Combination of GNSS and VLBI data for consistent estimation of Earth Orientation Parameters

Lisa Lengert\textsuperscript{1}, Daniela Thaller\textsuperscript{1}, Claudia Flohrer\textsuperscript{1}, Hendrik Hellmers\textsuperscript{1}, Alexander Kehm\textsuperscript{2}

\textsuperscript{1)} Bundesamt für Kartographie und Geodäsie (BKG), Frankfurt am Main
\textsuperscript{2)} Deutsches Geodätisches Forschungsinstitut (DGFI-TUM), Technische Universität München, München
Agenda

Motivation

Data Input – VLBI Intensive Sessions

1-day Inter-technique Combination

Multi-day Inter-technique Combination

Conclusion and Outlook
Motivation - Combination of GNSS and VLBI

Earth Rotation Parameters (ERPs)

<table>
<thead>
<tr>
<th></th>
<th>GNSS</th>
<th>VLBI Intensive</th>
<th>VLBI R1/R4</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polar motion</td>
<td>✓</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>UT1—UTC (dUT1)</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>LOD</td>
<td>✓</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

- Combination of GNSS and VLBI Intensives on normal equation level
  - Estimation of a **full and consistent** set of ERPs
  - Taking into account all correlations
  - Independent from **a priori values** of the ERPs

- Temporal resolution: **1 day**
- Shorter latency → **faster availability of a consistent set** of all ERPs
- **Stabilization** of the ERP estimation by multi-day solution
Data Input - VLBI Intensive Sessions

- Baselines optimized in **East-West direction** for the estimation of dUT1
- **One hour** observation per day, not equidistant in time
- **1-2 days** latency
1-day Inter-technique Combination
GNSS Rapids CODE + VLBI Intensives BKG

**Input used**

VLBI Intensives BKG

GNSS Rapids CODE

Input used

Parameter transformation

VLBI\textsubscript{INT} + GNSS\textsubscript{RAP}

VLBI\textsubscript{INT}

3 days x 24 h
# 1-day Inter-technique Combination – Scheme

<table>
<thead>
<tr>
<th>Parameterization</th>
<th><strong>GNSS\textsubscript{RAP}</strong></th>
<th><strong>VLBI\textsubscript{INT}</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>ERP</td>
<td>Offset + offset</td>
<td>Offset + drift → Offset + offset</td>
</tr>
<tr>
<td>Epoch</td>
<td>0h / 24h</td>
<td>INT mean epoch → 0h / 24h</td>
</tr>
<tr>
<td>a Priori</td>
<td>SINEX → IERS 14 C04</td>
<td>SINEX → IERS 14 C04</td>
</tr>
<tr>
<td>TRF</td>
<td>SINEX → ITRF2014</td>
<td>SINEX → ITRF2014</td>
</tr>
</tbody>
</table>

**Solution**

- dUT1 X/Y-pole

**Constraints**

- TRF VLBI
- TRF GNSS Core
- Satellite PCO
- Geocenter

- Fix
- NNR/NNT
- Fix to a Priori
- Fix to 0

**Applied transformations are indicated by →**
Scaling studies

**Imbalance** in the size of the NEQ elements of both techniques

GNSS too weak compared to VLBI → **Re-scaling of the NEQ needed**

<table>
<thead>
<tr>
<th>GNSS NEQ weight</th>
<th>dUT1 [ms]</th>
<th>x-pole [mas]</th>
<th>y-pole [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-technique</td>
<td>0.0265</td>
<td>0.0496</td>
<td>0.0423</td>
</tr>
<tr>
<td>100</td>
<td>0.0250</td>
<td>0.1545</td>
<td>0.0879</td>
</tr>
<tr>
<td>1 000</td>
<td>0.0251</td>
<td>0.0535</td>
<td>0.0428</td>
</tr>
<tr>
<td>5 000</td>
<td>0.0251</td>
<td>0.0491</td>
<td>0.0419</td>
</tr>
<tr>
<td>10 000</td>
<td>0.0254</td>
<td>0.0489</td>
<td>0.0420</td>
</tr>
<tr>
<td>15 000</td>
<td>0.0257</td>
<td>0.0489</td>
<td>0.0420</td>
</tr>
<tr>
<td>16 750</td>
<td>0.0259</td>
<td>0.0488</td>
<td>0.0420</td>
</tr>
<tr>
<td>20 000</td>
<td>0.0262</td>
<td>0.0488</td>
<td>0.0420</td>
</tr>
<tr>
<td>30 000</td>
<td>0.0271</td>
<td>0.0488</td>
<td>0.0420</td>
</tr>
<tr>
<td>50 000</td>
<td>0.0288</td>
<td>0.0488</td>
<td>0.0420</td>
</tr>
<tr>
<td>100 000</td>
<td>0.0323</td>
<td>0.0488</td>
<td>0.0421</td>
</tr>
<tr>
<td>300 000</td>
<td>0.0398</td>
<td>0.0488</td>
<td>0.0421</td>
</tr>
<tr>
<td>500 000</td>
<td>0.0442</td>
<td>0.0488</td>
<td>0.0421</td>
</tr>
</tbody>
</table>
Scaling studies

<table>
<thead>
<tr>
<th>Scale GNSS NEQ</th>
<th>NEQ ratio GNSS : VLBI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TRF</td>
</tr>
<tr>
<td>16750</td>
<td>72 : 1</td>
</tr>
<tr>
<td>450</td>
<td>2 : 1</td>
</tr>
</tbody>
</table>

Ratios of the average NEQ diagonal element per parameter group (TRF, pole, dUT1) between GNSS and VLBI

Scale 16750 \(\cong 72:1\) (TRF GNSS:VLBI)

→ Ratio of the observation periods (72h GNSS\(_{RAP}\) : 1h VLBI\(_{INT}\))

Scale 16750 was used for all following combination results
VLBI\textsubscript{INT} Single-technique – Results at 12:00 epochs

\[ \Delta \text{dUT1} \text{ w.r.t. IERS 14 C04} \]

\[ \text{RMS} = 0.029 \text{ mas} \quad \text{WRMS} = 0.026 \text{ mas} \]
1-Day Inter-technique Combination – Results at 12:00 epochs

\[
\begin{align*}
\text{RMS} &= 0.029 \text{ mas} \quad \text{WRMS} = 0.026 \text{ mas} \\
\text{RMS} &= 0.027 \text{ mas} \quad \text{WRMS} = 0.026 \text{ mas}
\end{align*}
\]
**GNSS\textsubscript{RAP} Single-technique – Results at 12:00 epochs**

![Graph showing X/Y-pole w.r.t. IERS 14 C04 with RMS and WRMS values](image)

- **X-Pole (mas)**
  - Year: 2015 to 2019
  - RMS: 0.050 mas
  - WRMS: 0.050 mas

- **Y-Pole (mas)**
  - Year: 2015 to 2019
  - RMS: 0.044 mas
  - WRMS: 0.042 mas
1-Day Inter-technique Combination – Results at 12:00 epochs

X/Y-pole w.r.t. IERS 14 C04

- RMS = 0.050 mas  WRMS = 0.050 mas
- RMS = 0.049 mas  WRMS = 0.049 mas
- RMS = 0.044 mas  WRMS = 0.042 mas
- RMS = 0.043 mas  WRMS = 0.042 mas
Multi-day Combination – Intra- and Inter-technique

Intra-technique

**VLBI Intensives**

1. Transformation of the ERPs from offset/drift parameterization at the observation mean epoch to offset/offset parameterization at day boundaries
2. Stacking of the normal equations of several days → continuity

**GNSS Rapids**

1. Extraction of the most recent day (pre-elimination of the first two days)
2. Stacking of the normal equations of several days → continuity

Inter-technique

**VLBI_{INT} + GNSS_{RAP}**

1. Combination of multi-day VLBI and GNSS normal equations
2. Systematic investigation for one up to seven combined solutions
Multi-day Combination

**Intra-technique combination**
- Extraction of the most recent day

**Inter-technique combination**

<table>
<thead>
<tr>
<th>Time</th>
<th>VLBI\textsubscript{INT}</th>
<th>VLBI\textsubscript{INT} + VLBI\textsubscript{INT} + VLBI\textsubscript{INT} + VLBI\textsubscript{INT}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h</td>
<td>3 x 24h</td>
<td>4 x VLBI\textsubscript{INT} + 4 GNSS\textsubscript{RAP}</td>
</tr>
<tr>
<td>1h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3h</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**GNSS\textsubscript{RAP}**
- GNSS\textsubscript{Rapids CODE}

**VLBI\textsubscript{INT}**
- VLBI Intensives BKG

**Int-technique combination**
- Combination of VLBI and GNSS

Lengert et al. Combination of VLBI and GNSS 06.05.2020 | Page 14
### Multi-day Inter-technique Combination – Results at 12:00 epochs

<table>
<thead>
<tr>
<th>#days</th>
<th>day</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0259</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0213 0.0202</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0188</td>
<td>0.0159</td>
<td>0.0195</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>0.0220</td>
<td>0.0160</td>
<td>0.0166</td>
<td>0.0212</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>0.0245</td>
<td>0.0171</td>
<td>0.0154</td>
<td>0.0177</td>
<td>0.0228</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.0256</td>
<td>0.0178</td>
<td>0.0149</td>
<td>0.0157</td>
<td>0.0187</td>
<td>0.0239</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.0263</td>
<td>0.0185</td>
<td>0.0150</td>
<td>0.0148</td>
<td>0.0163</td>
<td>0.0196</td>
<td>0.0250</td>
</tr>
</tbody>
</table>

The more days are stacked, the smaller the WRMS of the middle days w.r.t. IERS 14 C04 becomes

→ stabilization of the ERPs through continuity at the day boundaries

The more days (days > 2) are stacked, the larger the WRMS of the peripheral days w.r.t IERS 14 C04 becomes

→ due to the known effect of the GNSS LOD-Bias?
7-Day Inter-technique Combination – Day -3

\[ \Delta \text{dUT1 [ms]} \]

Year

RMS = 0.029 mas \ WRMS = 0.026 mas
RMS = 0.027 mas \ WRMS = 0.026 mas
RMS = 0.016 mas \ WRMS = 0.015 mas
## Multi-day Inter-technique Combination – Results at 12:00 epoch

### WRMS X-pole [mas]

<table>
<thead>
<tr>
<th>#days</th>
<th>day</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0488</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0451</td>
<td>0.0491</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0493</td>
<td>0.0452</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0449</td>
<td>0.0435</td>
<td>0.0452</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>0.0452</td>
<td></td>
<td>0.0440</td>
<td>0.0437</td>
<td>0.0441</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>0.0453</td>
<td>0.0441</td>
<td></td>
<td>0.0441</td>
<td>0.0433</td>
<td>0.0436</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.0452</td>
<td>0.0443</td>
<td>0.0441</td>
<td>0.0437</td>
<td>0.0428</td>
<td>0.0432</td>
<td>0.0465</td>
</tr>
</tbody>
</table>

### WRMS Y-pole [mas]

<table>
<thead>
<tr>
<th>#days</th>
<th>day</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0420</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0361</td>
<td>0.0420</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0421</td>
<td>0.0360</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0349</td>
<td>0.0351</td>
<td>0.0360</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>0.0345</td>
<td></td>
<td>0.0344</td>
<td>0.0358</td>
<td>0.0362</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>0.0347</td>
<td>0.0341</td>
<td></td>
<td>0.0349</td>
<td>0.0361</td>
<td>0.0363</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.0343</td>
<td>0.0344</td>
<td>0.0346</td>
<td>0.0351</td>
<td>0.0364</td>
<td>0.0367</td>
<td>0.0428</td>
</tr>
</tbody>
</table>
7-Day Inter-technique Combination – Day -3

X/Y-pole w.r.t. IERS 14 C04

RMS = 0.049 mas WRMS = 0.049 mas
RMS = 0.050 mas WRMS = 0.050 mas
RMS = 0.049 mas WRMS = 0.044 mas
RMS = 0.043 mas WRMS = 0.042 mas
RMS = 0.044 mas WRMS = 0.042 mas
RMS = 0.046 mas WRMS = 0.035 mas
Conclusion and Outlook

Conclusion

- Combination of GNSS Rapids and VLBI Intensives provides a full and consistent set of ERPs
- Faster availability of ERPs due to shorter latencies (1-2 days)

1-day Inter-technique Combination

- Slight improvement of dUT1 and pole

Multi-day Inter-technique Combination

- Stabilization of the ERP estimation through continuity at the day boundaries
- dUT1 significant improvement of dUT1
  larger WRMS of the peripheral days (GNSS LOD bias?)
- X-pole smaller WRMS of the middle and peripheral days
- Y-pole smaller WRMS of the oldest days (#days > 4)

Outlook

- Combination with bigger VLBI sessions (24h R1/R4) and other techniques (SLR)
- Study impact of GNSS LOD bias (first investigations showed a bias of only 8 micro-seconds)
- Improvement of datum definition by adding local ties
Thank you for your kind attention!

Federal Agency for Cartography and Geodesy
Section G1
Richard-Strauss-Allee 11
60598 Frankfurt am Main

Lisa Lengert
lisa.lengert@bkg.bund.de
www.bkg.bund.de
Phone +49 69 6333 - 449