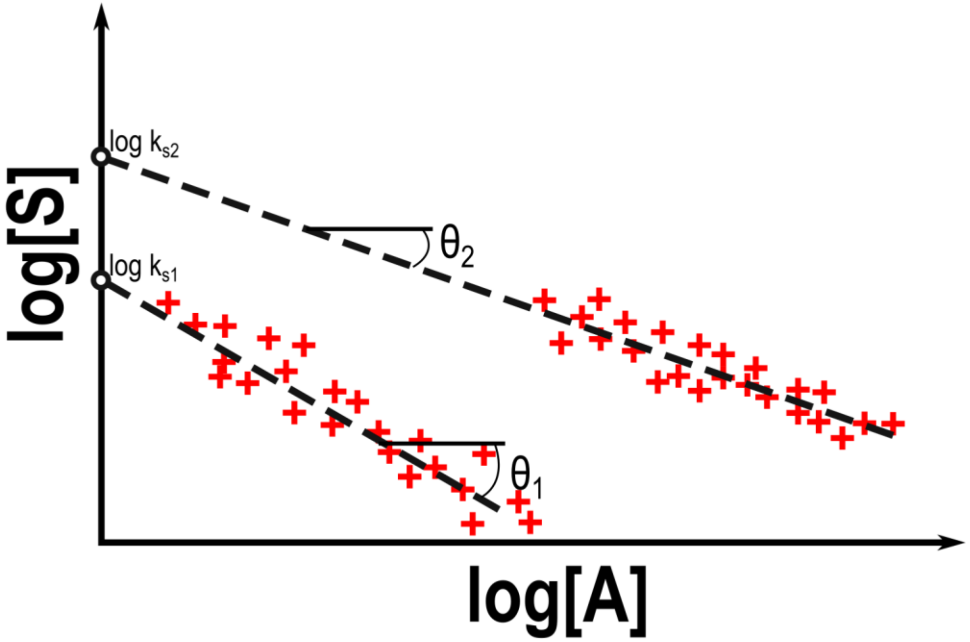
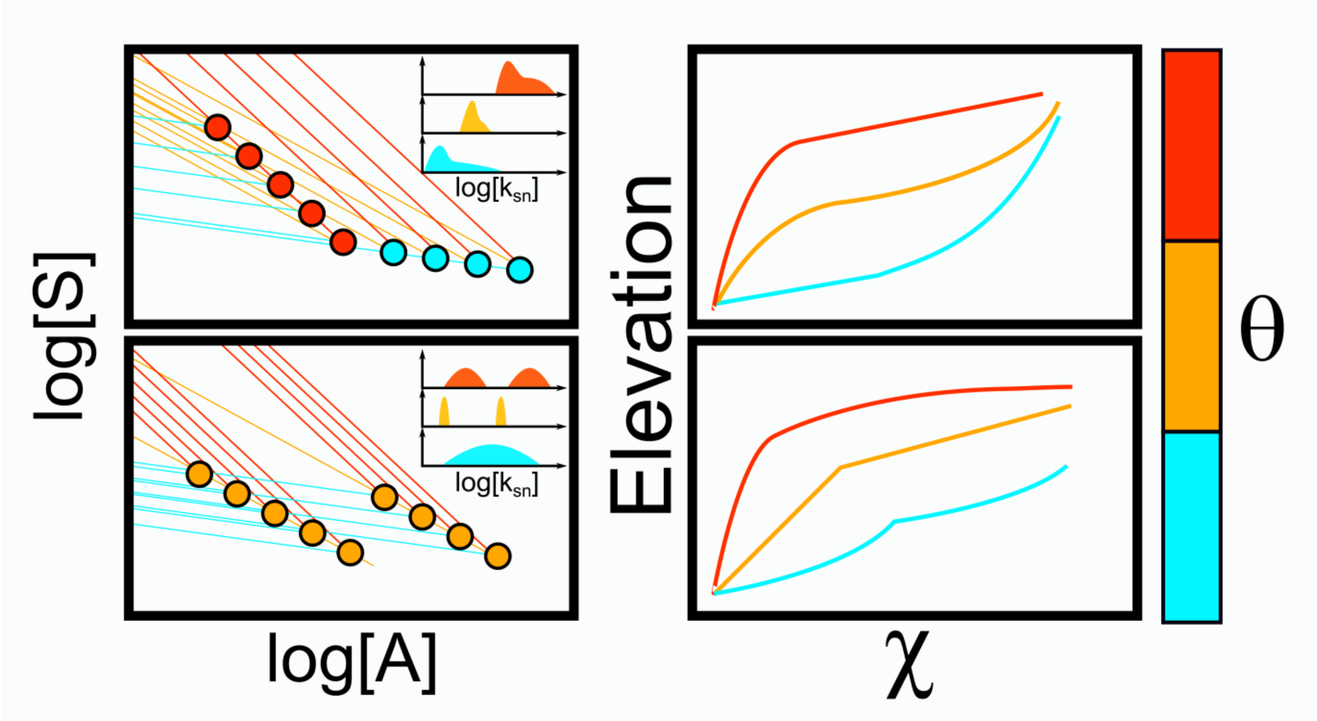
Importance of concavity for interpreting rates and patterns of landscape evolution from river profiles

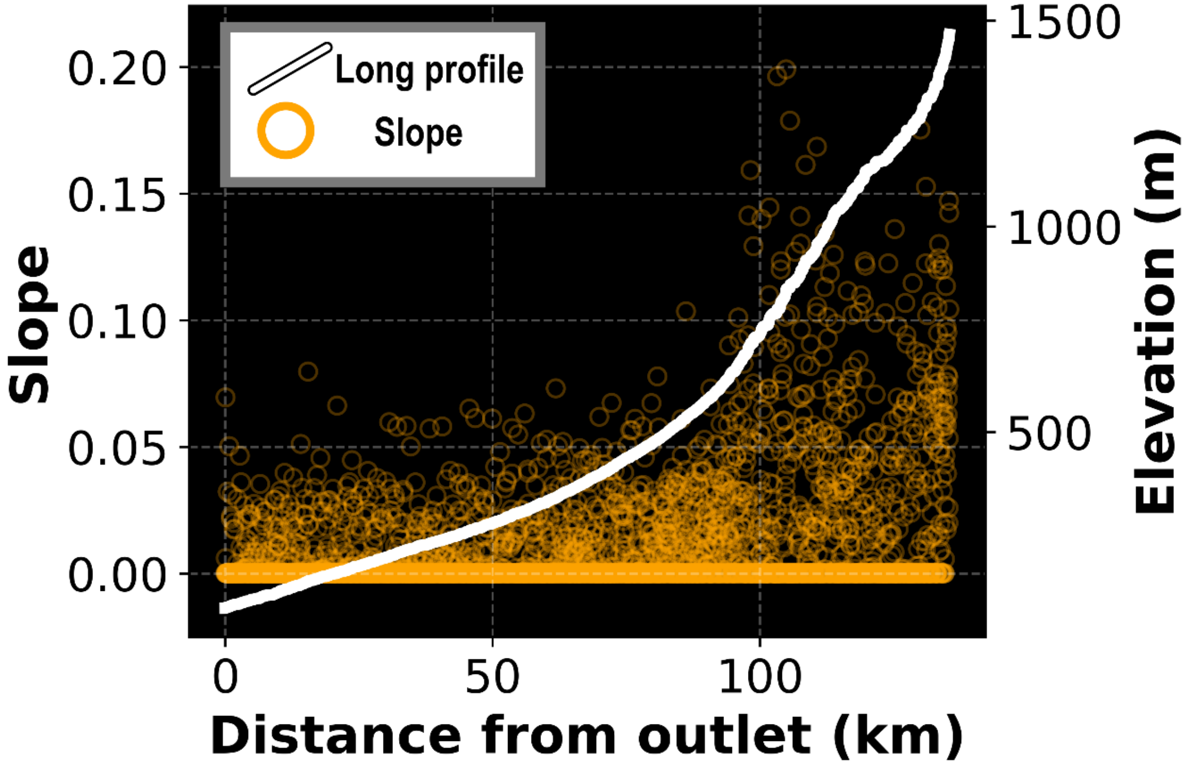
Boris Gailleton, Simon Mudd, Fiona Clubb, Martin Hurst, and Stuart Grieve





What are k_{sn}, χ and θ?

- Channel morphology responds to external forcings, e.g. tectonics, climate, lithology
- These forcings can be unravelled from long profiles, for example linking a steepened reach to fault activity
- However rivers do not yield information that easily: changes in discharge/drainage area affect channel gradient



A river long profile and its associated channel gradient (measured pixel-to-pixel). Note the noise in the gradient data, and the increasing gradient as one moves towards the headwaters. There is a suggestion of a steepened reach approximately 100km from the outlet. How do we tell if this is meaningful in the context of both noise in the gradient and the overall trend in the gradient? Outlet of the river 5 km North of Putna, Vrancea, Romania, extracted from SRTM 30 metres, processed with LSDTopoTools-lsdtopytools framework.

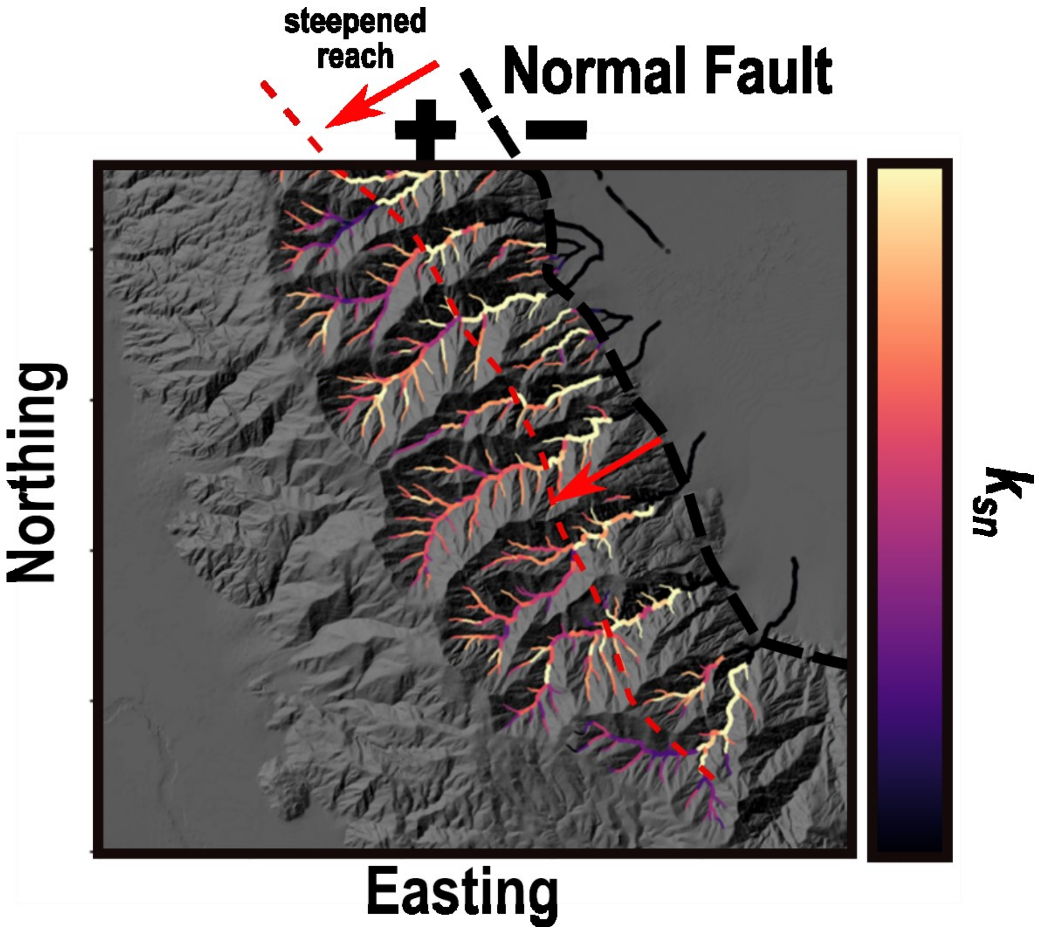
What are k_{sn}, χ and θ?

- Many river segments can be described by a power-law relating channel slope and drainage area
- θ, or concavity index, is the rate at which slope decreases as drainage area increases
- k_s, or steepness index, is the overall steepness of the slope patch (see equation below)
- These metrics have been widely related to tectonics, climate or lithology (see Whipple et al., 2013 for a review)

$$S = k_s A^{-\theta}$$

$$k_s = k_{sn} \ \ if\ \theta\ is\ constant$$

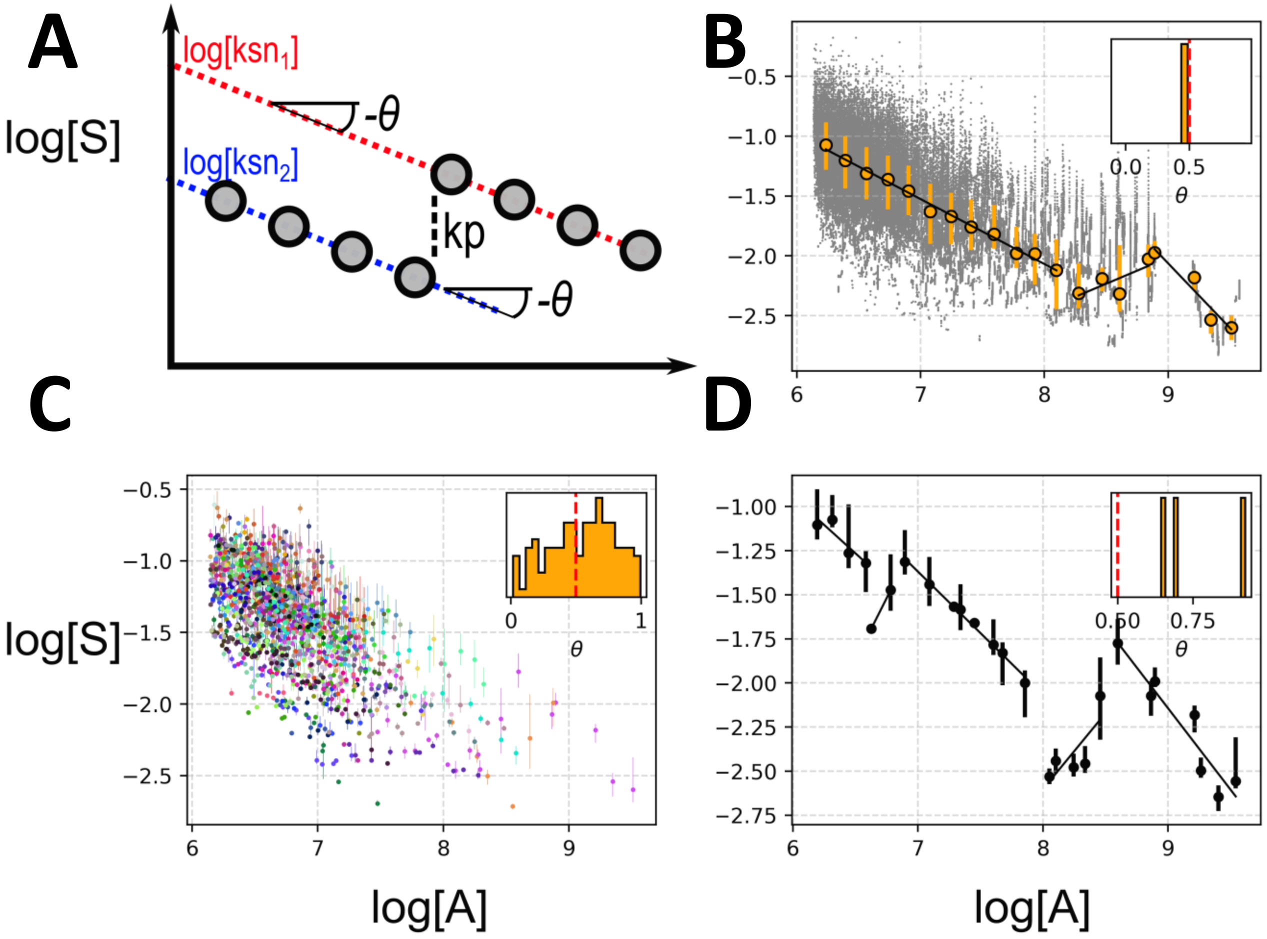
Wolmann (1955), Hack (1957), Morisawa (1962), Flint (1974)



Map view of k_{sn} values in the Saline Valley (CA,USA). A normal fault modifies base level and has "fired" a knickpoint and steepened slope patch upstream, expressed here as high k_{sn}. k_{sn} calculated with θ=0.25, constrained with Mudd et al., 2018 and k_{sn} calculated with Mudd et al., 2014 within lsdtopytools framework. Data from SRTM 30 metres. Geological context explained in Kirby et al., 2012.

What are k_{sn}, χ and θ?

- One limitation of the Slope—Area (S—A) relationship: Slope is frequently very noisy
- Data needs binning, smoothing or other procedures leading to data loss
- Interpretation of S—A data is sensitive to data processing method



Determination of k_s and θ from S—A plots. All the data in B,C, D are from the same base dataset (Buzau watershed, Romania). The histograms present the subsequent distribution of extracted θ.

A) Theory: θ is the slope of log S – log A plot and log k_s the intercept
B) Binning all points of the basin by log A
C) Binning the points by drainage area for each tributaries
D) Focusing on the main steam

Note how the extracted θ changes just depending on the processing method. Segmentation of points calculated using algorithm of Mudd et al., 2014.

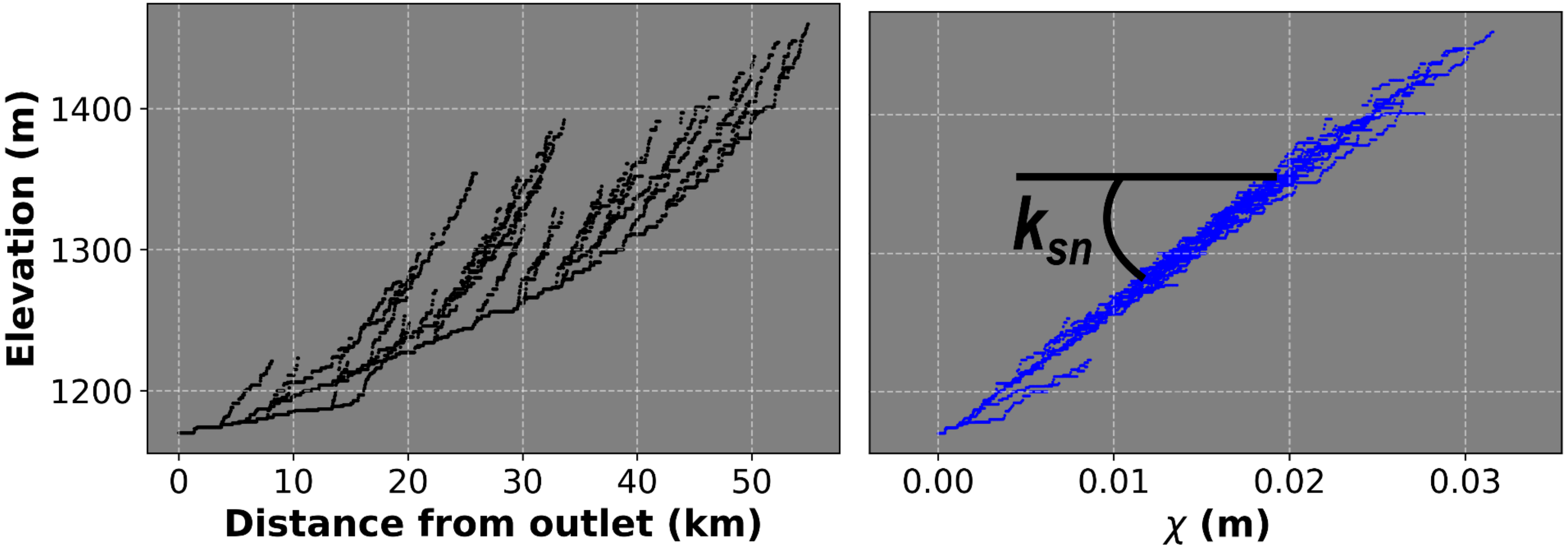
What are ksn, χ and θ?

- χ has been developed by Perron et al., 2013 to circumvent limitations of methods using gradient.
- It directly integrates drainage area, normalised to a value of θ, into a transformed coordinate
- It allows representation of rivers of different size into a same reference frame, and the gradient χ—elevation space is proportional to k_{sn}
- In incising landscapes, the most likely value of θ occurs when main stem and its tributaries are collinear (Niemann et al., 2001)
- Assuming uniform lithology, climate and erosional processes in two juxtaposed watersheds, χ can be used as a proxy for drainage divide migration or stream piracy (see Willett et al., 2014, Whipple et al., 2017).

$$z(x) = z(x_b) + \left(\frac{k_s}{{A_0}^{\theta}}\right)$$

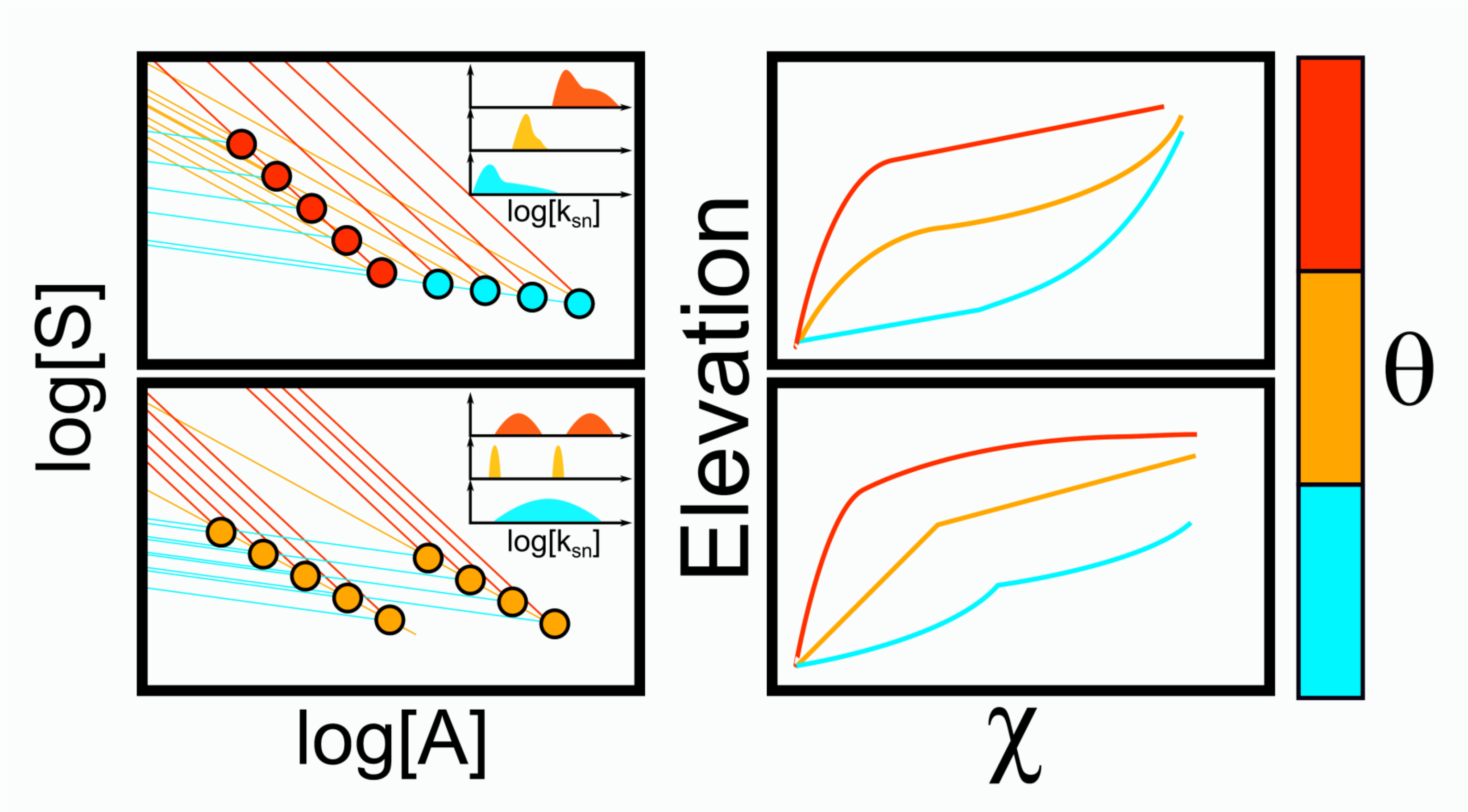
$$\chi = \int_{x_b}^{x} \left(\frac{A_0}{A(x)}\right)^{\theta}$$

Perron et Royden, 2013 (with SPL)
Whipple et al, 2017 (with Flint's law)



Limitations

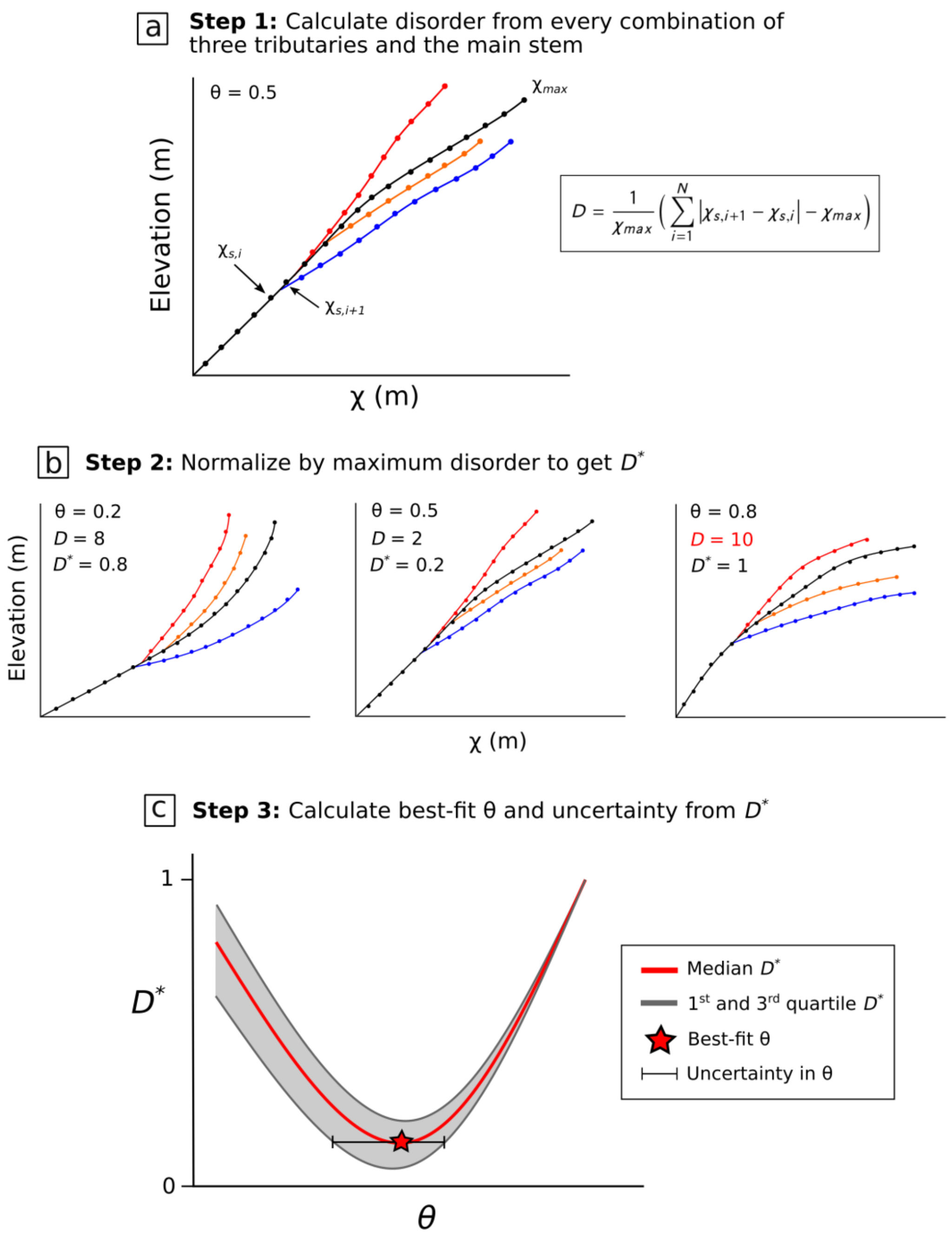
- To compare k_{sn} and χ at different locations, θ needs to be fixed to a reference value
- This value can be constrained, but over large areas k_{sn} and χ might be locally calculated with non-optimal θ
- Non-optimal θ can distort k_{sn} and χ . This distortion is the subject of this PICO
- How θ vary across scales? How does non-optimal θ affect ksn? And how does non-optimal θ affect χ?



Conceptual representation of potential distortion of k_{sn} and χ if calculated with different values of θ. Note the differences between the different scenarios, however all representing the same base dataset.

How does θ vary?

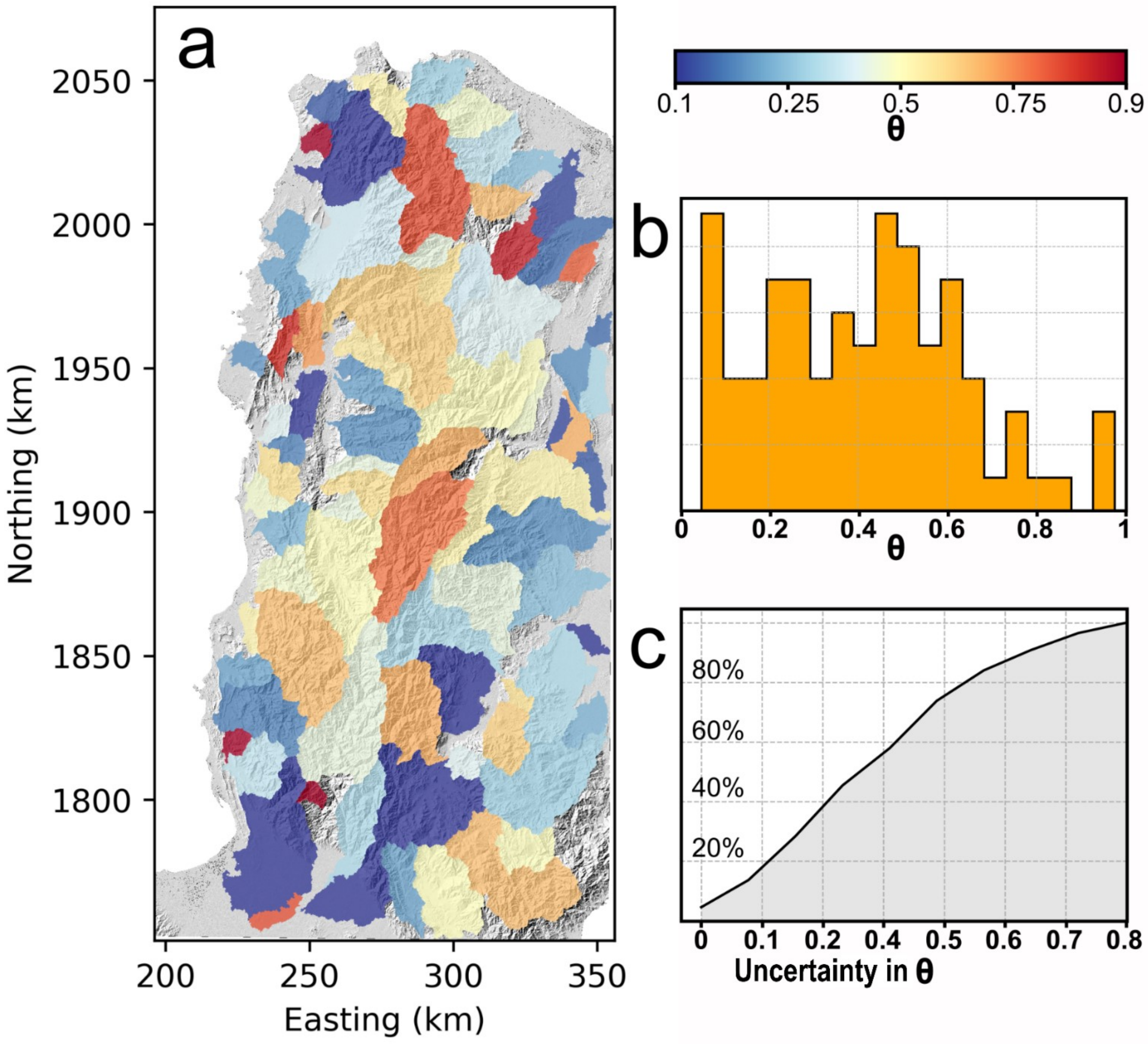
- We developed a method based on collinearity
- It suggest optimal θ based on which value minimises disorder, D, and maximises collinearity (Goren et al.,2014, Hergarten et al., 2016)
- It also provides an uncertainty estimation by estimating θ for different sets of tributaries



How does θ vary?

- Illustration on the NE of island of Luzon (Philippines)

a) best-fit θ for all the considered river basins. b) Distribution of θ values over the area, note that the relatively high concentration of θ=0.05 is due to 0.05 being the minimum tested value. c) Cumulative uncertainty in θ. Here for example, 80% of the basins have an uncertainty in θ of less than 0.5 (~ +/- 0.25).



How does θ vary?

- We compiled a number of sites across upland landscapes at global scale
- θ varies widely between 0.05 and 0.8
- The overall median is 0.425 and quartiles are 0.225 and 0.575
- Although globally roughly around the widely used value of 0.45, this observation is not true for many localities and most of them shows ranges of theta values, there is a strong case-to-case variation
- Most of the sites show error between 0.1 and 0.4, suggesting a single value is rarely suiting whole areas

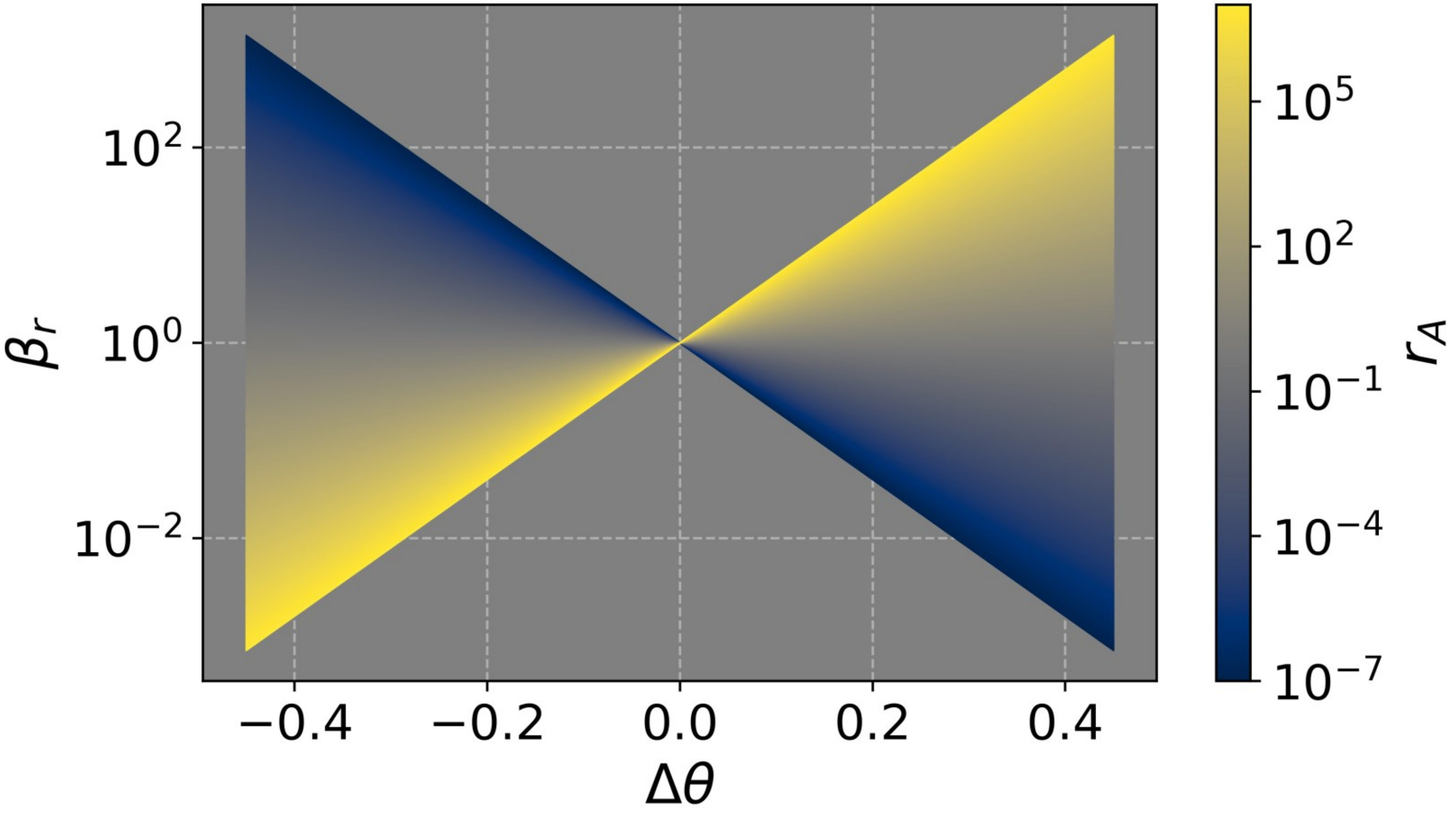
Site Name	N Basins	Median θ	Q1 θ	Q3 θ	Median R_θ	Q1 R_θ	Q3 R_θ
Chilean Andes	65	0.475	0.225	0.625	0.275	0.125	0.4
North Arkansas	11	0.65	0.525	0.663	0.3	0.2	0.412
Bureya Massif	75	0.45	0.325	0.55	0.225	0.175	0.325
Eastern Carpathians	876	0.5	0.325	0.65	0.275	0.175	0.375
Caucas Mountains	366	0.362	0.175	0.5	0.25	0.15	0.35
Sierra Madre, Mexico	94	0.45	0.306	0.525	0.25	0.131	0.375
Corsica	30	0.388	0.256	0.425	0.288	0.225	0.444
Ethiopian Highlands	111	0.3	0.2	0.4	0.175	0.125	0.275
Jebal Barez, Iran	54	0.2	0.106	0.275	0.175	0.125	0.25
Lesotho	78	0.475	0.35	0.569	0.175	0.1	0.275
Luzon	88	0.425	0.225	0.575	0.338	0.225	0.475
Edge of Mongolian Plateau	107	0.45	0.35	0.525	0.225	0.125	0.338
Basins along Nujang River	71	0.45	0.325	0.625	0.275	0.175	0.425
Oregon Coast Ranges	26	0.538	0.338	0.75	0.25	0.175	0.3
San Gabriel Mountains	34	0.325	0.275	0.444	0.212	0.125	0.3
Southern Altai Mountains	551	0.35	0.175	0.525	0.25	0.15	0.4
Southern Brazil	102	0.475	0.4	0.55	0.225	0.15	0.275
Western South Africa	634	0.25	0.125	0.425	0.225	0.15	0.35
Southern Wisconsin	60	0.562	0.45	0.625	0.2	0.144	0.325
Yemen	52	0.4	0.275	0.506	0.175	0.125	0.256
Atlas Mountains	26	0.4	0.275	0.5	0.225	0.175	0.325
Dolomites	28	0.538	0.35	0.756	0.338	0.225	0.5
Hida Mountains	51	0.5	0.3	0.575	0.3	0.225	0.438
Himalayas	645	0.4	0.25	0.525	0.275	0.175	0.4
Allegheny Plateau	118	0.7	0.556	0.819	0.25	0.175	0.394
Northern Appalachians, USA	177	0.525	0.4	0.675	0.35	0.225	0.45
Southern Appalachians, USA	277	0.5	0.3	0.625	0.35	0.225	0.45
Olympic Mountains	33	0.575	0.4	0.675	0.325	0.2	0.425
Pyrenees	61	0.475	0.3	0.575	0.325	0.225	0.4
Taiwan	97	0.45	0.15	0.575	0.275	0.2	0.375
Tien Shan	40	0.612	0.5	0.756	0.325	0.25	0.481
Zagros Mountains	49	0.475	0.3	0.625	0.25	0.125	0.4

How does non-optimal θ affect k_{sn}?

- To interpret k_{sn} values in a meaningful way, it is important to observe variations in ksn values
- Distortion of the aspect between two data point is function of the ratio of their drainage area and the Δθ from the optimal value. For example let's consider two points with a factor of 3 between the two ksn at optimal θ. If θ is overestimated by 0.2, the ratio will be exaggerated by 4 if there ration of drainage area is 1000.

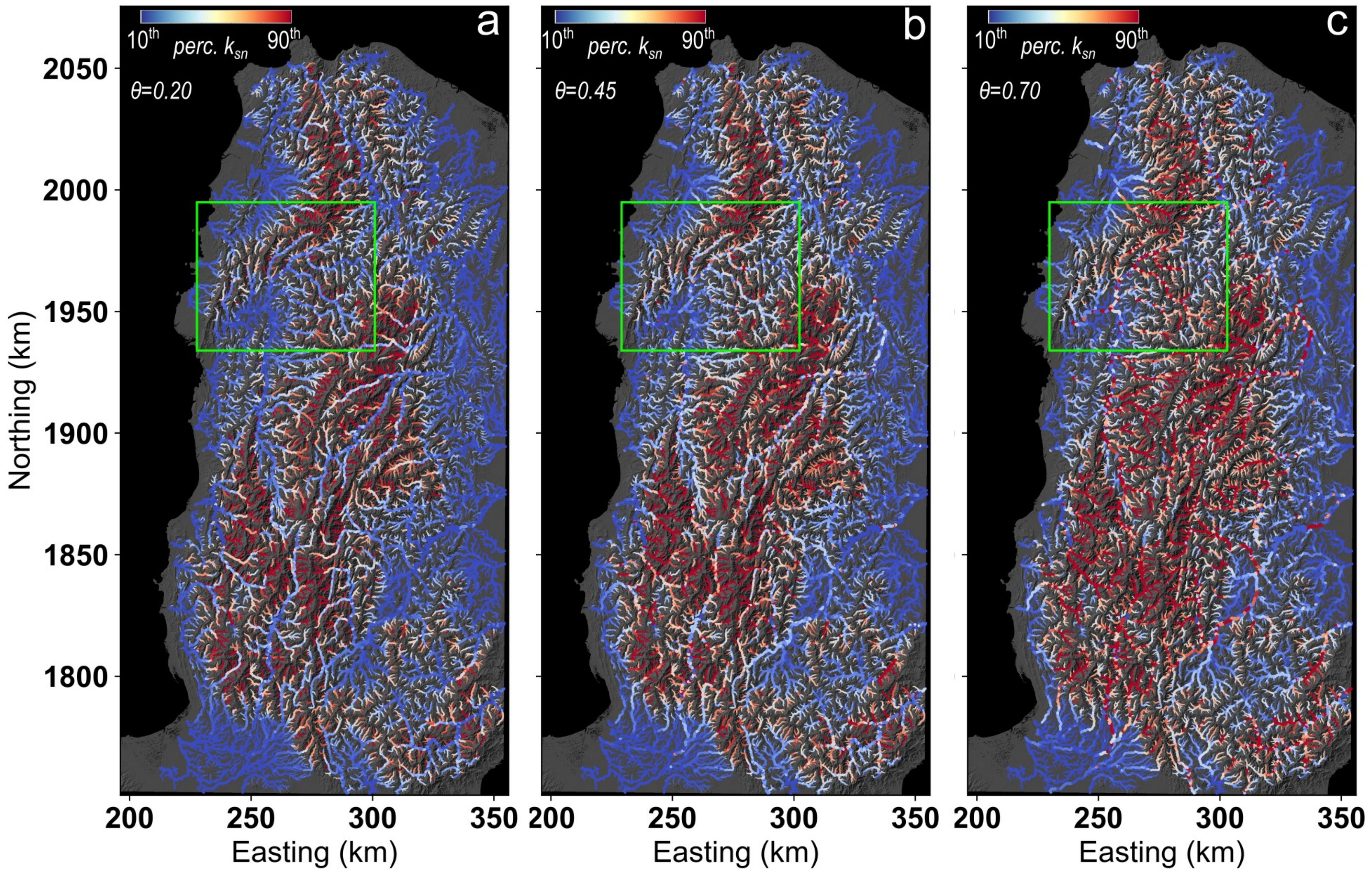
$$\beta_r(\Delta\theta) = \frac{r_{k,\theta_2}}{r_{k,\theta_1}} = r_A^{\Delta\theta}$$

- Contrasts in k_{sn} can be created, inverted, exaggerated or deleted.



How does non-optimal θ affect ksn?

- We calculated k_{sn} on the island of Luzon, which has high heterogeneity in θ, to demonstrate the scale of the distortion.

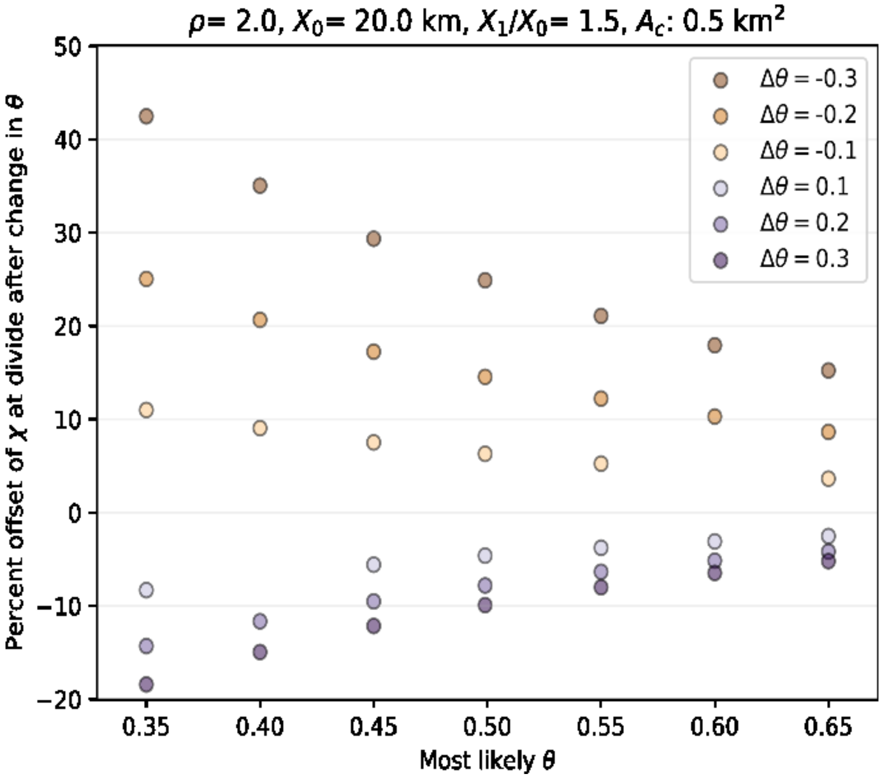


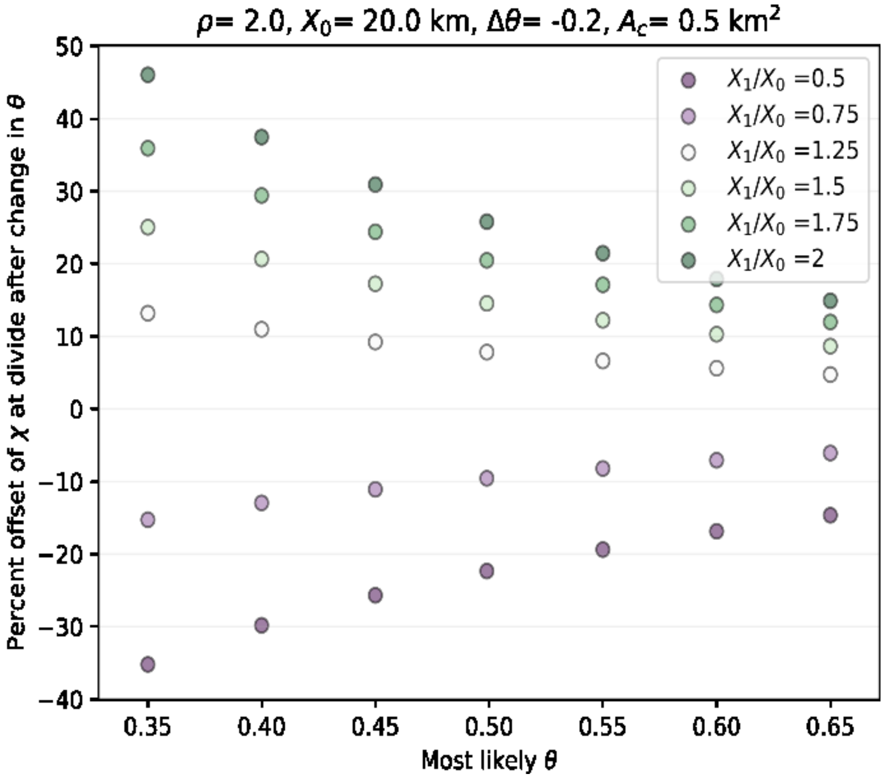
a) k_{sn} calculated for a) θ = 0.20 b) θ = 0.45 and c) θ = 0.70, both possible best fit for the area. Note how the observed k_{sn} changes with θ values.

How does non-optimal θ affect χ?

- χ contrasts across drainage divides have been used to infer divide migration (Willett et al., 2014). How badly are these contrasts affected by changing θ?
- To derive an analytical solution to the distortion, we used Hack's law to approximate A = f(x)

Distortion of χ coordinate across divide. These show basins where, for the most likely value of θ, there is no difference in χ across the divide. The basins are different shapes. The first basin has a length of X_0, and the second basin has a length of X_1. ρ is the exponent relating length and drainage area (a Hack exponent) in the first basin. This is computed for the second basin so χ is the same at the divide for most likely θ. A_c is the critical drainage area at which χ is compared. If the percent change is positive, it means the second basin has greater χ at the divide after changing θ.

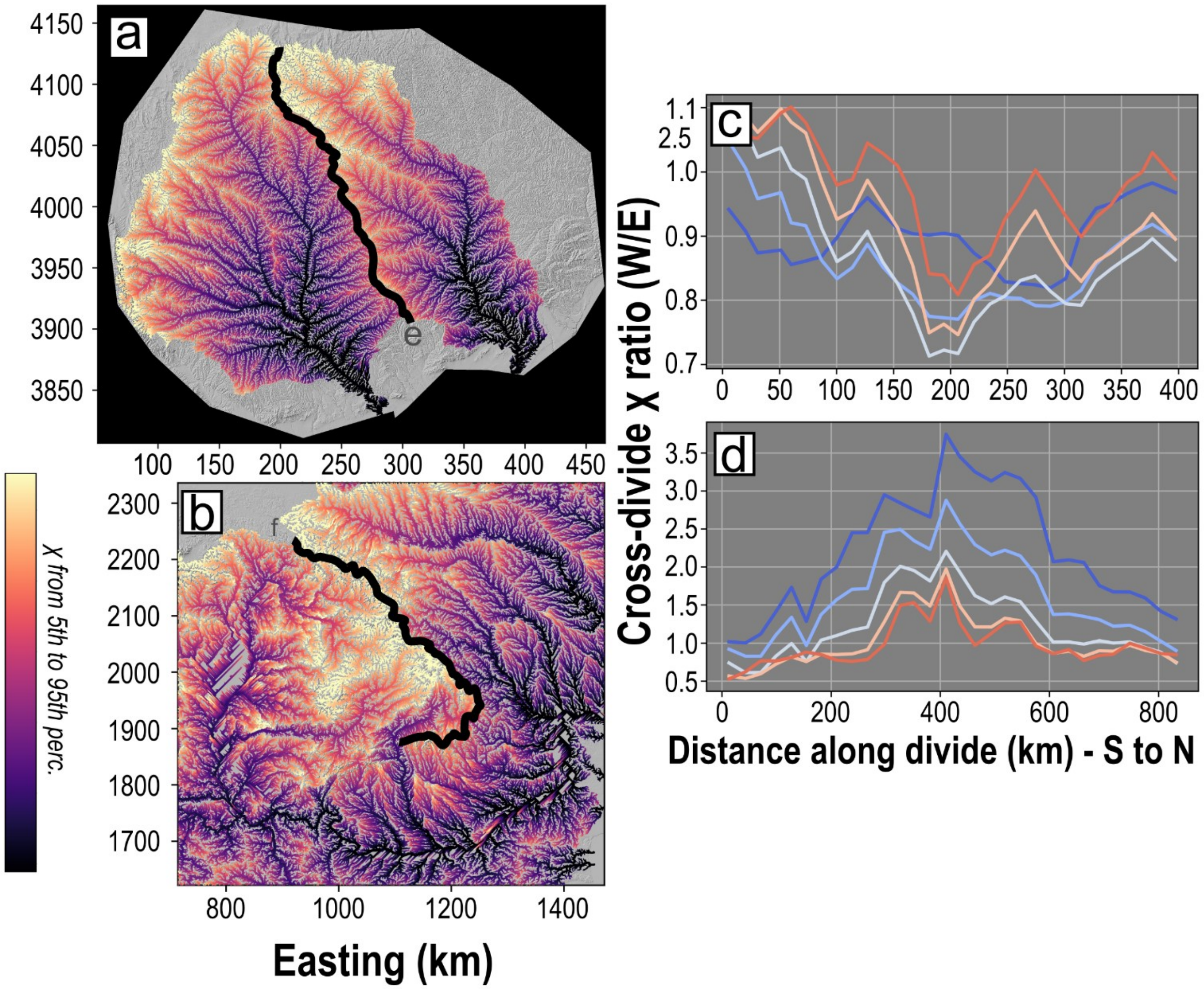




How does non-optimal θ affect χ?

- We calculated χ numerically over ranges of θ in different settings: Corsica, the Carpathians and Loess Plateau

χ maps for θ=0.45 for a) Loess plateau and b) the Carpathian range. d) and e) respectively show the ratio between χ values on the Western sides of the divides and χ values on the Eastern side. Note that each χ integrates all downstream points and can accumulate distortion if downstream region are in disequilibrium with their θ.



Take away message

- In upland landscapes θ varies globally from 0.1 to 0.8, with a lot of variations on a case-by-case basis

- **Calculating k_{sn} with non-optimal** θ distorts the relative values, function of drainage area differences

- χ is even more sensitive to **non-optimal** θ as it integrates all downstream distortions to its value

- Variability in θ can generate patterns of χ and **k_{sn}** s that can easily lead to spurious interpretations of tectonic activity, uplift and divide migration where non-optimal θ is used to calculate these metrics.

Additional information

All the analysis are made within the LSDTopoTools framework, a fully open-source framework for topographic analysis accessible via command-line, c++ or python:

https://lsdtopotools.github.io/

https://github.com/LSDtopotools/LSDTopoTools2
https://github.com/LSDtopotools/lsdtopytools (early development python version)

Corresponding author: Boris Gailleton - b.gailleton@sms.ed.ac.uk – Scholar page