coupling function**

$$C_i\big(x_1(t), \ldots, x_n(\mathrm{t})\big) = \sum_{j=1}^{n} d_{ji} x_j(t)$$ with $i \neq j$

d_{ji}: coupling strength

for simplicity

A Simple Model of Interacting Tipping Elements

1

A critical transition takes place

a) Uncoupled:

for c_i crossing **intrinsic tipping point**

$$c_{i_{crit}}(a_i, b_i) = \pm 2\sqrt{\frac{1}{b_i}\left(\frac{a_i}{3}\right)^3}$$

b) Master-Slave System ($X_1 \rightarrow X_2$)

for c_2 crossing **effective tipping point**

$$c_2 = -d_{12}x_1^* \pm 2\sqrt{\frac{1}{b_2}\left(\frac{a_2}{3}\right)^3}$$

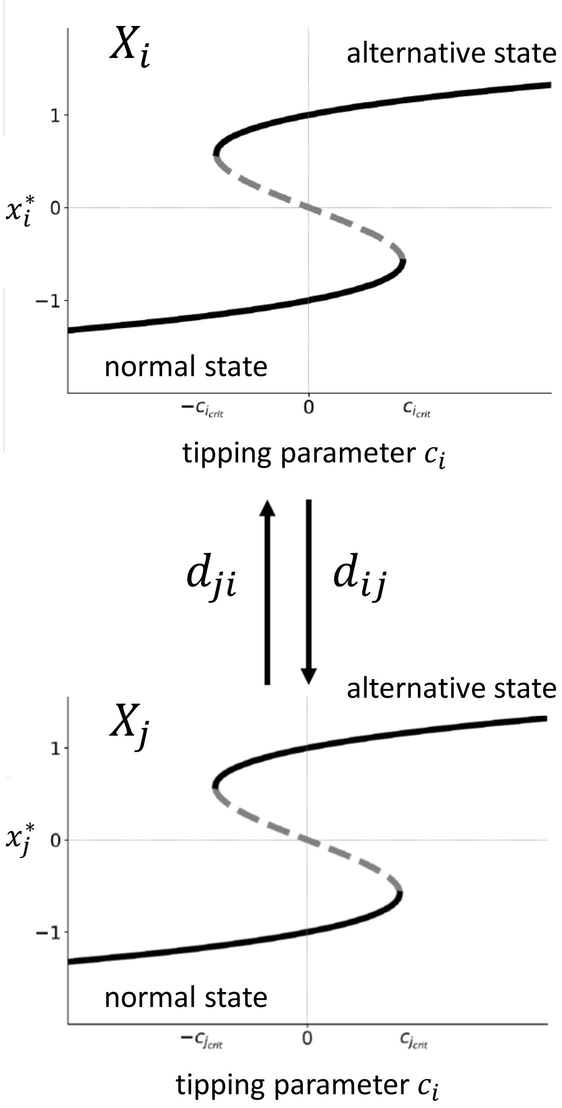

Fig. 1: Schematic coupled tipping elements

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Klose et al. (in review), arXiv:1910.12042

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2 Classification of Emerging Tipping Behavior in a simple Master-Slave System

$X_1 \rightarrow X_2$ with $d_{12} > 0$

Facilitated tipping (Fig. 2)

With X_1 in its alternative state, X_2 is pushed towards its tipping point in our model and may undergo a critical transition before its intrinsic tipping point is crossed.

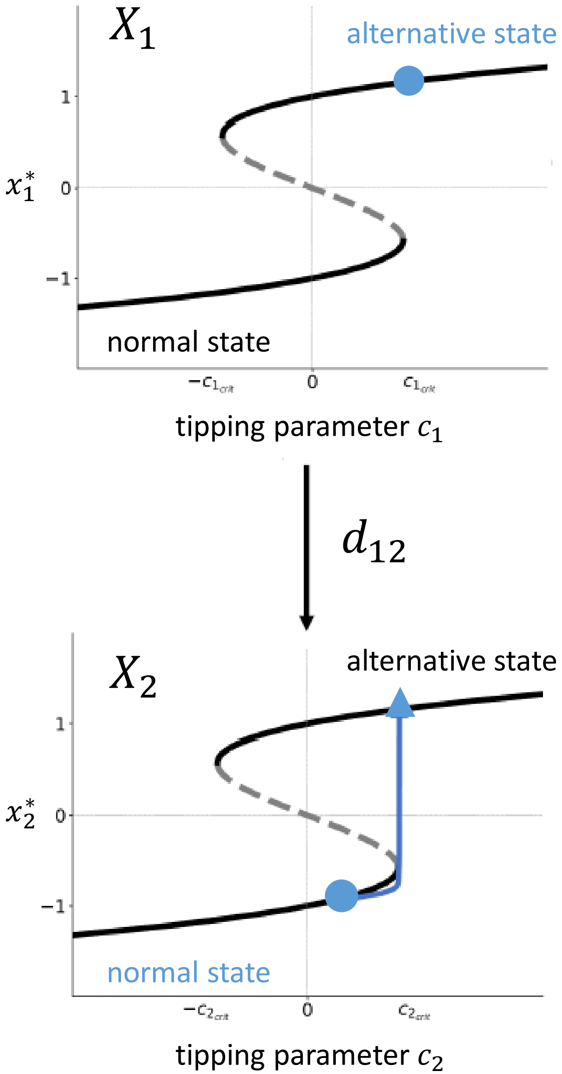

Fig. 2: Schematic representation of facilitated tipping behavior

Classification of Emerging Tipping Behavior in a simple Master-Slave System

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$X_1 \rightarrow X_2$ with $d_{12} > 0$

Impeded tipping (Fig. 3)

With X_1 is in its normal state, X_2 is pulled away from its tipping point in our model. It may undergo a critical transition at an effective tipping point which is higher than the intrinsic tipping point.

Back-tipping

With X_1 is in its normal state and X_2 occupying its alternative state, X_2 can tip back to the normal state (for high coupling strengths d_{12}, small values of the control parameter c_2).

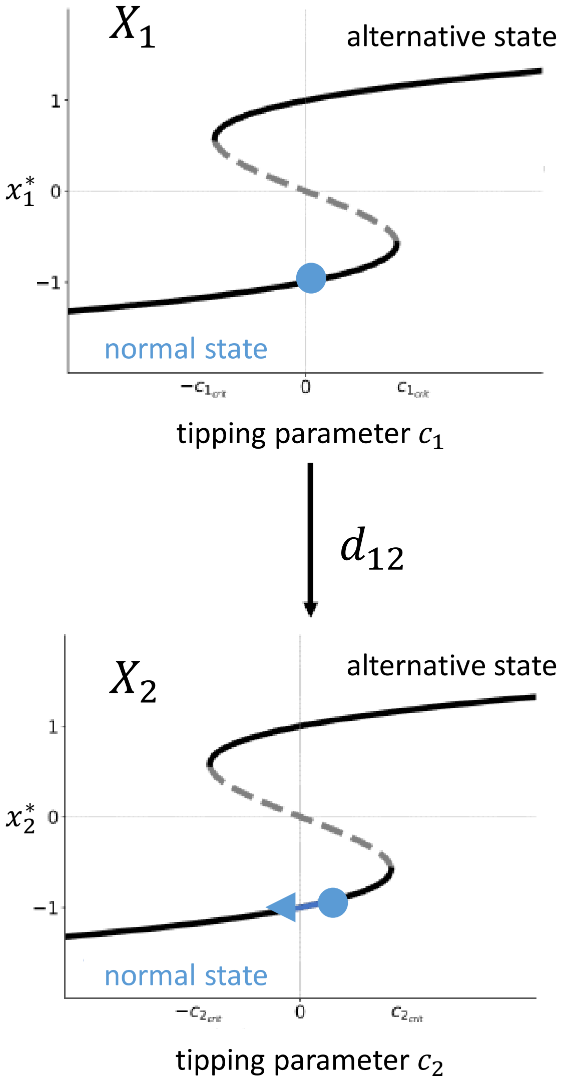
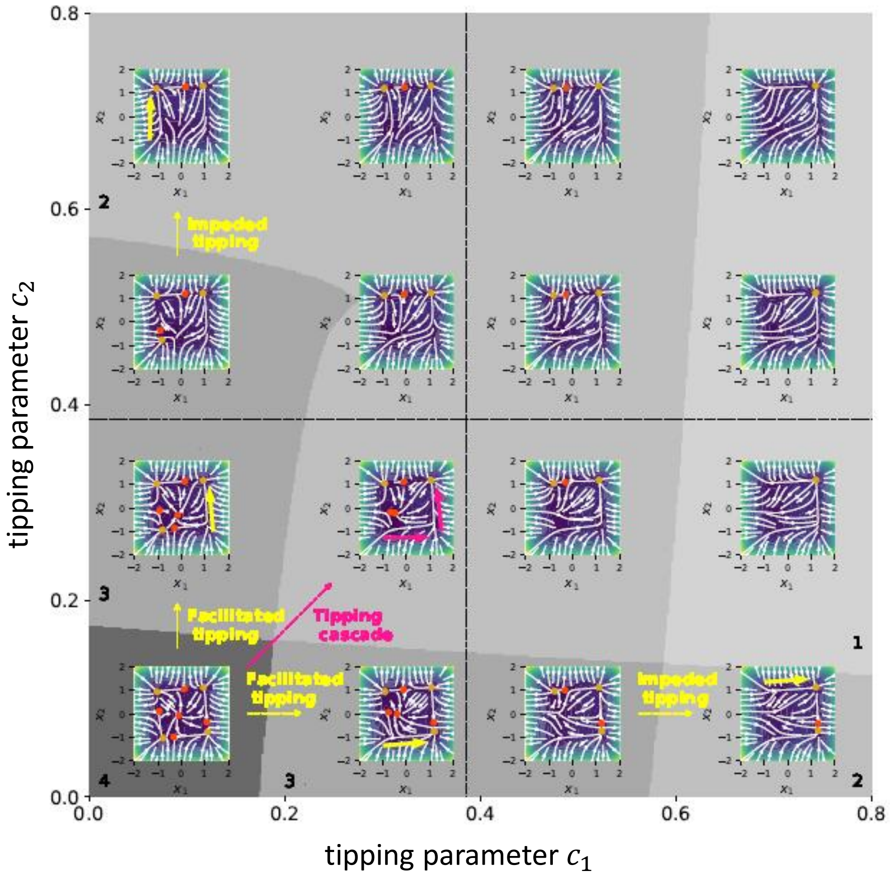


Fig. 3: Schematic representation of impeded tipping behavior

Application to a Real-World Example: Greenland ice sheet-AMOC

$X_1 \overset{\rightarrow}{\underset{\leftarrow}{}} X_2$ with $d_{12} > 0, d_{21} < 0$ positive-negative coupling



as assumed for the interaction between Greenland ice sheet (GIS, X_1) and Atlantic Meridional Overturning Circulation (AMOC, X_2)

GIS → AMOC: via meltwater influx [5]

AMOC → GIS: via cooling around Greenland [2]

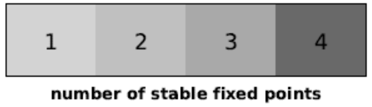


Fig. 4: Number of stable fixed points and phase space portraits (with unstable fixed points in red, stable fixed points in orange) for d21 = -0.2 < 0 and d12 = 0.2 > 0 depending on the control parameters c1 and c2

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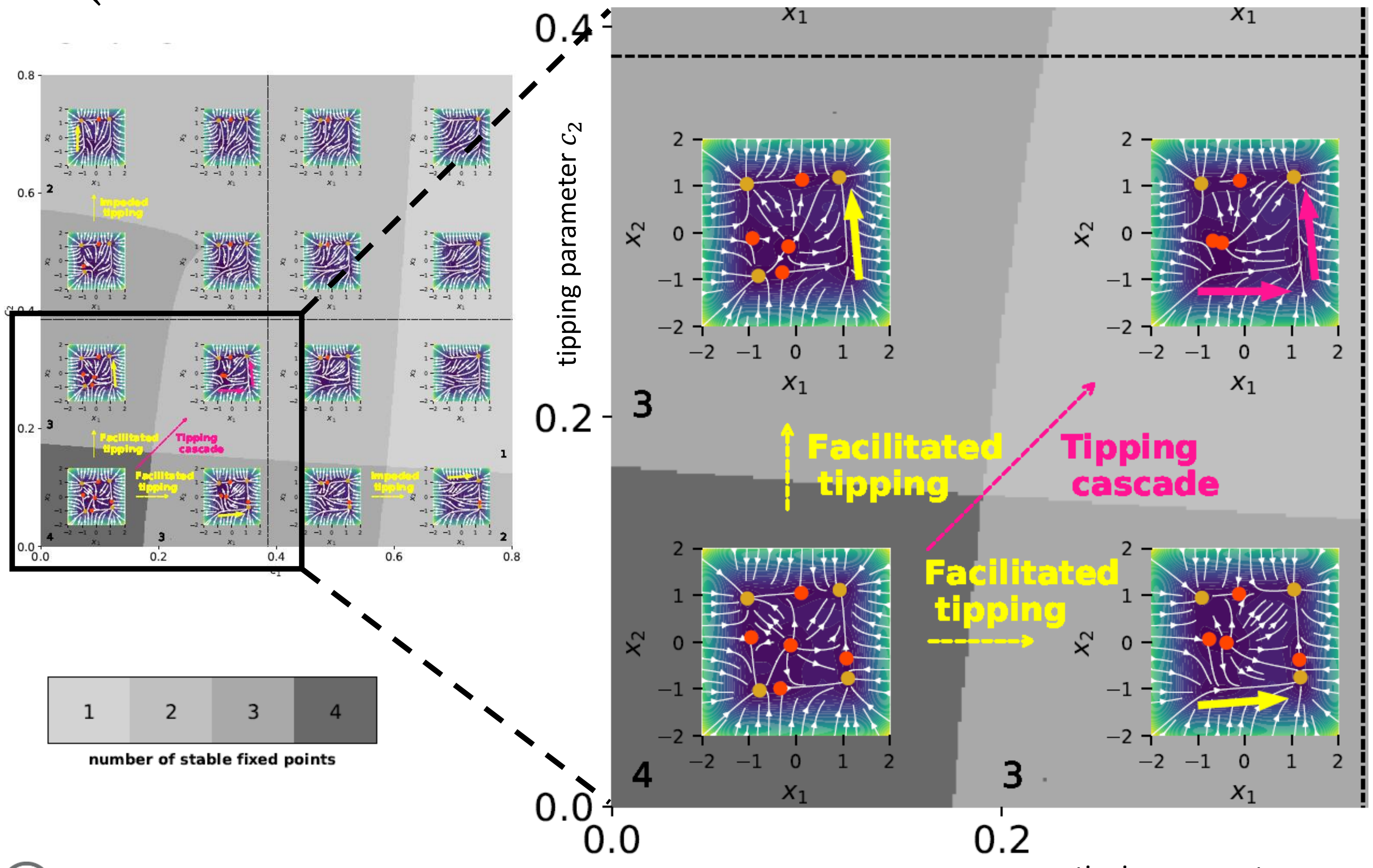
Application to a Real-World Example: Greenland ice sheet-AMOC

$X_1 \underset{\leftarrow}{\overset{\rightarrow}{}} X_2$ with $d_{12} > 0, d_{21} < 0$



Critical transition to alternative state in both subsystems possible for slight increase of control parameters before intrinsic tipping point crossed (**tipping cascade**) (Fig. 5).

Fig. 5: Zoom into Fig. 4

Klose et al. (in review), arXiv:1910.12042

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Limitations

- **Idealized description** of tipping elements & simple coupling
- Assumption: **sufficiently slowly varying control parameters**
- Isolated analysis of **pairs** (chains) instead of networks of tipping elements (see Displays EGU2020-5412 & EGU2020-21507 for further work)

Outlook

- Further **classification of tipping cascades** / types of multiple tipping
- Necessary conditions and likelihood for **tipping cascades in more system specific but still conceptual models,** e.g. Greenland ice sheet-AMOC
- **Early warning signals and predictability of tipping cascades**: Assessment of critical slowing down in systems of interacting tipping elements by eigenvectors and eigenvalues

References

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