Shape asymmetry of rogue waves

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Rogue Waves: Main approaches

- In-situ Measurements
- Laboratory Experiments
- Numerical Modeling

Rogue wave dynamics

Probabilistic description

Study: mechanisms of freak wave generation, conditions, characteristics of individual waves
Application: evaluation of wave impact on ships and marine stationary structures, safety methods,…

Study: probabilistic models, relations between spectral and statistical parameters
Application: forecast of hazardous weather and geophysical conditions, risk assessment of extreme waves, safety requirements of ship design and marine structures
Rogue Waves: Main approaches

Problems:

1. In-situ measurements:
   - statistical inhomogeneity
   - data unreliability
   - one-point measurements

2. Approximate probabilistic models, kinetic equations:
   - the use of approximations and assumptions questionable in the case of extreme waves

3. Laboratory experiments: expensive, time consuming, size limitation

4. Stochastic modelling
   Direct numerical simulations of ensemble of quasi-random wave realizations:

   + Fully controllable conditions
   + Full information on waves
   + Accurate
   + Fast
   + Cheap
Simulated by the HOSM (potential Euler eqs, West et al, 1987) with $M = 3$, RK4 for the steps in time. The domain is double-periodic in space, $50 \times 50$ dominant wave periods.

$t = 0$: Irregular waves with a given averaged JONSWAP spectrum and random phases. $H_s \approx 3.5\text{÷}7$ m, $T_p = 10$ s

$t = 0\cdots200$ s: Slow nonlinearity adjustment according to Dommermuth (2000)

$t = 200\cdots1400$ s: The simulation. The data is recorded each 0.5 s of the simulated dynamics.

The spectrum corresponds to the North Sea conditions (JONSWAP)

The output: sheets of wave fields in the domain 10 km x 20 min with high resolution both in time and space

1 realization: 60 waves during 120 wave periods.
1 sea state: from 100 to 1000 realizations

A. Slunyaev, A. Sergeeva, I. Didenkulova, Rogue events in spatiotemporal numerical simulations of unidirectional waves in basins of different depth. Natural Hazards 84(2), 549-565 (2016).
DOI: 10.1007/s11069-016-2430-x

I. Unidirectional Waves

Initial conditions

Objectives:

- Detection of rogue waves ($H > H_s$).
- Probability functions
- Variety of rogue wave shapes
- Assembling rogue waves into ‘rogue events’
- Joint statistics of surface elevation and velocity fields

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I. Unidirectional Waves

Portrait of rogue wave: Zero-crossing analysis for time series and space series

In-situ measurements at Brazilian coast
U. Ferreira, P. C. Liu and C. E. Parente GEOFIZIKA, V.21, 2004

in total 305 rogue waves

Rear slope waves:
197 waves (~65%) in numerical simulations ~ 63%

Front slope waves:
108 waves (~35%) in numerical simulations ~ 37%

[Hs = 3.5 m] [Hs = 7 m]

[Sergeeva & Slunyaev, 2013; Slunyaev et al, 2016]
I. Unidirectional Waves

Rogue wave evolution: Rogue events

Step 1: detection of rogue waves \((H>2H_s)\)

Step 2: gathering rogue waves in events

Individual rogue waves are within 2.5 wave periods and 2.5 wave lengths from each other

\[ T_0 \sim 10 \text{ s} \]

Duration of a rogue event is up to 10 min!

More intense waves live for longer time
II. Directional Waves

Examples of 3D rogue wave events

Snapshots of the surface in the co-moving frame and the corresponding longitudinal wave sections during a long-living rogue wave event.

The sections are taken along the points where rogue waves are detected, shown on the surfaces by circles with strokes.

The circles show locations of the rogue wave crests; the strokes show directions to the deep troughs.

Irregular waves with a given averaged JONSWAP spectrum ($T_p = 10\text{s}$, $\gamma = 3, 6$), random Fourier amplitudes and random phases $H_s \approx 3.5 - 7\text{ m}$.

The directional spreading is according to the cos$^2$ distribution with $\theta = 5^\circ, 12^\circ, 62^\circ$ (following Xiao et al, 2013)

\[ D(\chi) = \begin{cases} \frac{2}{\theta} \cos^2 \left( \frac{\pi \chi}{\theta} \right), & |\chi| \leq \frac{\theta}{2} \\ 0, & |\chi| > \frac{\theta}{2} \end{cases} \]

Simulated by the HOSM (West et al, 1987) with $M = 1, 3, 4$. The domain is double-periodic in space, it is of the size $50 \times 50$ dominant wave periods.
Series E₁₂

Lasts for 40 \( T_p \)

Travels 3 km

lateral size grows with \( A/ \)
Series $E_3^{62}$

Lasts for $24 T_p$


II. Directional Waves

Life-time distributions for 3D rogue events

Probability distribution of the duration of rogue-wave events $T_{RW}$

$$P(T_n) = \frac{n}{N_{ev} + 1}$$

$$n = 1, 2, \ldots, N_{ev}$$


Dependence between rogue event lifetimes $T_{RW}$ and amplification factor $AI$
III. Typical shapes of rogue waves

The procedure:

1) cut along the direction of wave propagation

2) zero-crossing analysis of the space series $\rightarrow$ individual waves

3) selection of rogue waves according to the height criterion, $H > 2H_s$,
   $H = A_{cr} + A_{tr}$, $A_{tr} = \max(|A_{tr-}|,|A_{tr+}|)$

4) asymmetry properties:
   - vertical asymmetry: $A_{cr} > A_{tr}$ or $A_{cr} < A_{tr}$
   - horizontal asymmetry: $A_{tr+} > A_{tr-}$ or $A_{tr+} < A_{tr-}$
III. Typical shapes of rogue waves

[Slunyaev & Kokorina, 2020]
Conclusions

- Distribution of life-times of rogue waves is exponential in the linear framework.

- Life-time distribution of short-crested rogue waves approximately agree with the linear limit.

- Rogue waves under the conditions of large wave steepness and narrow angle spectrum live remarkably longer.

- Vertical and horizontal asymmetries increase when waves are steeper or have broader angle spectrum.

- For $H_s = 7$ m, $\theta = 62^\circ$, $\gamma = 3$ three fourths of rogue waves are represented by crests which are higher than troughs with deeper troughs behind.