Screening the coupled atmosphere-ocean system based on covariant Lyapunov vectors

Vera Melinda Galfi[1],
Lesley de Cruz[2], Valerio Lucarini[3], Sebastian Schubert[1]
Email: vera.melinda.galfi@uni-hamburg.de

[1] University of Hamburg, [2] Royal Meteorological Institute of Belgium, [3] University of Reading

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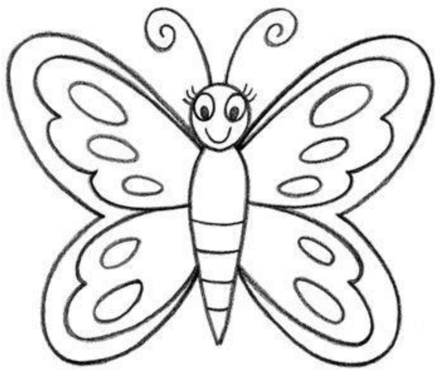
TRR181

What is the effect of infinitesimal perturbations to chaotic systems?



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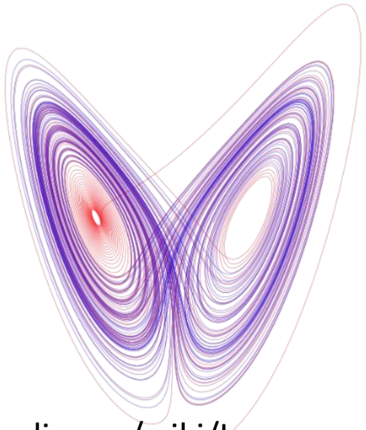


wikipedia.org/wiki/Lorenz_system

?

Lyapunov exponents

How fast does the perturbation grow / decay?

Covariant Lyapunov vectors (CLVs)

Which are the directions of growth / decay of perturbations?

Can be used for model reduction, data analysis, prediction of critical transitions, identification of coherent structure.

We analyse a coupled ocean-atmosphere quasi-geostrophic system based on CLVs.

Vannitsem and Lucarini (2016) performed a CLV-based analysis in a low-order version of the model.

Here, we analyse in more detail the regime dynamics of the low-order, coupled ocean-atmosphere quasi-geostrophic model MAOOAM.

We consider also the dynamics at a higher resolution.

CLVs reveal the local structure of MAOOAM‘s attractor, they give insight into the regime behaviour of the system at low resolution.

For a higher model resolution they point out a very different dynamics.

From tangent linear equations to CLVs

- Nonlinear dynamical system

$$\dot{\mathbf{x}} = f(\mathbf{x}, t)$$

- Infinitesimal perturbation:
  tangent linear equation

$$\dot{\mathbf{y}} = \mathbf{J}(\mathbf{x}, t)\mathbf{y}$$

- Propagation of infinitesimal perturbations

$$\mathbf{y}(t_2) = \mathcal{F}(t_1, t_2)\mathbf{y}(t_1)$$

- The growth/decay of perturbation vectors
  is determined by

$$\mathcal{F}(t_1, t_2)^\top \mathcal{F}(t_1, t_2)$$

  - eigenvalues are related to singular values
  - eigenvectors are called singular vectors

From tangent linear equations to CLVs

- The asymptotic growth/decay of perturbation vectors is determined by

$$\mathrm{W}^{+}(t) = \lim_{t_2 \to \infty} \left[\mathcal{F}(t, t_2)^{\top} \mathcal{F}(t, t_2)\right]^{1/(2(t_2 - t))}$$

$$\mathrm{W}^{-}(t) = \lim_{t_1 \to -\infty} \left[\mathcal{F}(t_1, t)^{-\top} \mathcal{F}(t_1, t)^{-1}\right]^{1/(2(t - t_1))}$$

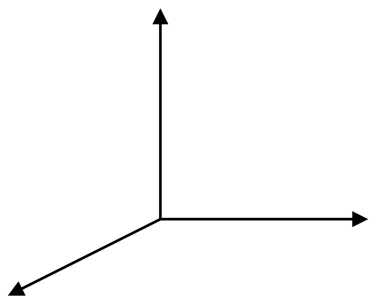
- logarithms of eigenvalues are called Lyapunov exponents
- eigenvectors are called Forward or Backward Lyapunov vectors

- The FLVs and BLVs are not covariant with the dynamics, but the subspaces spanned by them (Oseledec subspaces) are covariant.

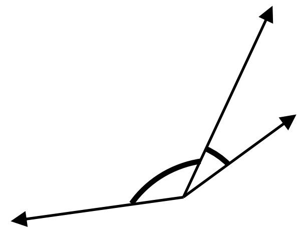
- At the intersection of Oseledec subspaces one finds the **Covariant Lyapunov Vectors (CLVs)**. We use the Ginelli et al. (2007) algorithm.

Covariant Lyapunov vectors point into the direction of perturbations, they are physically relevant.

Forward and backward Lyapunov vectors



Covariant Lyapunov vectors

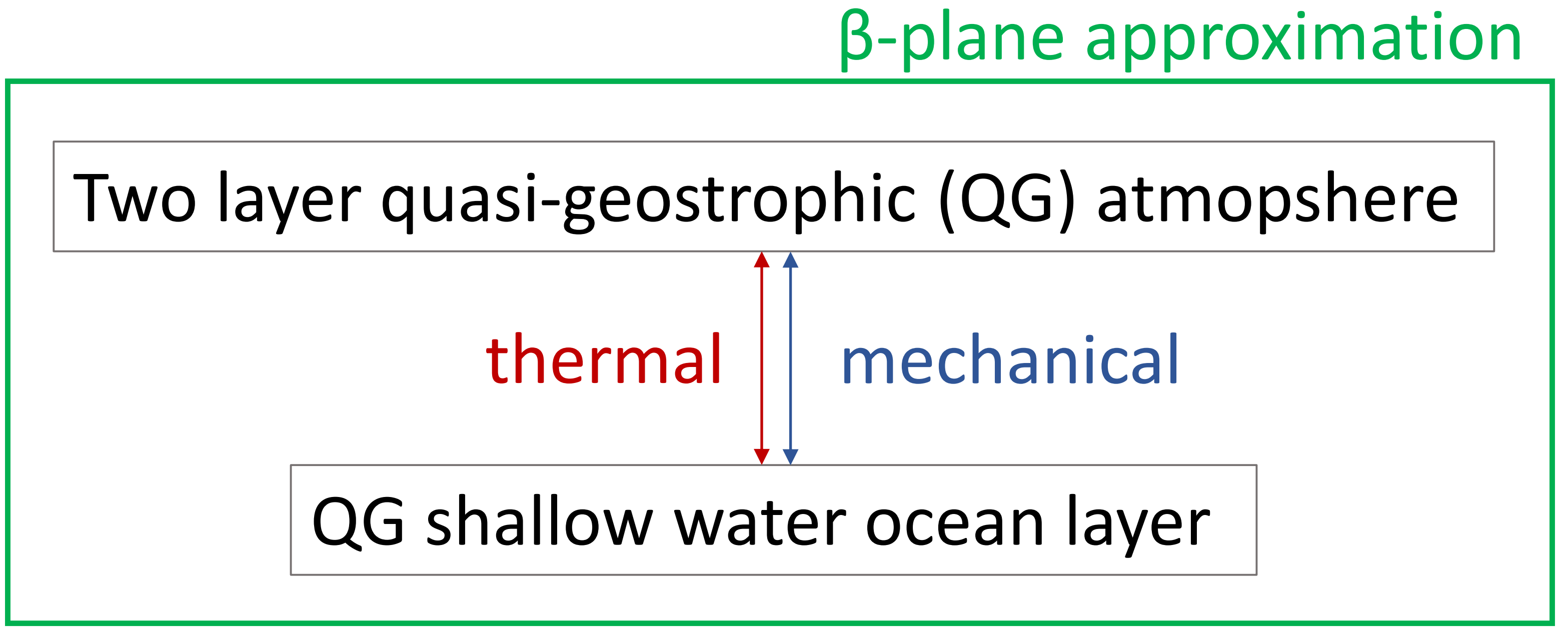


- Orthonormal
- Not covariant with the dynamics
- Norm-dependent

- Not orthogonal
- Covariant with the dynamics
- Norm-independent

We analyse the dynamics of the coupled ocean-atmosphere model MAOOAM based on CLVs.

The Modular Arbitrary Order Ocean-Atmosphere Model (MAOOAM):
(model equations in Appendix)



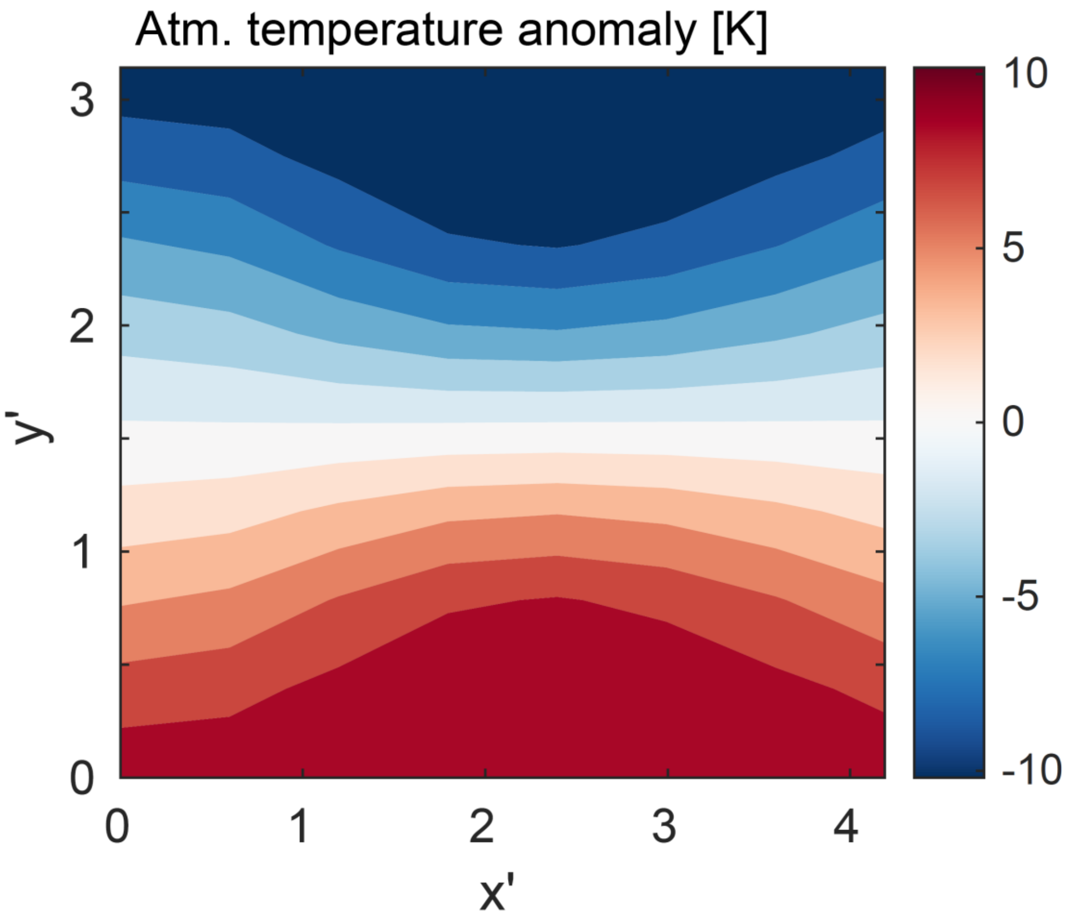
We have 4 categories of spectral model variables

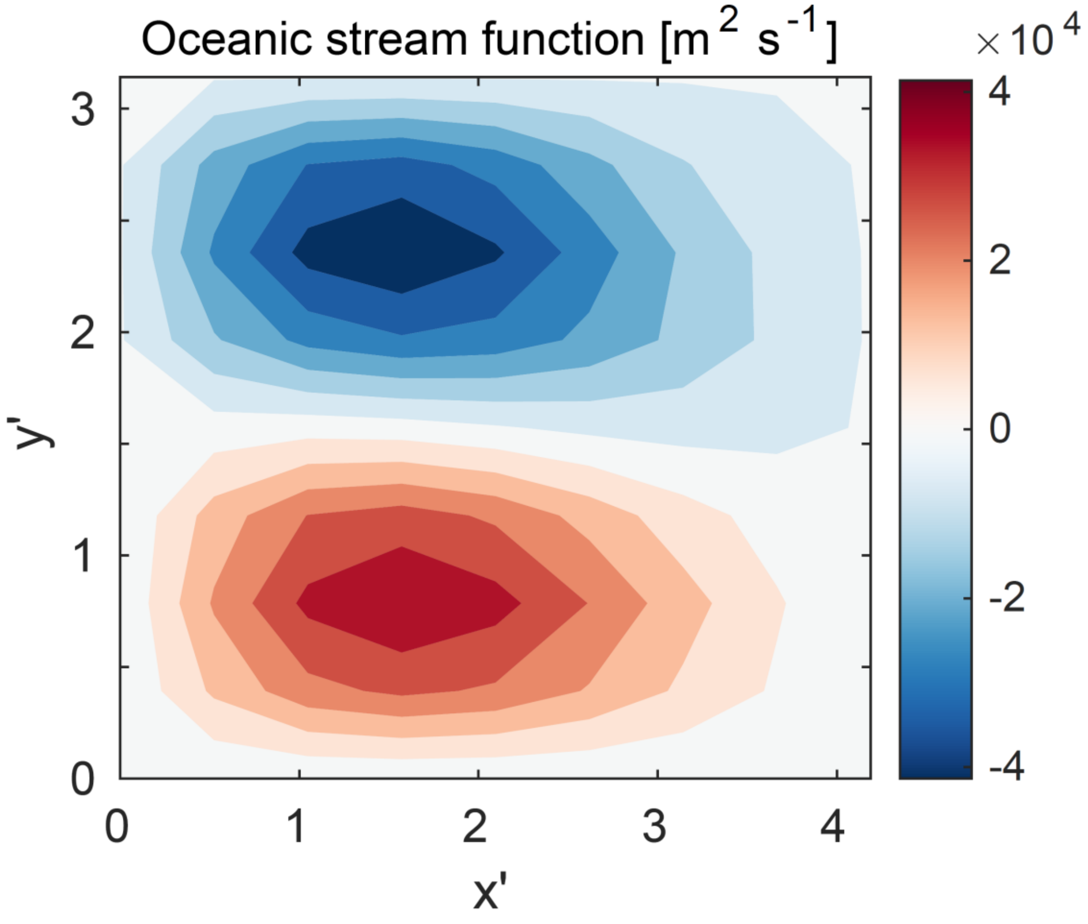
1. Atmospheric dynamic variables
   -> barotropic stream function
2. Atmospheric thermodynamic variables
   -> baroclinic stream function or temperature
3. Oceanic dynamic variables
   -> oceanic stream function
4. Oceanic thermodynamic variables
   -> oceanic temperature

The dimension of the system is $n = n_a + n_o$

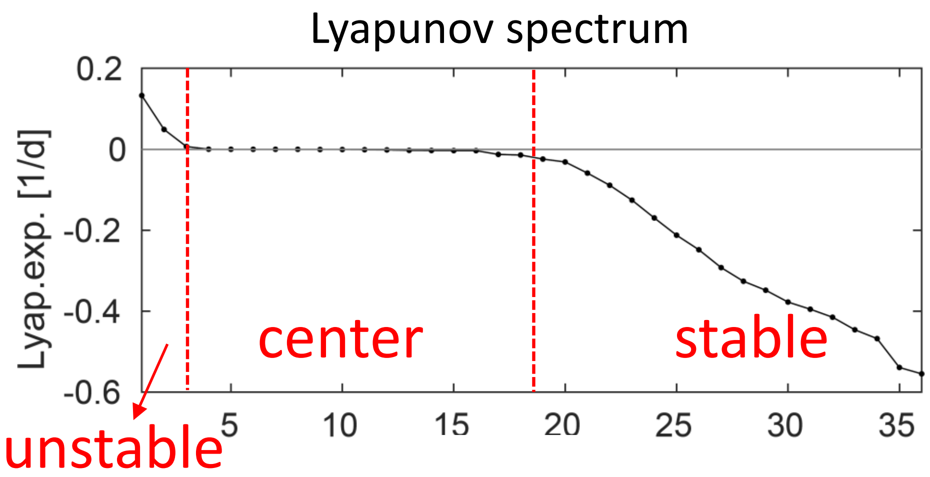
We start with a low resolution of the model:
10 atmospheric modes and 8 oceanic modes

MAOOAM simulates the atmospheric dynamics at mid-latitudes and a double-gyre in the ocean.

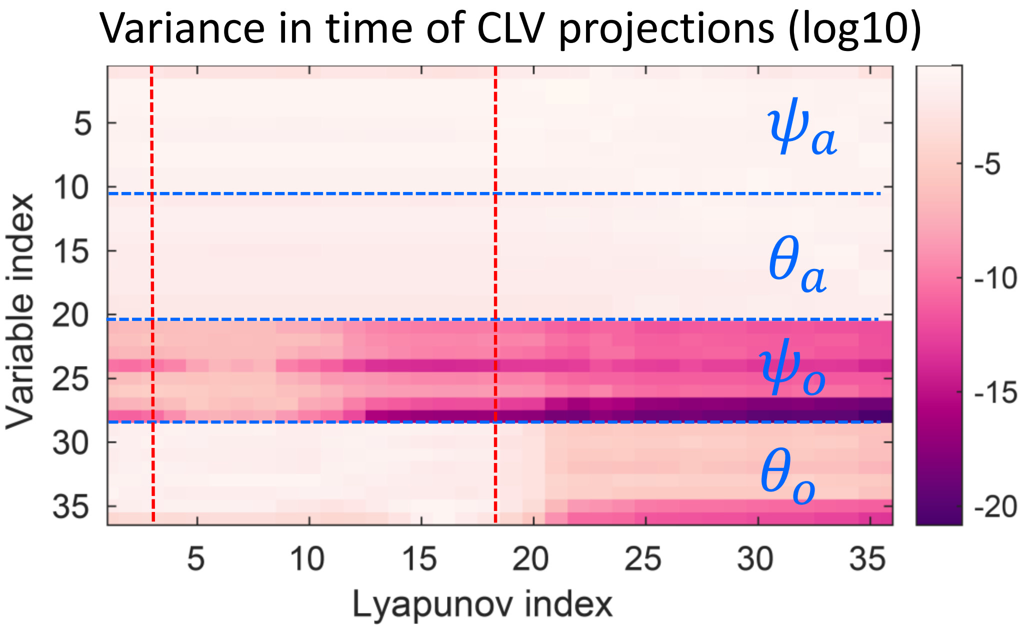




We find an extended centre subspace corresponding to the slow dynamics od the system

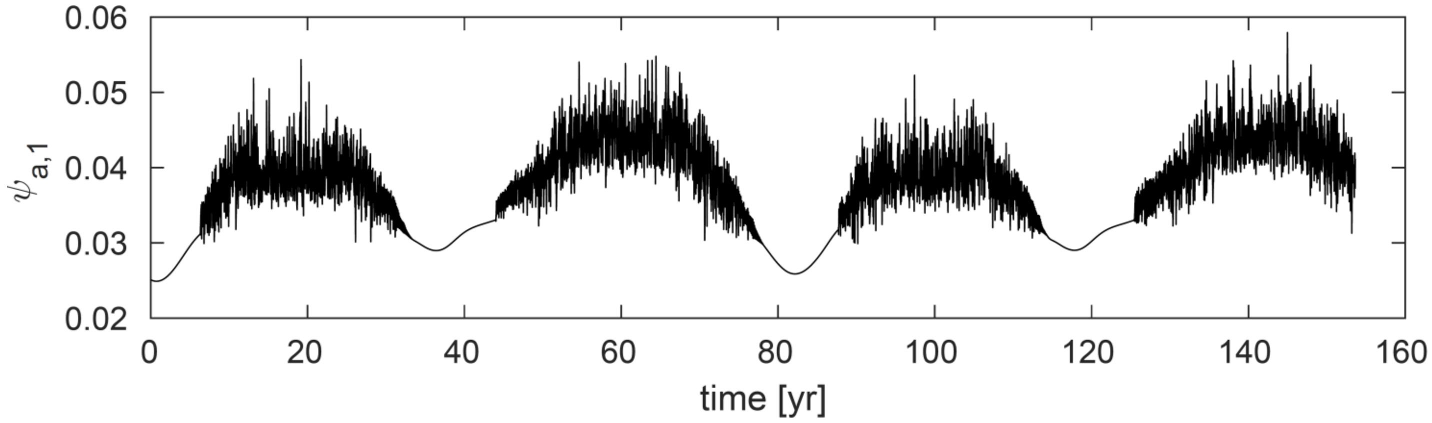


Similar to Vannitsem and Lucarini (2016)

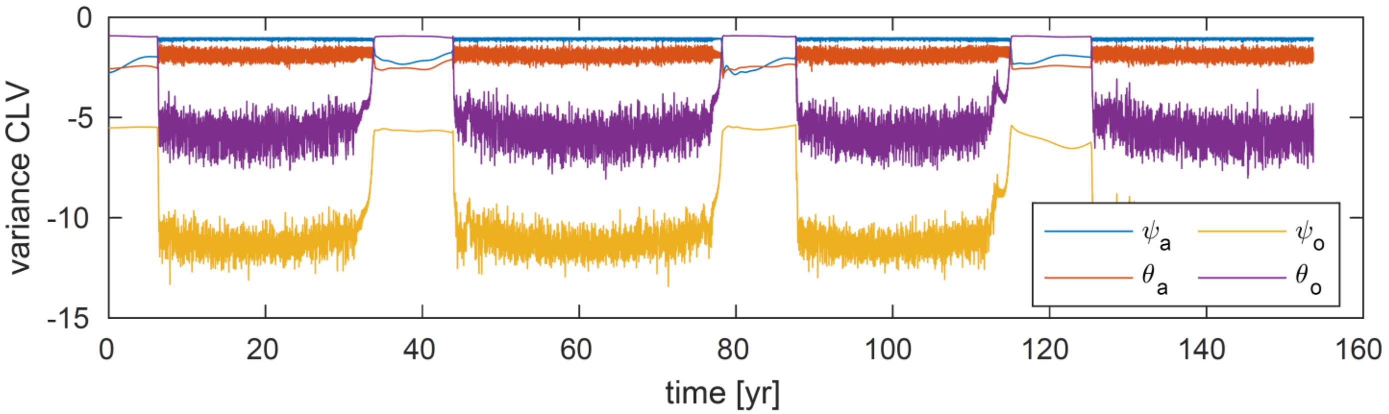


The dynamics of the centre subspace is dominated by the atmospheric and the oceanic thermodynamic variables.

CLVs point out the different regime dynamics.



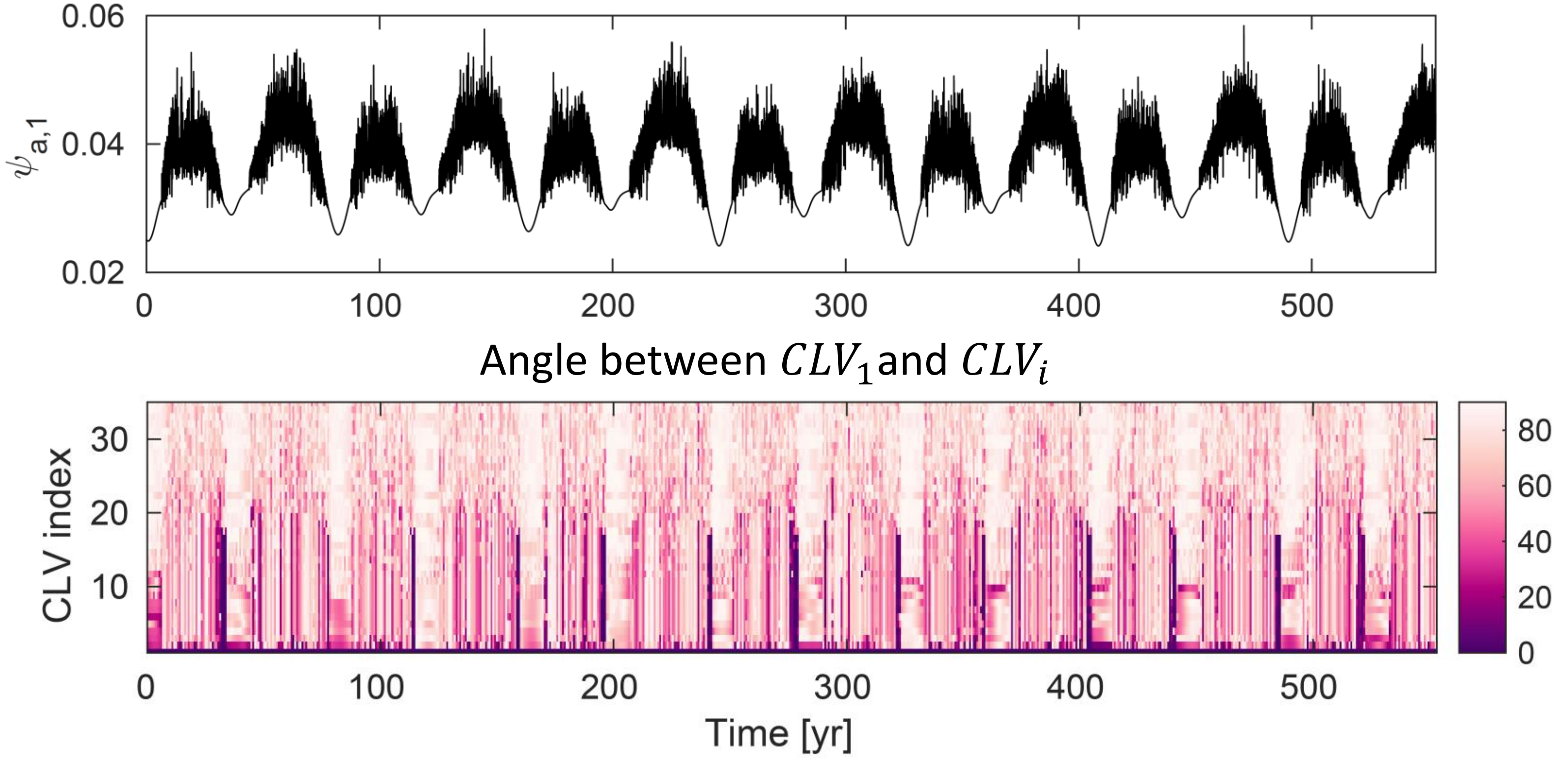
Atmospheric dynamic variable, 1. mode

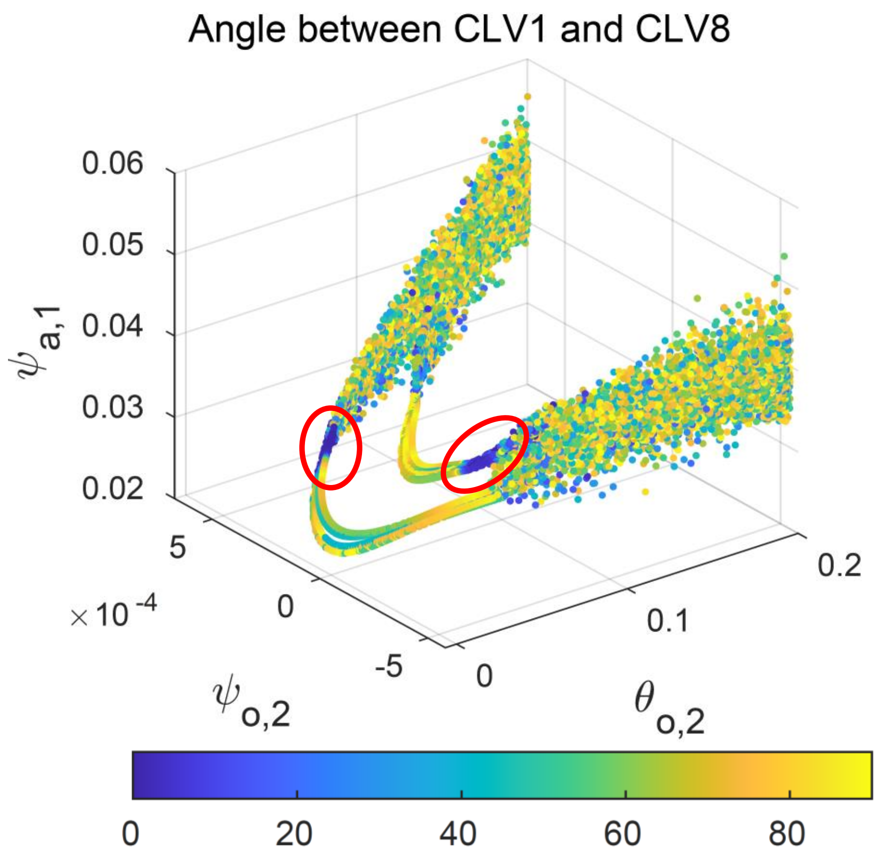


Variance of CLV[1], averaged for each variable type

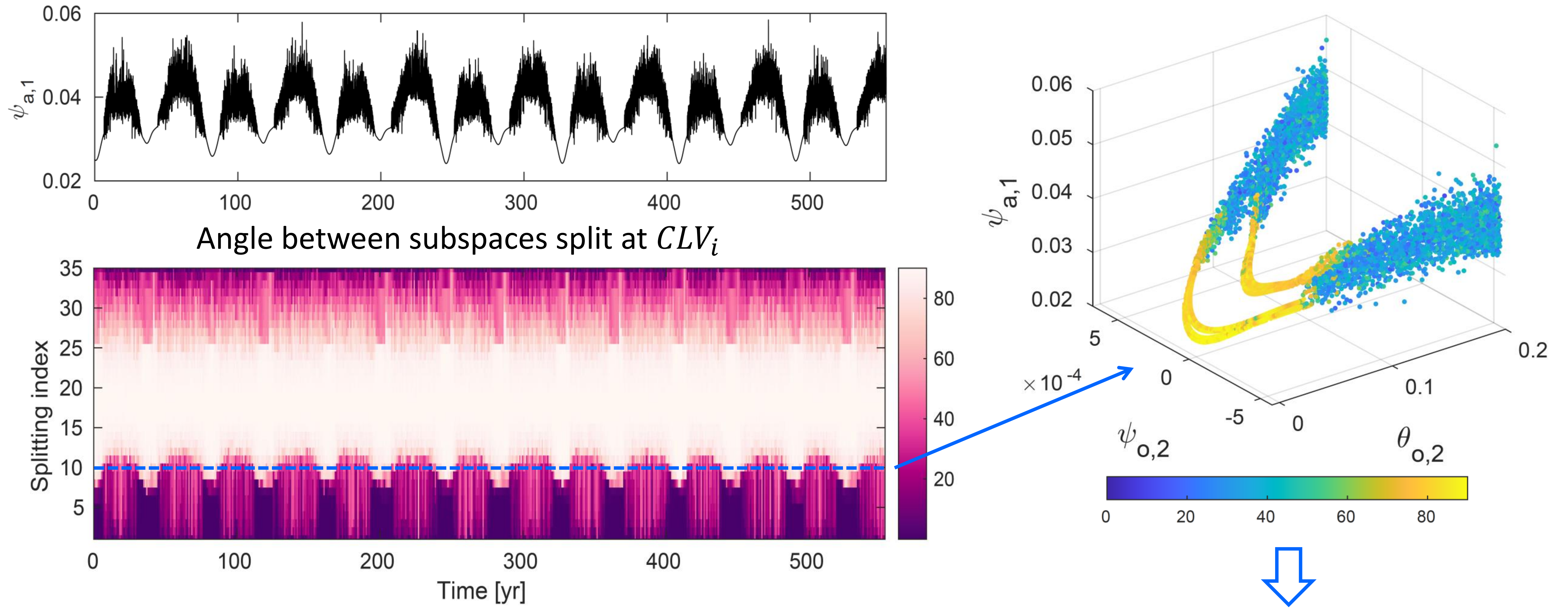
During the weakly chaotic regime the contribution of the oceanic thermodynamic variables is dominant for the development of instabilities.

At the beginning of the weakly chaotic regime, CLV_1 points into the direction of the flow





The angle between subspaces (max. principle angle) changes substantially between the two regimes.

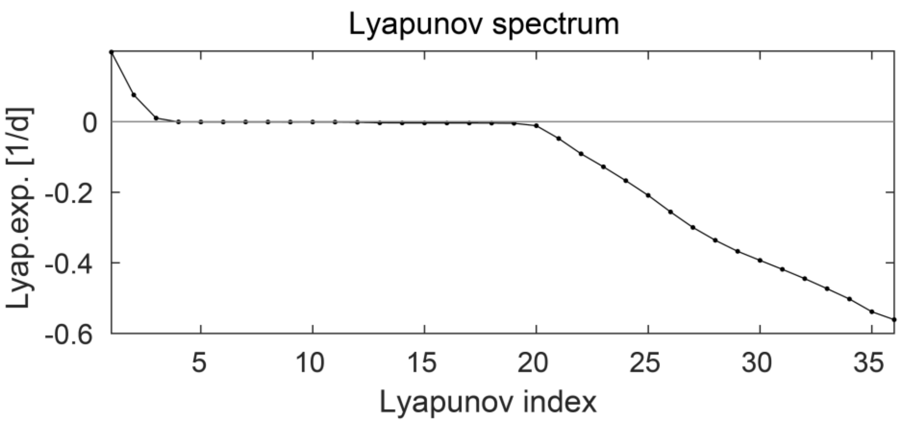


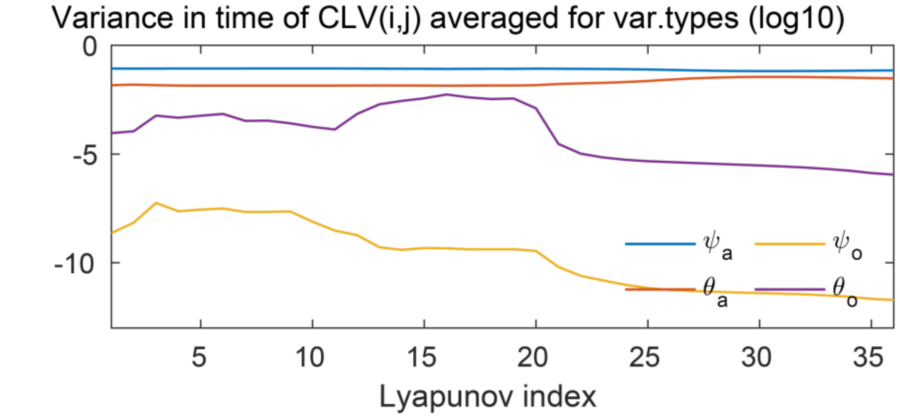
We can separate the 2 regimes

We can separate the two regimes based on the angle between subspaces split at CLV_{10}

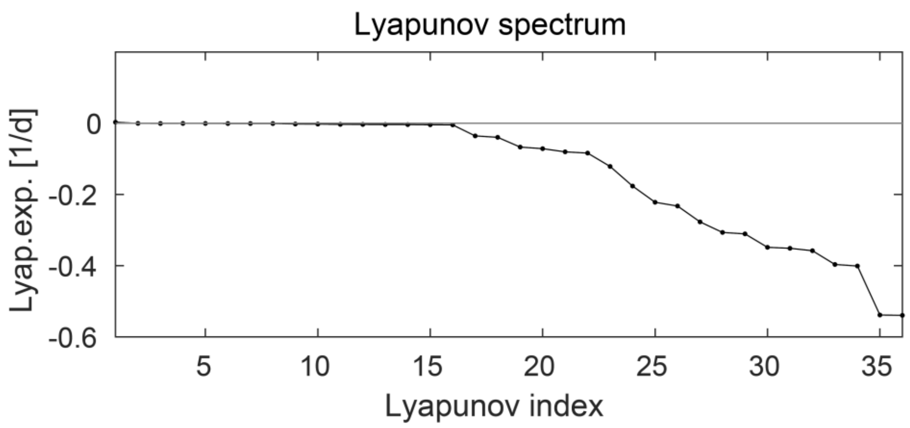
The Lyapunov spectrum and the CLVs are very different in the two regimes.

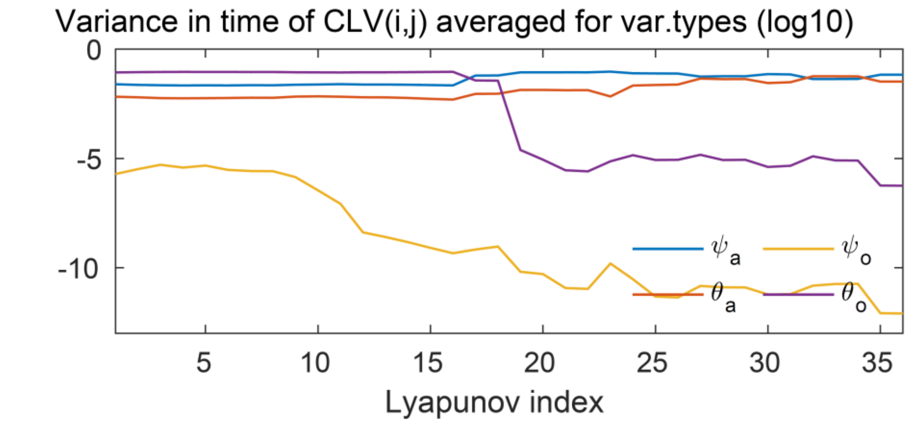
Chaotic regime: Atmospheric variables dominate (blue and red lines)





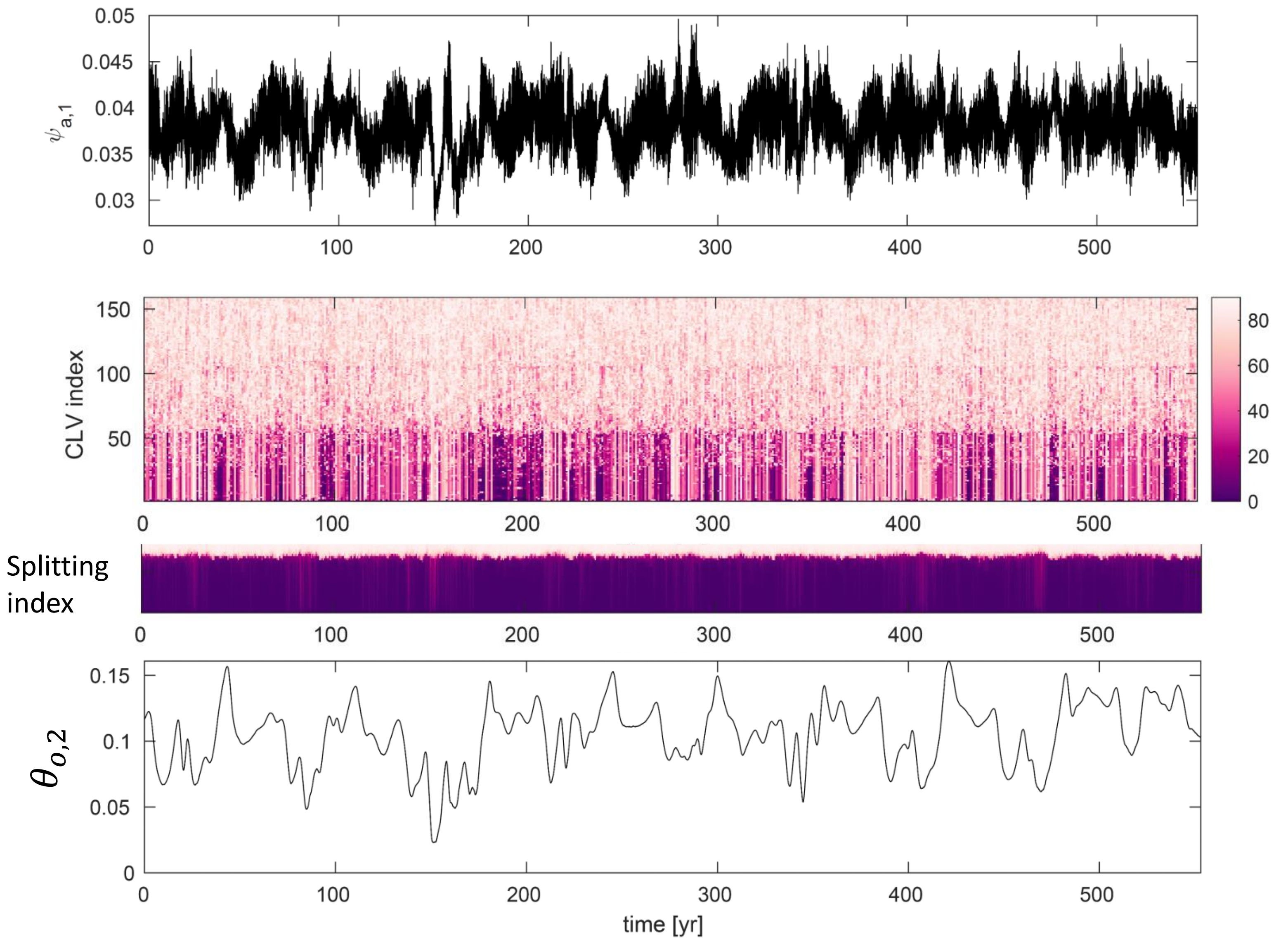
Weakly chaotic regime: Oceanic thermodynamic variables dominate (purple line)





How do things look like in a higher resolution of the model with 55 atmospheric modes and 25 oceanic modes?

The connection between angles of CLVs (subspaces) and the dynamics of the system is less trivial



The connection between the dynamics and the angles seems to be sometimes in contradiction with the low resolution case.

Further investigations are needed!

CLVs help us to understand the regime dynamics in MAOOAM in case of the low-order version:

- The first CLV points in the direction of the flow when the system enters the weakly chaotic state
- In the weakly chaotic state the oceanic thermodynamic variables dominate
- Based on the angle between subspaces split at CLV_{10} we can separate the two regimes

In the higher resolution case, the connection between angles of CLVs (or subspaces) and the dynamics of our system is much more complex, and needs further investigations.

References

Ginelli, F., Poggi, P., Turchi, A., Chaté, H., Livi, R., and Politi, A.: Characterizing dynamics with covariant Lyapunov vectors, *Phys. Rev. Lett.*, **99**, 130601 (2007), https://doi.org/10.1103/PhysRevLett.99.130601

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Appendix: model equations

Equation of motion - atmosphere:

Layer 1: $$\frac{\partial}{\partial t}(\nabla^2\psi_a^1) + J(\psi_a^1, \nabla^2\psi_a^1) + \beta\frac{\partial\psi_a^1}{\partial x} = -k_d'\nabla^2(\psi_a^1 - \psi_a^2) + \frac{f_0}{\Delta p}\omega$$

Friction between atm. - ocean

Layer 2: $$\frac{\partial}{\partial t}(\nabla^2\psi_a^2) + J(\psi_a^2, \nabla^2\psi_a^2) + \beta\frac{\partial\psi_a^2}{\partial x} = k_d'\nabla^2(\psi_a^1 - \psi_a^2) - \frac{f_0}{\Delta p}\omega - k_d\nabla^2(\psi_a^1 - \psi_o)$$

Equation of motion - ocean:

Impact of the wind stress

$$\frac{\partial}{\partial t}(\nabla^2\psi_o - \frac{\psi_o}{L_R^2}) + J(\psi_o, \nabla^2\psi_o) + \beta\frac{\partial\psi_o}{\partial x} = r\nabla^2\psi_o - \frac{C}{\rho h}\nabla^2(\psi_a^2 - \psi_o)$$

Appendix: model equations

Thermodynamic equation - atmosphere:

$$\gamma_a \left(\frac{\partial T_a}{\partial t} + J(\psi_a, T_a) - \sigma\omega \frac{p}{R} \right) = -\lambda(T_a - T_o) + \epsilon_a \sigma_B T_o^4 - 2\epsilon_a \sigma_B T_a^4 + R_a$$

Heat transfer
atm.-ocean

Thermodynamic equation - ocean:

$$\gamma_o \left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right) = -\lambda(T_o - T_a) - \sigma_B T_o^4 + \epsilon_a \sigma_B T_a^4 + R_o$$