How to predict hydraulic conductivity of unsaturated soils from an incomplete SWCC

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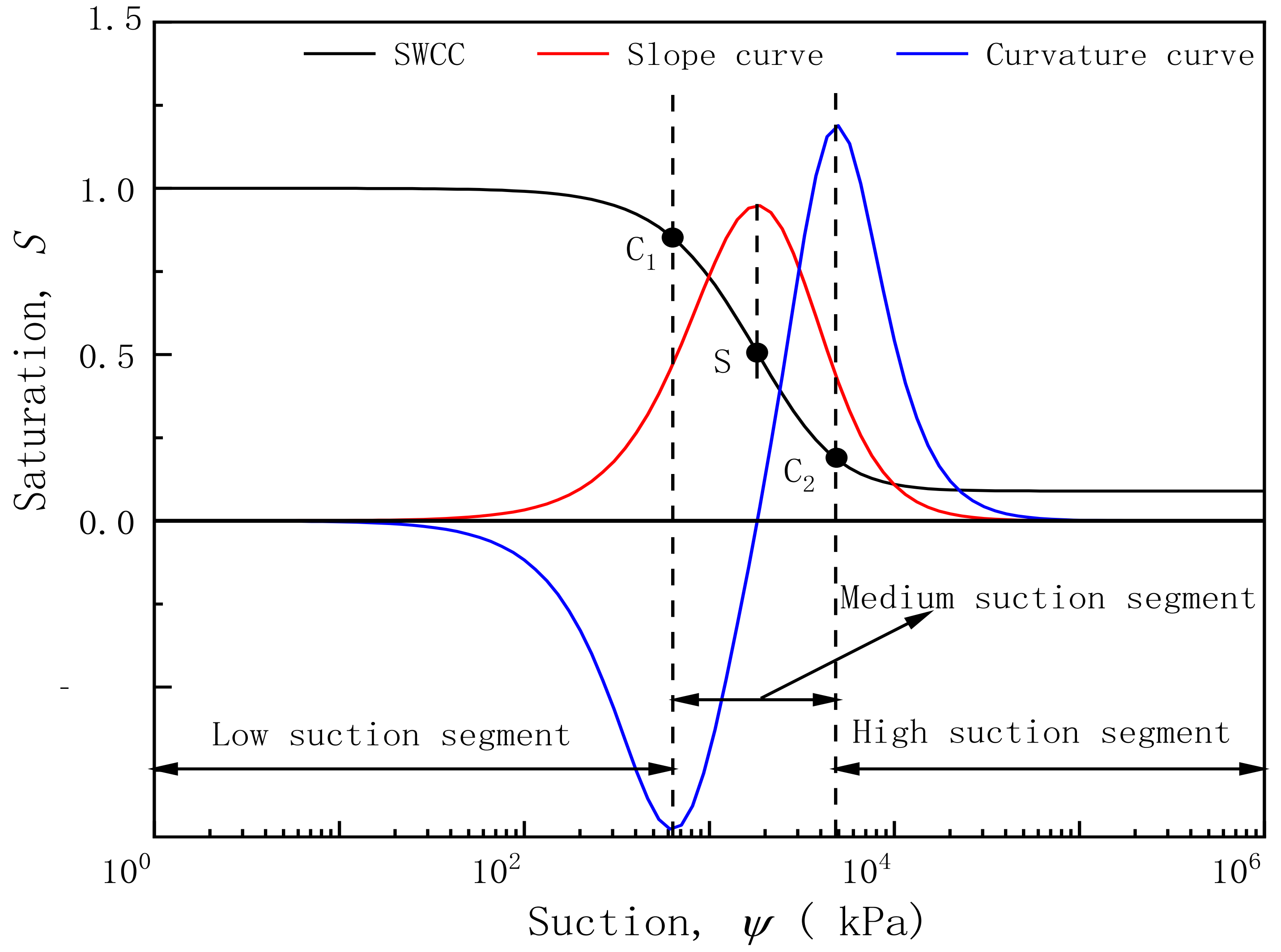
➢ What is SWCC (soil water characteristic curve)?



➢ Summary of SWCC models

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Author	SWCC Models	Parameters	Author	SWCC Models	Parameters
Gardner W.R. , 1958	$\Theta = 1/(1 + q\psi^n)$	q, n	**McKee and Bumb, 1984**	$\Theta = Aexp(a\psi - B)$	A, a, B
Brooks and Corey, 1964	$\Theta = (a/\psi)^\lambda, \psi < a$	a, λ	**McKee and Bumb, 1987**	$\Theta = 1/[1 + Aexp(a\psi - B)]$	A, a, B
Brussaert, 1966	$\Theta = a/(a + \psi^n)$	a, n	**Fredlund and Xing, 1994**	$S = 1/ln[e + (\psi/a)^n]^m$	a, m, n
Farrel and Larson, 1972	$\Theta = b(ln\psi - lna)$	a, b		$S = C(\psi)/ln[e + (\psi/a)^n]^m$	
Van Genuchten, 1980	$\Theta = 1/[1 + (\alpha\psi)^n]^m$	α, n, m		$C(\psi) = 1 - \frac{ln(1 + \psi/\psi_r)}{ln(1 + 10^6/\psi_r)}$	
WillIams et al., 1983	$ln\Theta = A + Bln\psi$	A, B	**Assouline S, 1998**	$\Theta = 1 - exp[-\alpha(\psi^{-1} - L^{-1})]^\mu$	α, L, μ

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How to determine the point S, C_1 and C_2 ?

➢ How to determine the point S, C_1, C_2?

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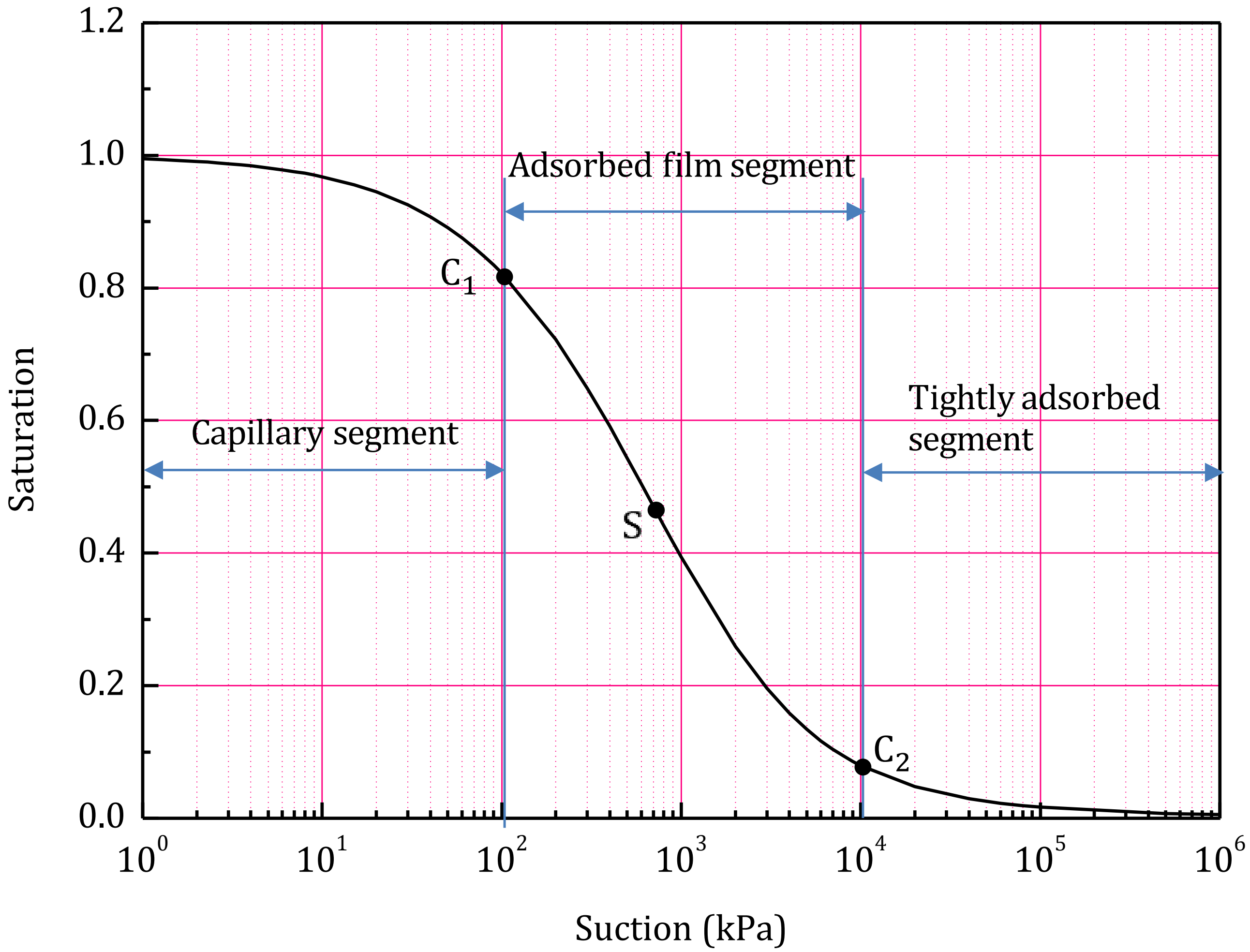
VG model

in terms of water content

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha\psi)^n]^m}$$

in terms of saturation

$$S = S_r + \frac{1 - S_r}{[1 + (\alpha\psi)^n]^m}$$



➢ How to determine the point S, C_1, C_2?

Determination S: inflection point, maximum slope

$$S' = \frac{dS}{d\lg(\psi)} = \frac{\partial S}{\partial \psi} \times \psi \times \ln 10 = -\frac{m \times n \times \left(\psi/a\right)^{n-1} \times \lg\left[e + \left(\psi/a\right)^{n}\right]^{-m-1}}{a \times \left[e + \left(\psi/a\right)^{n}\right]} \times \psi \times \ln 10$$

$$S'' = \frac{dS'}{d\lg(\psi)} = \frac{\partial S'}{\partial \psi} \times \psi \times \ln 10$$

$$= \{(1+m) \times n \times (\psi/a)^{n} + [e - e \times n + (\psi/a)^{n}] \times \lg[e + \left(\psi/a\right)^{n}]\}$$

$$\times \frac{m \times n \times \left(\psi/a\right)^{n} \times \lg\left[e + \left(\psi/a\right)^{n}\right]^{-m-2}}{\psi^2 \left[e + \left(\psi/a\right)^{n}\right]^2} \times \psi^2 \times \ln^2 10 = 0$$

➢ How to determine the point S, C_1, C_2?

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Determination of C_1, C_2: point with the maximum curvature

$$C=\frac{S''}{\left(1+S'^2\right)^{\frac{3}{2}}}=\left\{(1+m)\times n\times(\psi/a)^n+\left[e-e\times n+(\psi/a)^n\right]\times \lg\left[e+(\psi/a)^n\right]\right\}$$

$$\times\frac{m\times n\times(\psi/a)^n\times \lg\left[e+(\psi/a)^n\right]^{-m-2}}{\psi^2\left[e+(\psi/a)^n\right]^2}$$

$$\times\psi^2\times \ln^2 10\Bigg/\left\{1+-\frac{m\times n\times(\psi/a)^{n-1}\times \lg\left[e+(\psi/a)^n\right]^{-m-1}}{a\times\left[e+(\psi/a)^n\right]}\times\psi\times\ln 10\right\}^{\frac{3}{2}}$$

$$C_1\ or\ C_2=\frac{dC}{d\lg(\psi)}=\frac{\partial C}{\partial\psi}\times\psi\times\ln 10=0$$

Is there a geometric relationship between point S, C_1 and C_2 ?

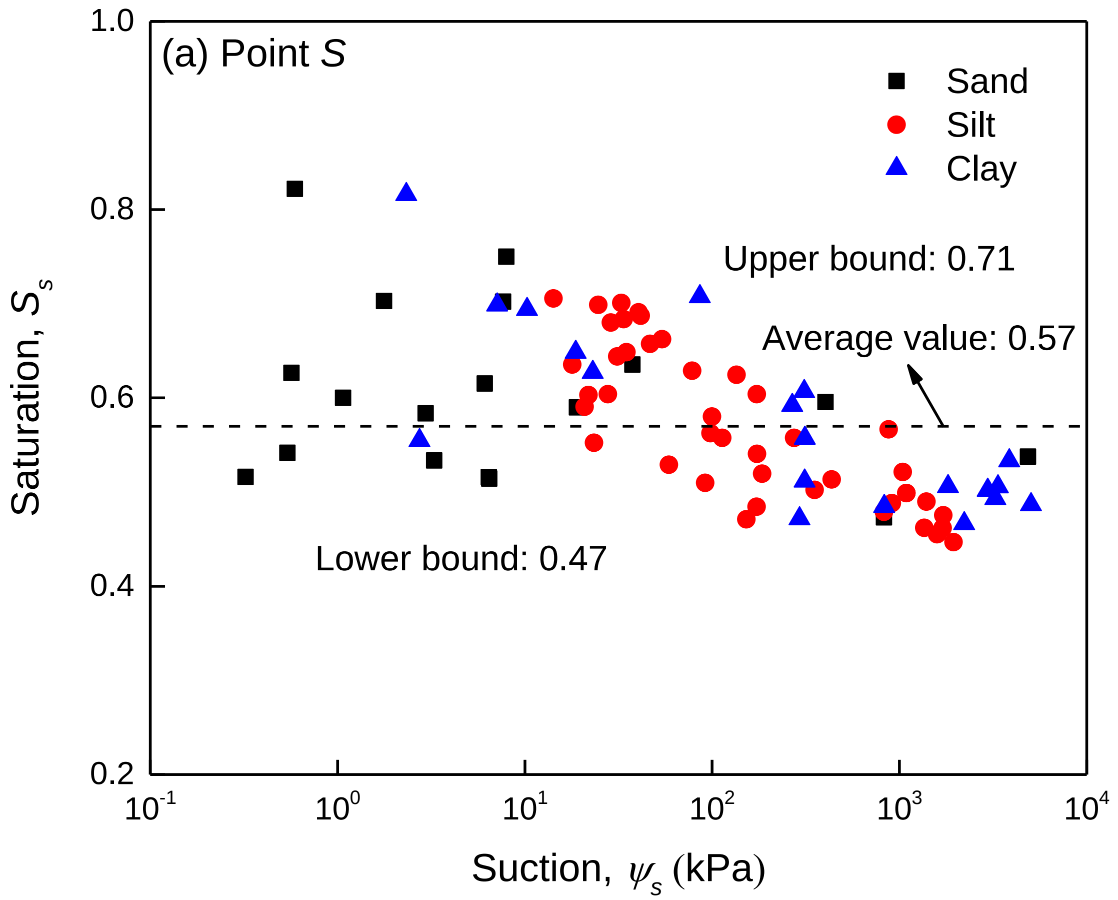
NO	Soil type	Reference	Soil name	Parameters				Inflection Point S		Point C1		Point C2		$\frac{lg\,\psi_{C_1} - lg\,\psi_S}{lg\,\psi_S - lg\,\psi_{C_2}}$	$\frac{S_{C_1} - S_S}{S_S - S_{C_2}}$
				S_r	α	m	n	$lg\,\psi_S$	S_S	$lg\,\psi_{C_1}$	S_{C_1}	$lg\,\psi_{C_2}$	S_{C_2}		
1	Sand	Bruch (1993)	fine sand	0.101	0.42	0.70	1.63	0.47	0.58	-2.9e-3	0.87	0.96	0.28	0.95	0.97
2		Fredlund and Xing (1994)	A sand	0.017	1.24	0.44	2.84	0.03	0.60	-0.32	0.91	0.44	0.22	0.83	0.84
3		Fredlund and Xing (1994)	Kidd creek tailings	0.013	4e-3	0.45	1.58	2.61	0.60	2.08	0.88	3.16	0.28	0.91	0.93
4		Dane and Hruska (1983)	A sand	0.382	2.92	0.18	3.09	-0.23	0.82	-0.55	0.95	0.13	0.67	0.86	0.91
5		Vanapalli et al. (1999)	A till	0	3.3e-4	0.70	0.76	3.69	0.54	2.81	0.83	4.58	0.24	0.98	0.99
6		Bruch (1993)	beaver creek sand	0.227	0.167	0.98	1.12	0.79	0.62	0.20	0.86	1.37	0.37	1.00	1.00
7		Vachaud (1966)	sable de rivière	0.087	0.133	1.44	2.42	0.81	0.51	0.45	0.89	1.15	0.17	1.08	1.08
8		Barbour (1998)	fine sand	0.23	1.84	0.86	3.47	-0.25	0.63	-0.52	0.92	0.04	0.32	0.96	0.96
9		Pachepsky et al. (1984)	sandy loam	0.03	0.10	0.50	1.19	1.28	0.59	0.64	0.87	1.94	0.30	0.94	0.96
10		Fujimaki and Inoue (2003)	Masa loamy sand	0.048	0.28	0.22	1.99	0.88	0.70	0.40	0.92	1.41	0.45	0.86	0.92
11		Nemes et al. (2001)	UNSODA, 1010	0.23	8.7e-3	56.05	1.40	0.81	0.52	0.37	0.83	1.17	0.27	1.26	1.21
12		Nemes et al. (2001)	UNSODA, 1020	0.259	7.5e-3	69.43	1.15	0.52	0.53	0.04	0.82	0.93	0.30	1.21	1.15
13		Nemes et al. (2001)	UNSODA, 3310	0	0.093	0.32	0.91	1.57	0.64	0.69	0.88	2.48	0.37	0.93	0.98
14		Nemes et al. (2001)	UNSODA, 2120	0.16	0.76	2.87	0.75	-0.49	0.52	-1.21	0.80	0.20	0.25	1.07	1.03
15		Nemes et al. (2001)	UNSODA, 2540	0	6.1e-3	1.35	0.44	2.92	0.47	1.64	0.78	4.19	0.17	1.02	1.00
16		Nemes et al. (2001)	UNSODA, 3132	0.17	3.14	0.20	0.50	0.90	0.75	-0.90	0.92	2.71	0.56	0.93	1.00
17		Nemes et al. (2001)	UNSODA, 3141	0.24	1.73	0.39	0.84	0.25	0.70	-0.65	0.90	1.16	0.50	0.95	0.99
18		Nemes et al. (2001)	UNSODA, 3142	0.13	1.62	1.35	2.25	-0.27	0.54	-0.64	0.89	0.08	0.22	1.06	1.06

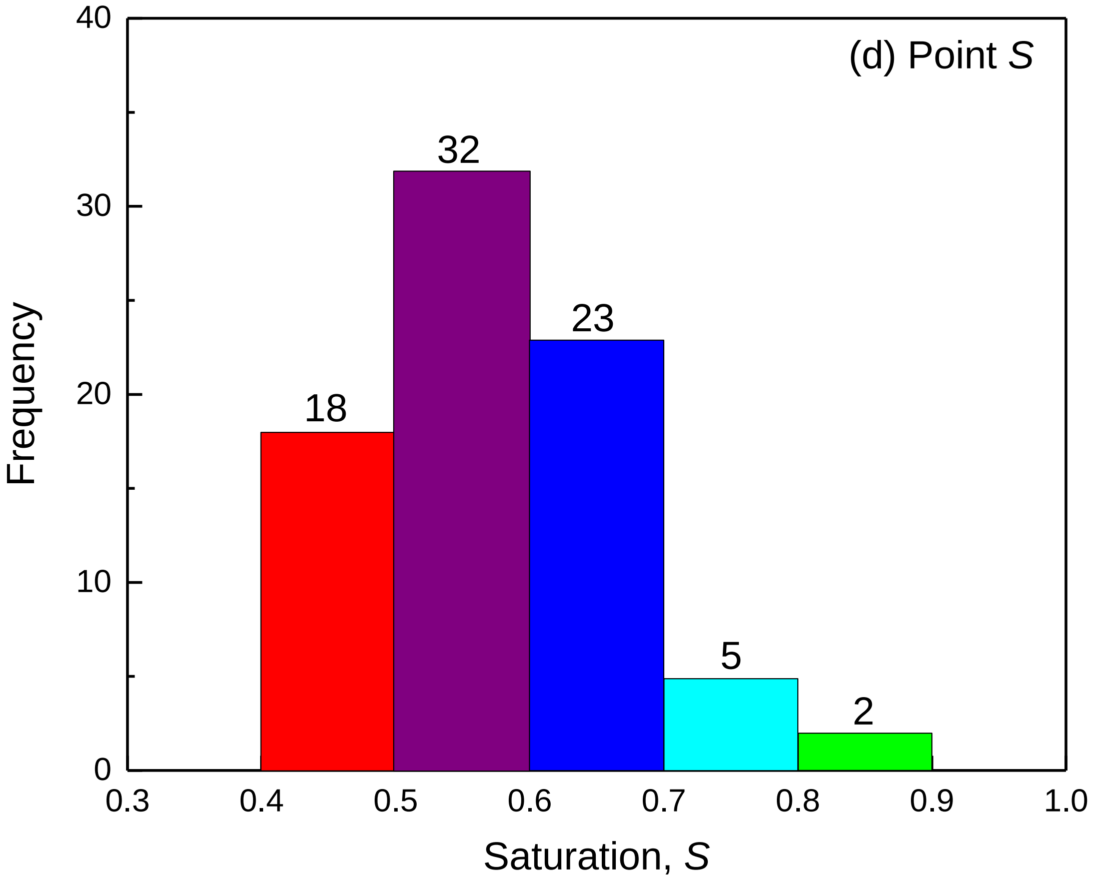
NO	Soil type	Reference	Soil name	Parameters				Inflection Point S		Point C1		Point C2		$\frac{lg\,\psi_{C_1} - lg\,\psi_S}{lg\,\psi_S - lg\,\psi_{C_2}}$	$\frac{S_{C_1} - S_S}{S_S - S_{C_2}}$
				S_r	α	m	n	$lg\,\psi_S$	S_S	$lg\,\psi_{C_1}$	S_{C_1}	$lg\,\psi_{C_2}$	S_{C_2}		
19	Silt	Fredlund and Xing (1994)	a silt	0.302	0.084	0.21	3.29	1.34	0.60	0.96	0.91	1.78	0.24	0.83	0.85
20		Brooks and Corey (1964)	a Silty Loam	0.235	0.016	0.10	15.25	1.37	0.55	0.48	0.83	2.26	0.27	0.98	0.99
21		Aubertin et al. (1998)	BE tailings	0	3.6e-4	1.56	0.91	3.23	0.46	2.55	0.80	3.90	0.14	1.04	1.03
22		Aubertin et al. (1998)	SE tailings	0.17	5.9e-8	8333.30	0.98	3.23	0.48	2.69	0.79	3.72	0.21	1.20	1.14
23		Aubertin et al. (1998)	SI tailings 0.746	0.088	3.4e-4	2.12	1.01	3.14	0.49	2.54	0.81	3.72	0.19	1.07	1.05
24		Aubertin et al. (1998)	SI tailings 0.802	0.181	2.8e-4	3.47	1.01	3.02	0.52	2.45	0.81	3.55	0.26	1.10	1.07
25		Bruch (1993)	natural silt	0.045	2.2e-3	1.11	2.28	2.64	0.51	2.26	0.89	3.01	0.15	1.02	1.02
26		Van Dam (1992)	a Silt Loam	0	0.07	0.27	1.09	1.67	0.66	0.88	0.89	2.48	0.40	0.92	0.97
27		Nemes et al. (2001)	UNSODA, 1224	0	3.9e-3	1.39	0.63	2.18	0.47	1.25	0.79	3.10	0.16	1.02	1.01
28		Nemes et al. (2001)	UNSODA, 1225	0	4.6e-3	1.18	0.73	2.24	0.48	1.40	0.80	3.07	0.17	1.01	1.01
29		Nemes et al. (2001)	UNSODA, 1230	0	0.11	0.45	0.96	1.32	0.59	0.53	0.86	2.13	0.30	0.94	0.97
30		Nemes et al. (2001)	UNSODA, 1231	0	0.094	0.41	0.94	1.44	0.60	0.62	0.87	2.28	0.32	0.94	0.97
31		Nemes et al. (2001)	UNSODA, 1280	0	0.092	0.23	1.51	1.46	0.68	0.85	0.91	2.11	0.42	0.89	0.94
32		Nemes et al. (2001)	UNSODA, 1281	0	8.9e-3	1.01	0.36	3.04	0.50	1.41	0.79	4.66	0.20	1.00	1.00
33		Nemes et al. (2001)	UNSODA, 1282	0	2.4e-3	1.56	0.39	3.13	0.46	1.75	0.77	4.51	0.16	1.02	1.01
34		Nemes et al. (2001)	UNSODA, 1330	0.043	0.055	0.24	1.71	1.62	0.69	1.08	0.91	2.20	0.43	0.88	0.93
35		Nemes et al. (2001)	UNSODA, 1331	0	8.9e-4	1.01	0.36	3.04	0.50	1.41	0.79	4.66	0.20	1.00	1.00
36		Nemes et al. (2001)	UNSODA, 1340	0	0.047	0.34	0.84	1.89	0.63	0.94	0.88	2.86	0.36	0.94	0.98
37		Nemes et al. (2001)	UNSODA, 1341	0	0.11	0.29	0.92	1.54	0.65	0.64	0.89	2.47	0.39	0.93	0.98
38		Nemes et al. (2001)	UNSODA, 1342	0	0.13	0.30	0.86	1.49	0.64	0.54	0.88	2.47	0.39	0.93	0.98

NO	Soil type	Reference	Soil name	Parameters				Inflection Point S		Point C1		Point C2		$\frac{lg\,\psi_{C_1} - lg\,\psi_S}{lg\,\psi_S - lg\,\psi_{C_2}}$	$\frac{S_{C_1} - S_S}{S_S - S_{C_2}}$
				S_r	α	m	n	$lg\,\psi_S$	S_S	$lg\,\psi_{C_1}$	S_{C_1}	$lg\,\psi_{C_2}$	S_{C_2}		
39		Nemes et al. (2001)	UNSODA, 1350	0	0.26	0.19	1.27	1.15	0.71	0.41	0.92	1.93	0.47	0.90	0.97
40		Nemes et al. (2001)	UNSODA, 1351	0	0.074	0.26	0.97	1.73	0.66	0.85	0.89	2.64	0.41	0.92	0.98
41		Nemes et al. (2001)	UNSODA, 1352	0	0.114	0.22	1.11	1.53	0.68	0.72	0.91	2.36	0.44	0.91	0.97
42		Nemes et al. (2001)	UNSODA, 1490	0	0.028	0.35	0.80	2.13	0.62	1.14	0.87	3.14	0.36	0.94	0.98
43		Nemes et al. (2001)	UNSODA, 2000	0	8.4e-4	1.13	0.47	2.96	0.49	1.73	0.79	4.18	0.19	1.01	1.00
44		Nemes et al. (2001)	UNSODA, 2001	0	7e-4	1.26	0.42	2.92	0.48	1.57	0.78	4.26	0.18	1.01	1.00
45		Nemes et al. (2001)	UNSODA, 2002	0	1.7e-4	1.70	0.41	3.20	0.46	1.88	0.77	4.51	0.15	1.03	1.01
46		Nemes et al. (2001)	UNSODA, 2010	0	0.003	0.98	0.40	2.55	0.50	1.07	0.80	4.02	0.21	1.00	1.00
47		Nemes et al. (2001)	UNSODA, 2011	0	1.1e-4	1.93	0.43	3.29	0.45	2.04	0.76	4.53	0.14	1.03	1.01
48		Nemes et al. (2001)	UNSODA, 2012	0	0.1	0.59	0.53	2.44	0.56	1.16	0.83	3.72	0.27	0.97	0.99
49	**Silt**	Nemes et al. (2001)	UNSODA, 4560	0	0.15	0.20	1.23	1.39	0.70	0.63	0.91	2.18	0.46	0.90	0.97
50		Nemes et al. (2001)	UNSODA, 4071	0	0.021	0.56	0.79	1.99	0.56	1.11	0.84	2.89	0.27	0.96	0.98
51		Nemes et al. (2001)	UNSODA, 4080	0	0.01	0.68	0.70	2.24	0.54	1.29	0.83	3.20	0.24	0.98	0.99
52		Nemes et al. (2001)	UNSODA, 3210	0	0.013	0.41	1.11	2.24	0.60	1.53	0.87	2.97	0.32	0.93	0.96
53		Nemes et al. (2001)	UNSODA, 4081	0	7.6e-3	0.83	0.56	2.27	0.52	1.15	0.81	3.39	0.22	0.99	1.00
54		Nemes et al. (2001)	UNSODA, 4182	0	0.027	0.75	0.61	1.77	0.53	0.72	0.82	2.83	0.23	0.98	0.99
55		Nemes et al. (2001)	UNSODA, 4510	0	0.076	0.21	1.38	1.61	0.69	0.93	0.91	2.31	0.45	0.90	0.96
56		Nemes et al. (2001)	UNSODA, 3380	0	0.018	0.59	0.75	2.05	0.56	1.13	0.84	3.00	0.27	0.97	0.99
57		Nemes et al. (2001)	UNSODA, 4031	0	0.22	0.32	0.83	1.25	0.64	0.28	0.88	2.24	0.37	0.93	0.98
58		Nemes et al. (2001)	UNSODA, 4670	0.018	0.011	0.99	0.71	1.96	0.51	1.09	0.81	2.84	0.21	1.00	1.00
59		Nemes et al. (2001)	UNSODA, 4671	0	0.022	0.49	0.91	2.00	0.58	1.19	0.86	2.83	0.29	0.95	0.98

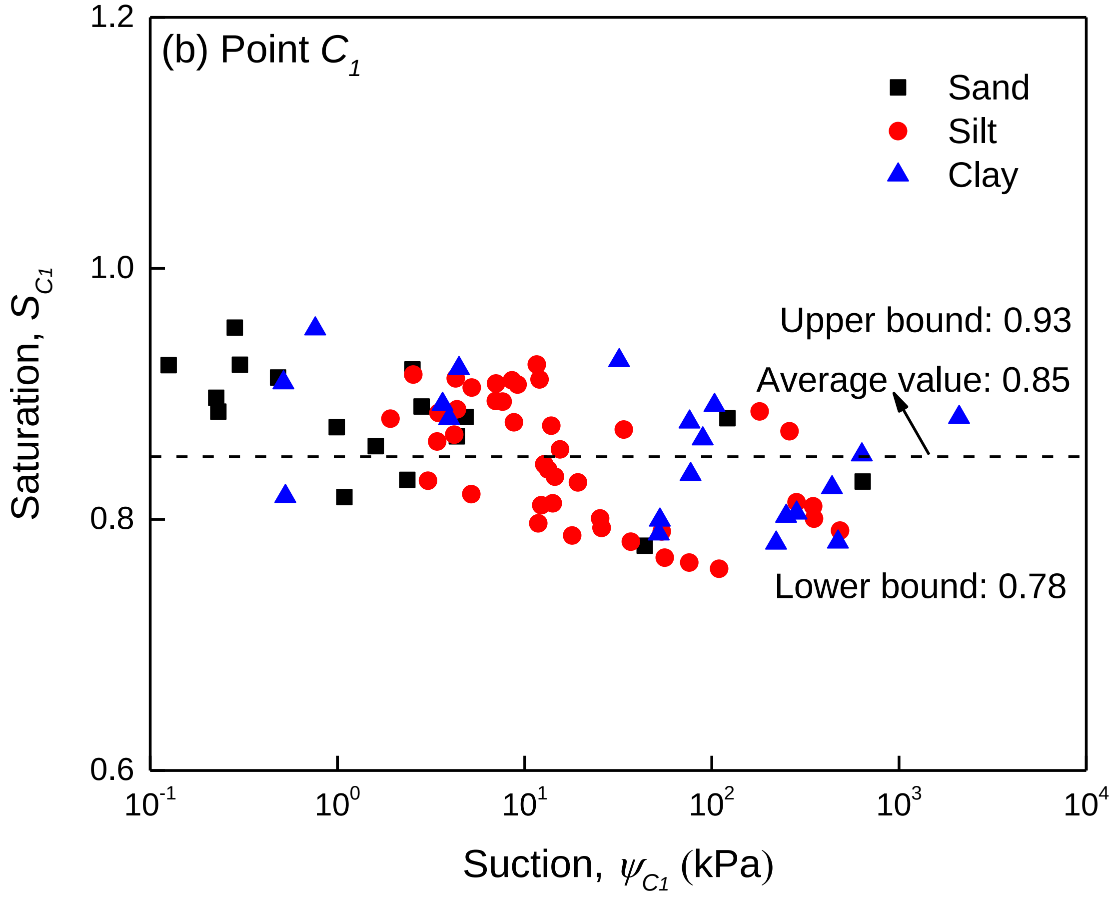
NO	Soil type	Reference	Soil name	Parameters				Inflection Point S		Point C1		Point C2		$\frac{lg\,\psi_{C_1} - lg\,\psi_S}{lg\,\psi_S - lg\,\psi_{C_2}}$	$\frac{S_{C_1} - S_S}{S_S - S_{C_2}}$
				S_r	α	m	n	$lg\,\psi_S$	S_S	$lg\,\psi_{C_1}$	S_{C_1}	$lg\,\psi_{C_2}$	S_{C_2}		
60	Clay	Chen and Gong (2014)	SC1	0.008	0.023	0.19	2.38	1.94	0.71	1.51	0.93	2.42	0.44	0.82	0.88
61		Miao et al. (2006)	GX expansive soil 1	0	2.3e-3	1.38	0.83	2.47	0.47	1.72	0.80	3.20	0.15	1.03	1.02
62		Miao et al. (2006)	GX expansive soil 3	0	6.4e-3	0.44	1.50	2.43	0.59	1.88	0.88	3.01	0.28	0.91	0.94
63		Miao et al. (2006)	HF expansive soil 1	0.004	3.5e-3	0.90	1.16	2.49	0.51	1.89	0.84	3.11	0.18	0.99	0.99
64		Miao et al. (2006)	HF expansive soil 2	0.002	4.6e-3	0.59	1.43	2.50	0.56	1.95	0.86	3.07	0.23	0.94	0.95
65		Miao et al. (2006)	HF expansive soil 3	0	5.3e-3	0.40	1.83	2.49	0.61	2.01	0.89	3.02	0.28	0.88	0.91
66		Fredlund and Xing (1994)	a till	0	4.1e-4	0.73	0.70	3.59	0.53	2.64	0.83	4.54	0.23	0.98	0.99
67		Agus (2005)	PEBSM	0	8.6e-4	1.18	0.48	2.92	0.48	1.72	0.79	4.12	0.18	1.01	1.00
68		Vanapalli et al. (1998)	Guadalix Red silty clay	0	3.5e-4	0.98	0.57	3.47	0.50	2.40	0.80	4.55	0.20	1.00	1.00
69		Ye et al. (2006)	SH clay	0.125	5.1e-4	3.07	0.62	3.51	0.49	2.67	0.78	4.33	0.22	1.06	1.03
70		Fleureau et al. (1993)	white clay	0.052	1e-4	1.62	2.24	3.70	0.49	3.32	0.88	4.05	0.13	1.10	1.10
71		Fleureau et al. (1993)	white clay	0.25	1.09	0.13	2.19	0.37	0.82	-0.12	0.95	0.87	0.66	0.88	0.96
72		Fleureau et al. (1993)	yellow clay	0.19	0.17	1.83	0.79	0.44	0.55	-0.28	0.82	1.14	0.30	1.04	1.02
73		Vanapalli et al. (1999)	a clay	0	3e-4	0.94	0.57	3.53	0.51	2.45	0.81	4.60	0.21	1.00	1.00
74		Mualem (1976b)	pachapa fine sandy clay	0.17	0.15	0.33	2.56	1.01	0.69	0.64	0.92	1.42	0.43	0.85	0.88
75		Brooks and Corey (1964)	beit netofa clay	0	2.3e-4	1.47	0.57	3.35	0.47	2.34	0.78	4.34	0.16	1.02	1.01
76		Nemes et al. (2001)	2690	0	1.05	0.20	0.80	0.85	0.70	-0.29	0.91	2.01	0.47	0.92	0.99
77		Nemes et al. (2001)	2392	0	0.12	0.34	1.06	1.36	0.63	0.60	0.88	2.15	0.35	0.92	0.97
78		Nemes et al. (2001)	2391	0	0.15	0.29	1.20	1.27	0.65	0.56	0.89	2.01	0.38	0.91	0.96

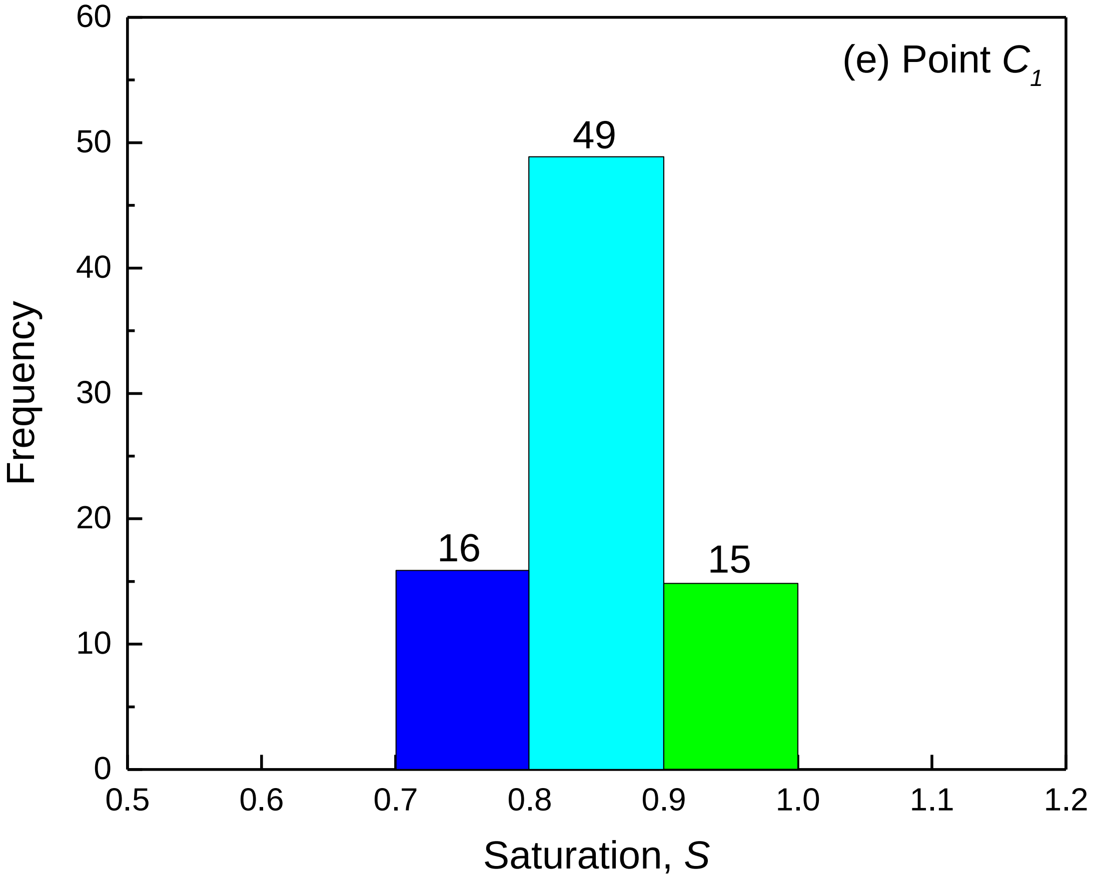
➢ The statistics on the values of S



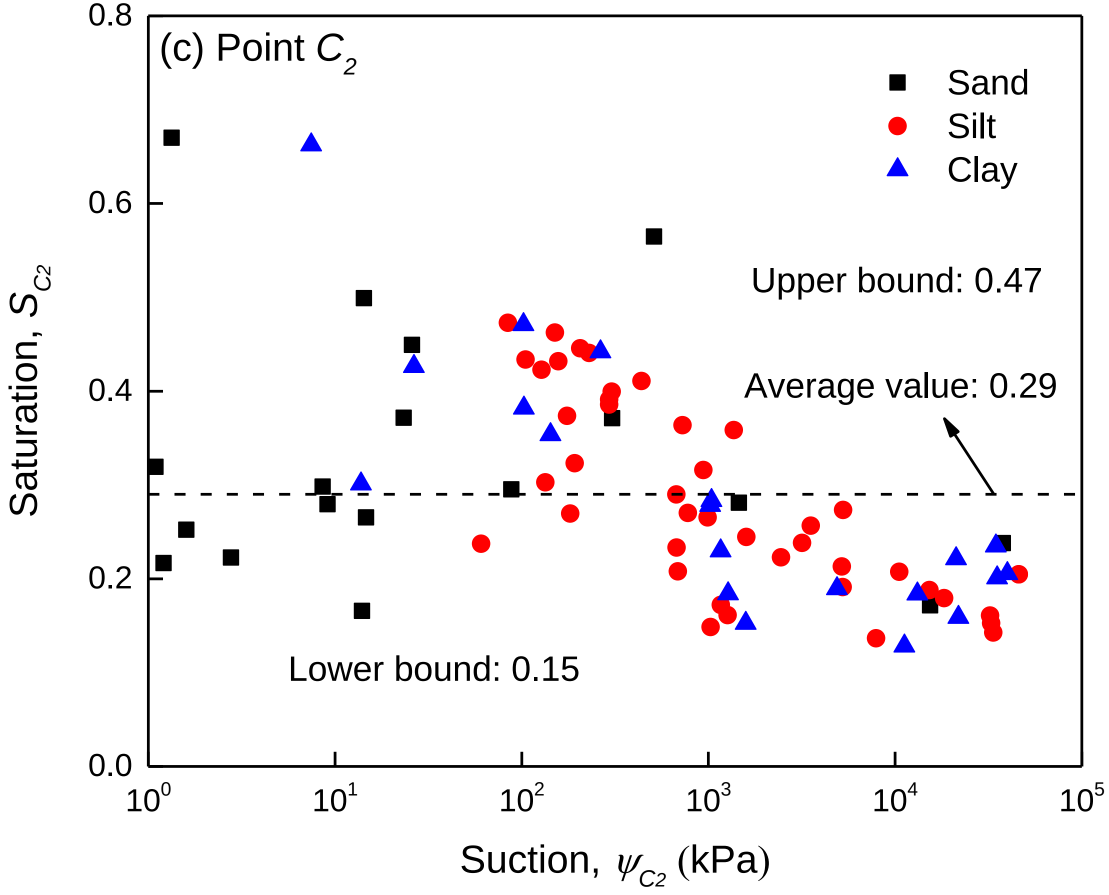


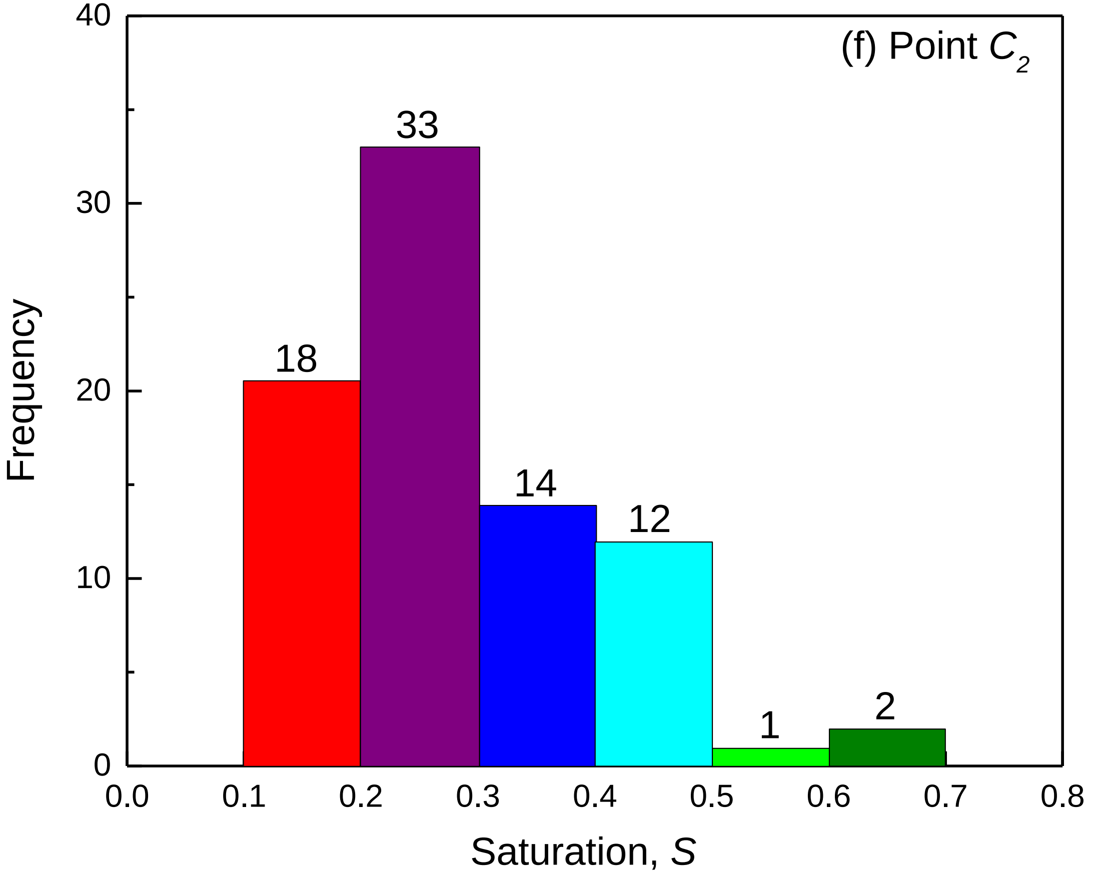
➢ The statistics on the values of C_1



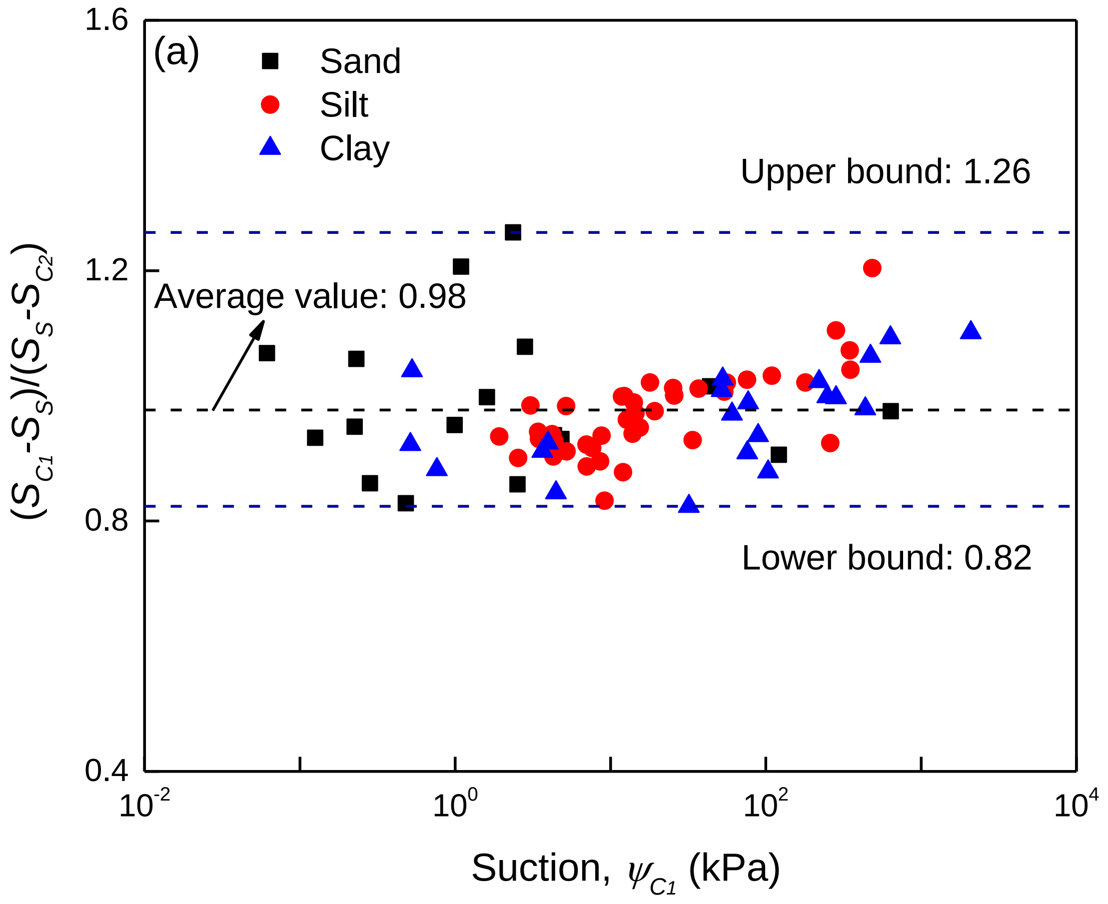


➢ The statistics on the values of C_2

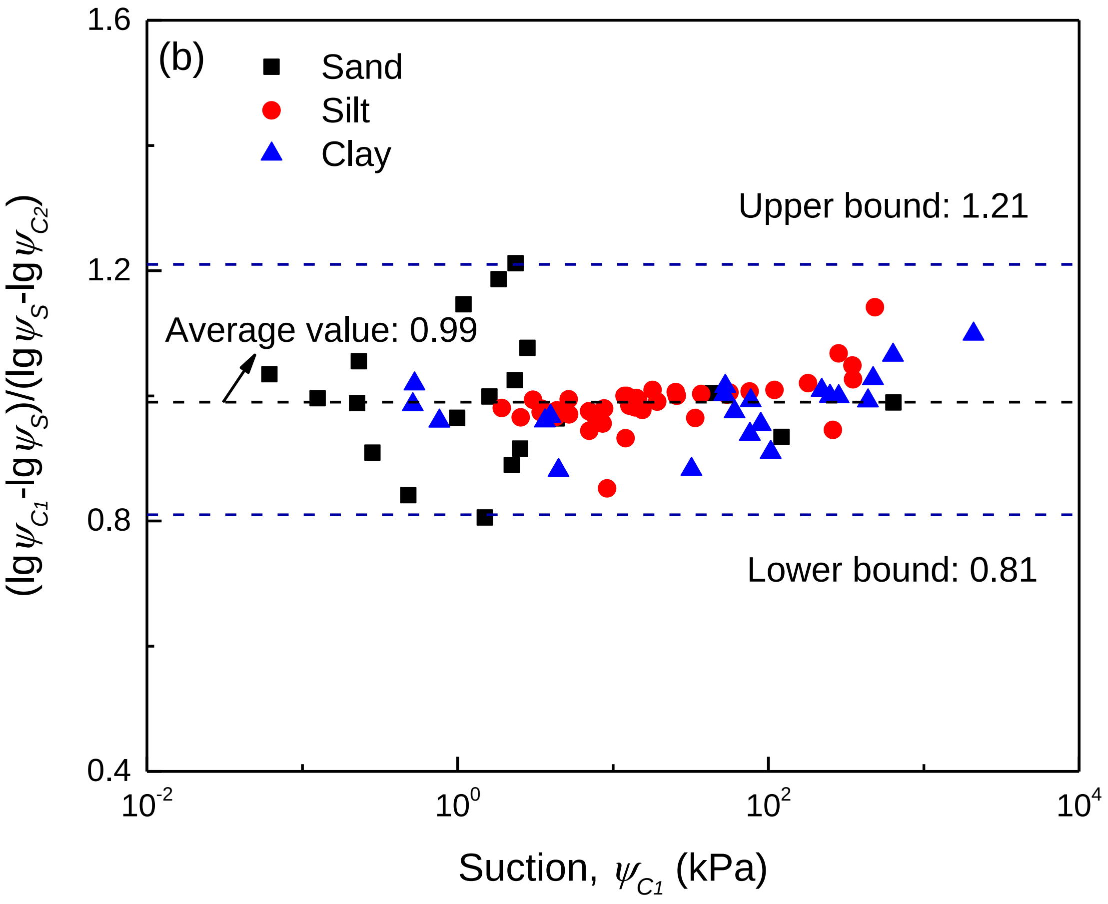




➢C_1 and C_2 are symmetric about the center of point of S.



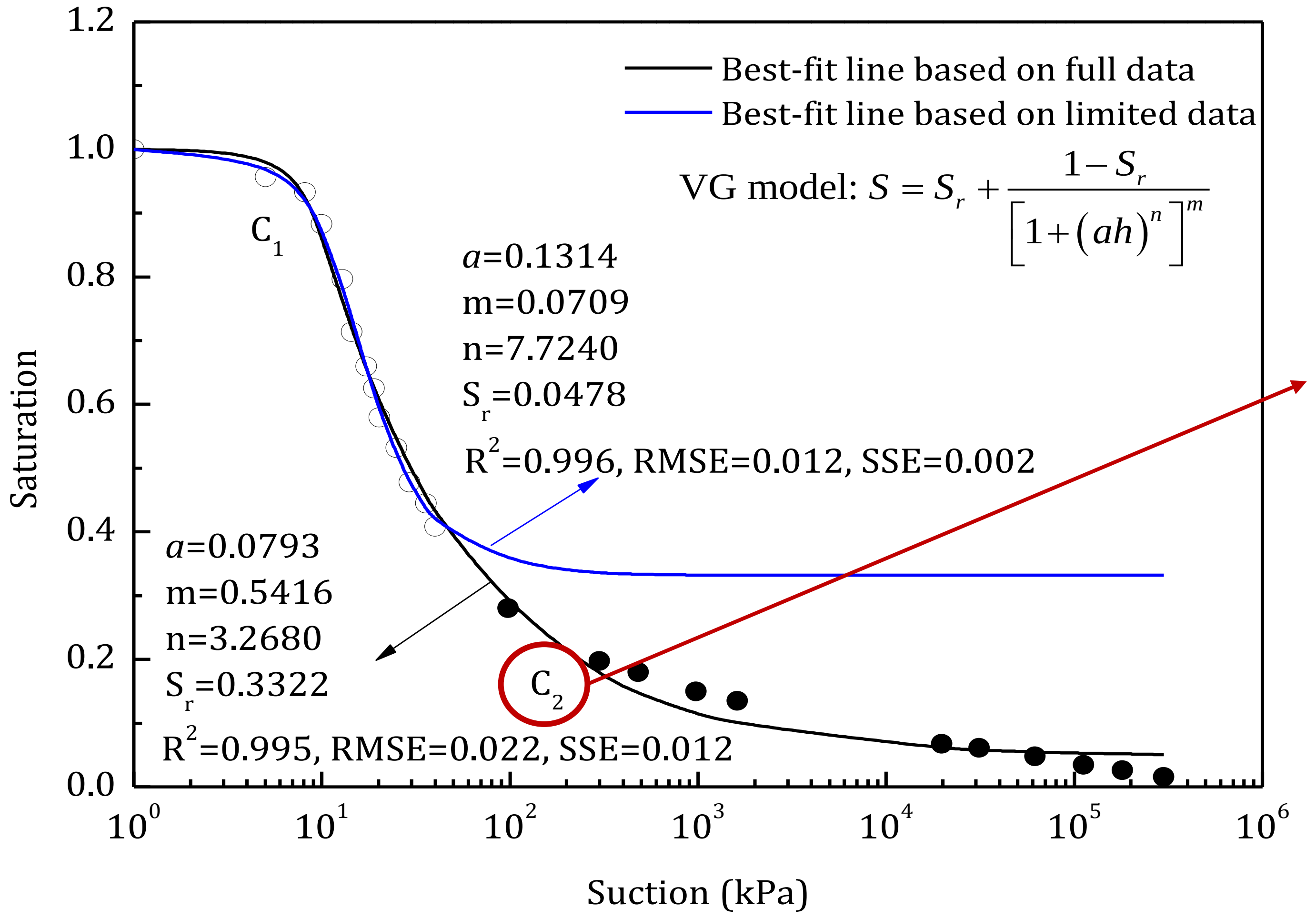
The values of $\frac{S_{C_1}-S_S}{S_S-S_{C_2}}$ of different soils.



The values of $\frac{\lg\psi_{C_1}-\lg\psi_S}{\lg\psi_S-\lg\psi_{C_2}}$ of different soils.

Complete data or Limited data?

➢ Complete data or Limited data?



Without C_2

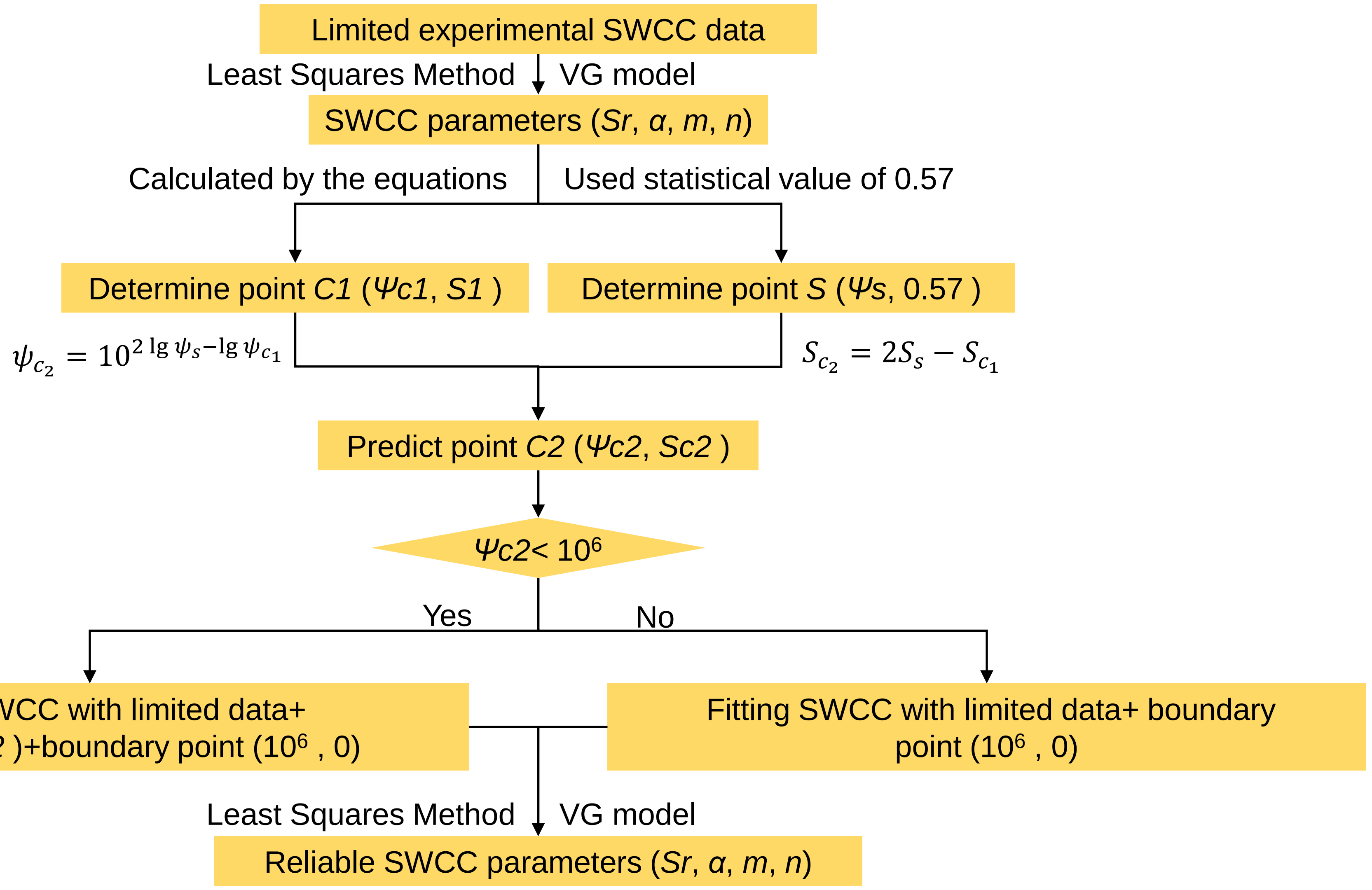
can not get residual state

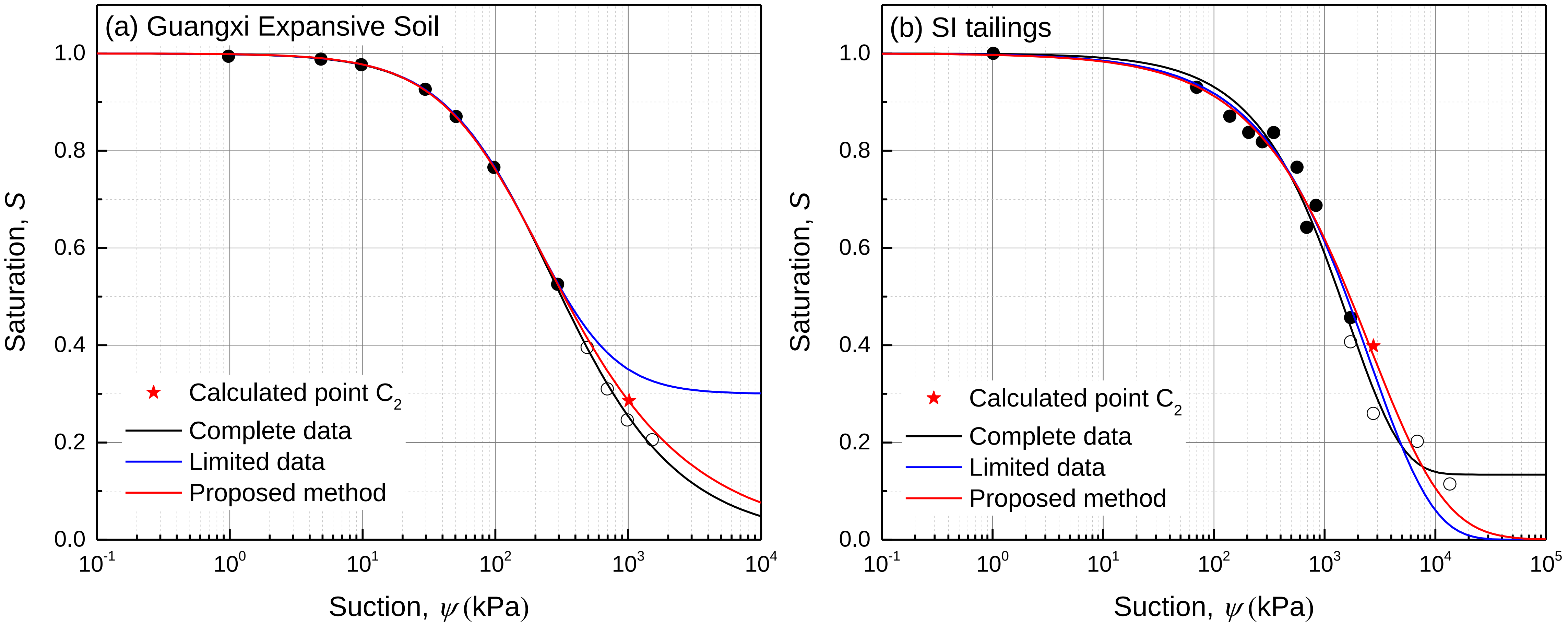
Comparison of VG model fitting to full data and limited data.

艰苦樸素 求真务實 温家寶

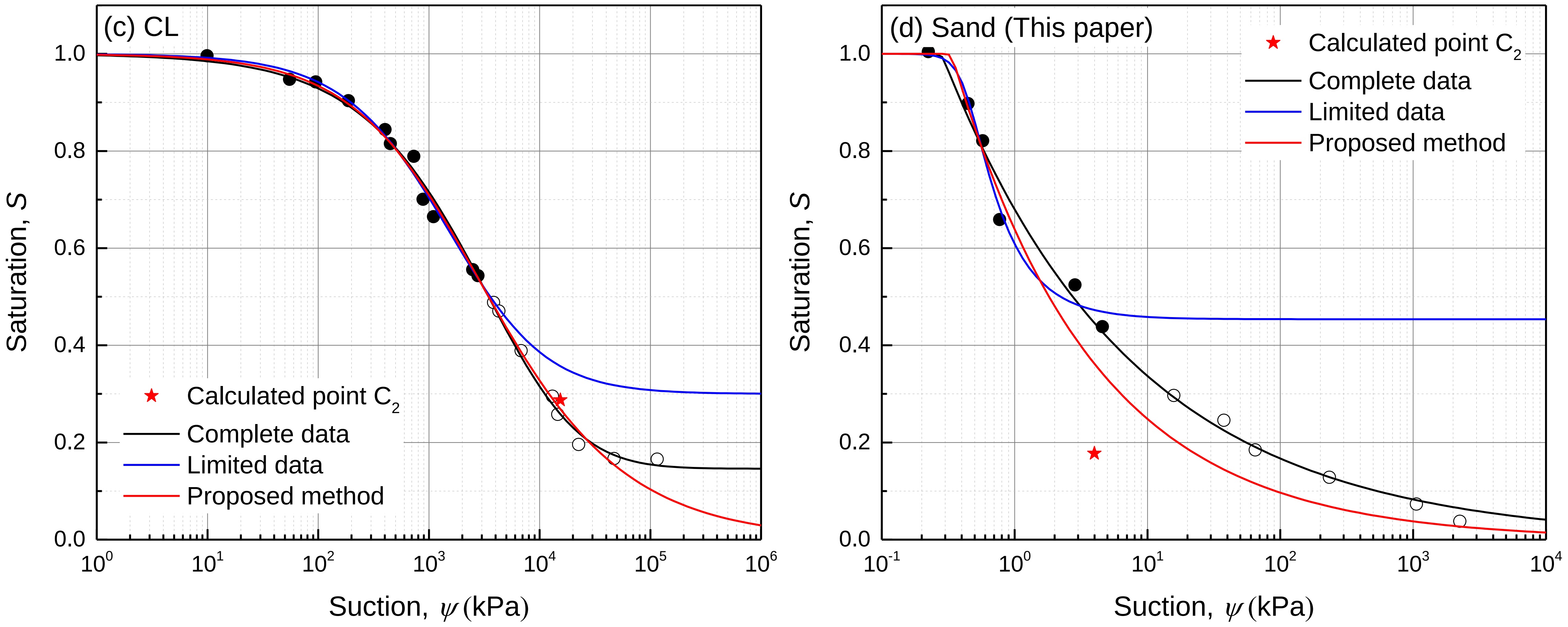
How to estimate of SWCC parameters with **limited** experimental data?

➢How to estimate of SWCC parameters with limited experimental data?

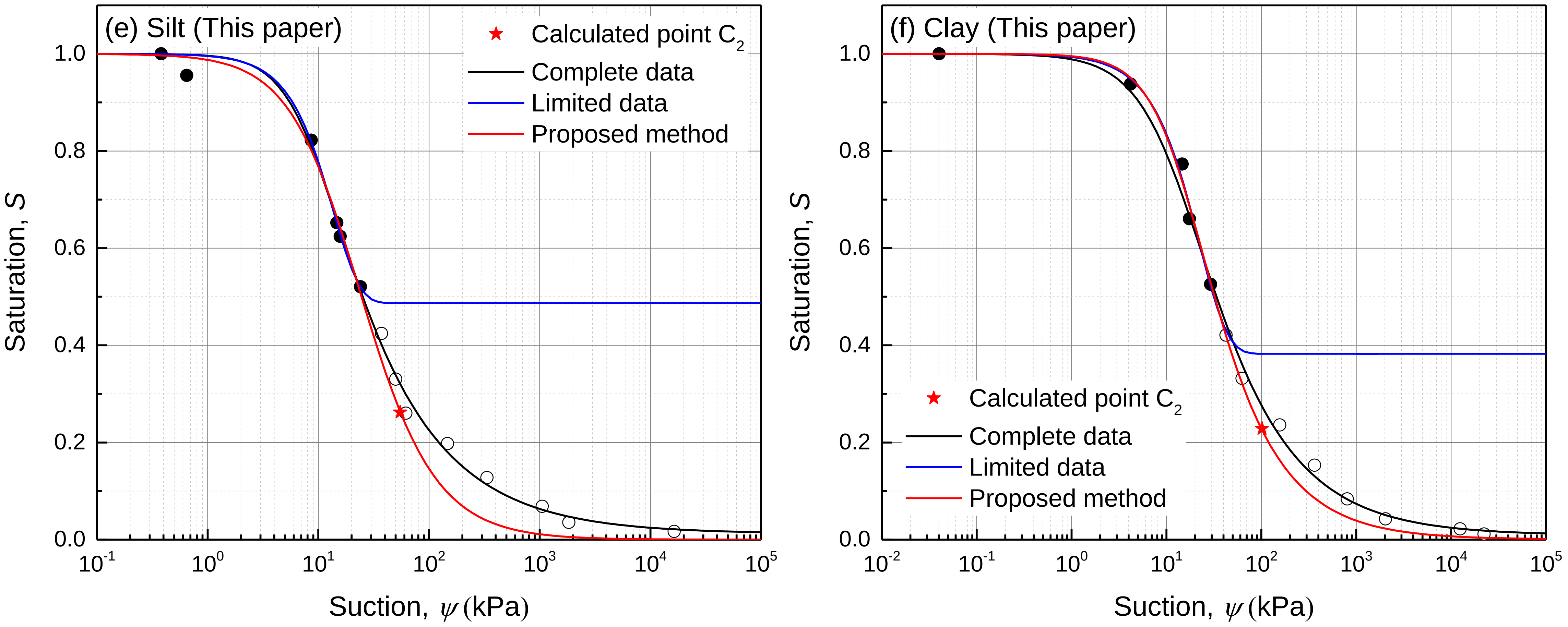




Comparison of best-fitting SWCC with complete data, limited data and proposed method



Comparison of best-fitting SWCC with complete data, limited data and proposed method



Comparison of best-fitting SWCC with complete data, limited data and proposed method

➢Summary

(1) C_1 and C_2 are symmetric about the center of point of S. Thus, the point C_2 can be obtained from the point S and C_1.

(2) A statistical datum $S_S = 0.57$ was obtained.

(3) A new method to estimate SWCC parameters was proposed in this paper. The above results show that proposed method presents a good capability to estimate SWCC parameters of fine-grained soils, whose residual state is difficult to be achieved in laboratory.

X.W., Ren, J. Kang, J.Ren, X. Chen, M. Zhang, 2020. A method for estimating soil water characteristic curve with limited experimental data. Geoderma, 360, 114013. https://doi.org/10.1016/j.geoderma.2019.114013

➢Hydraulic Conductivity Curve (K)

FXWJD Model:

$$K = K_S S_n^l \left[1 - \left(1 - \Gamma^{1/m}\right)^{1-1/n}\right]^2$$

$$\Gamma(h) = \left(\ln\left(e + |\alpha h|^n\right)\right)^{-m}$$

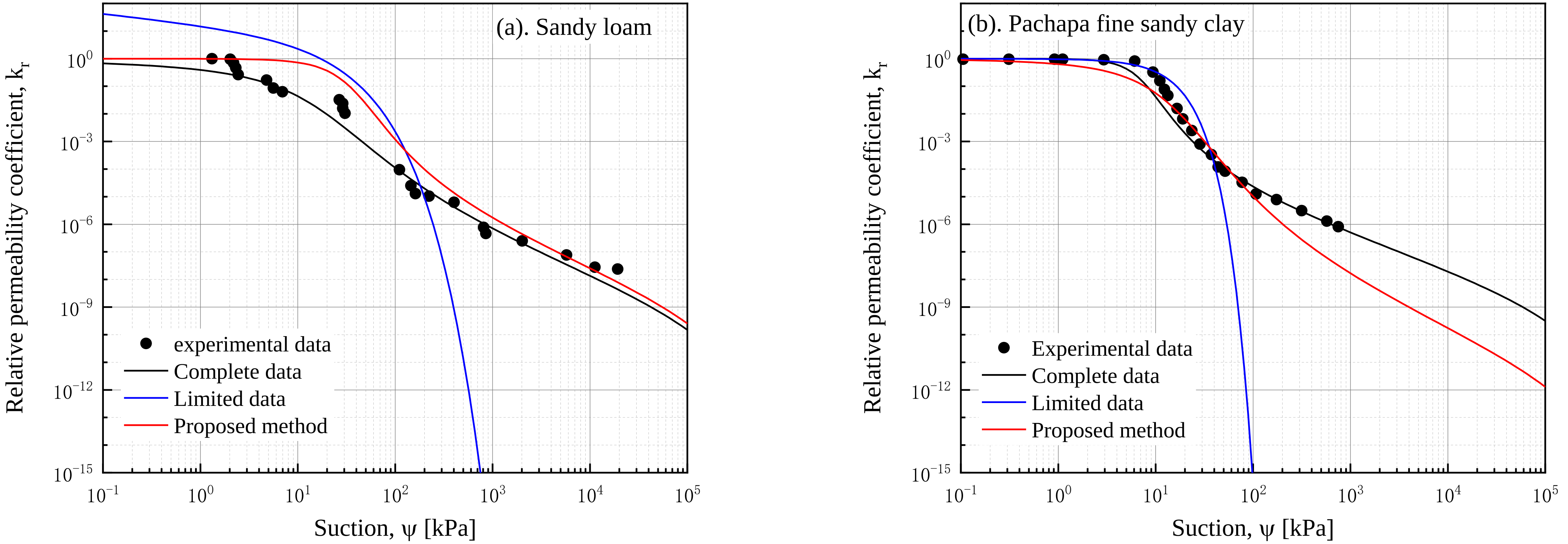
$$S_n(h) = \frac{\Gamma(h) - \Gamma(h_0)}{1 - \Gamma(h_0)}$$

h_0=630000kPa

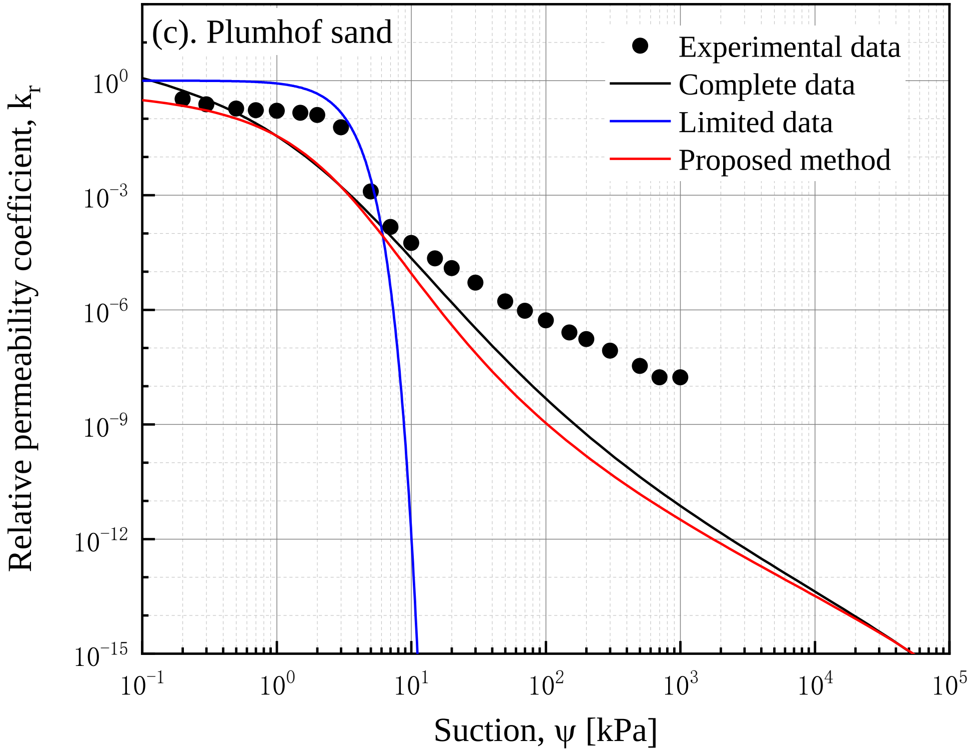
$l = 3.5$

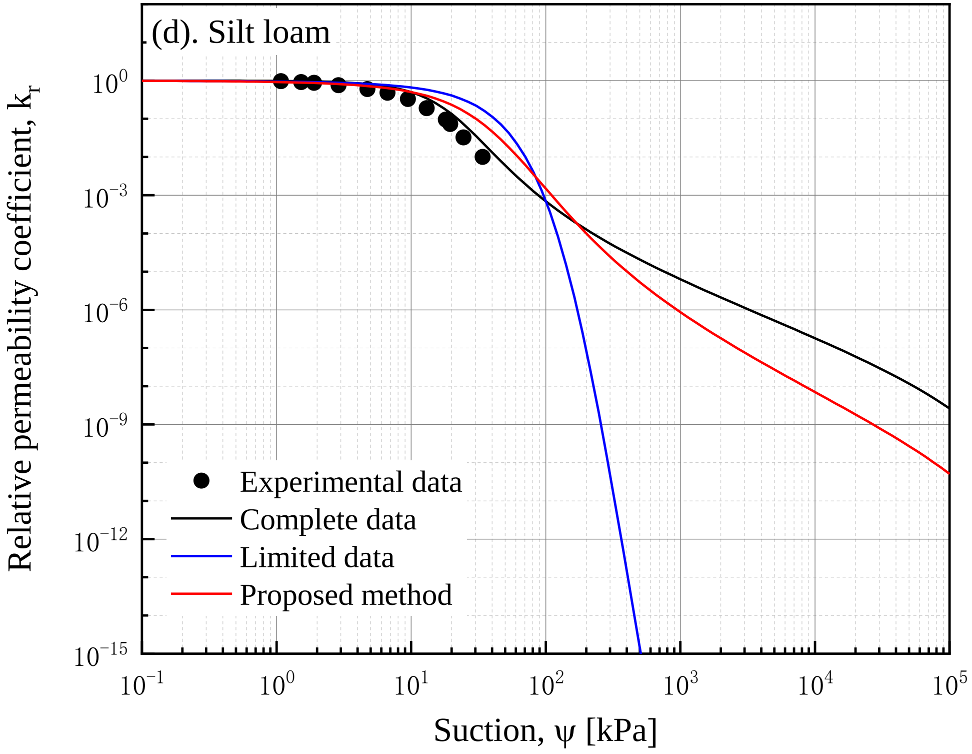
➢Hydraulic Conductivity Curve (K)

艰苦樸素 求真务实



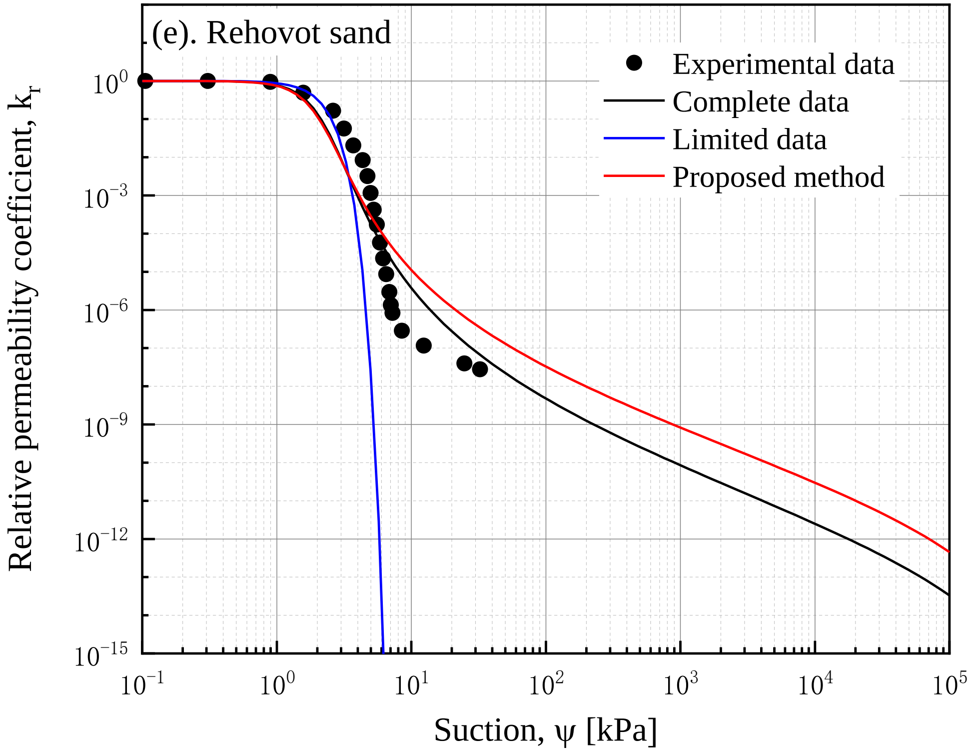
➢Hydraulic Conductivity Curve (K)

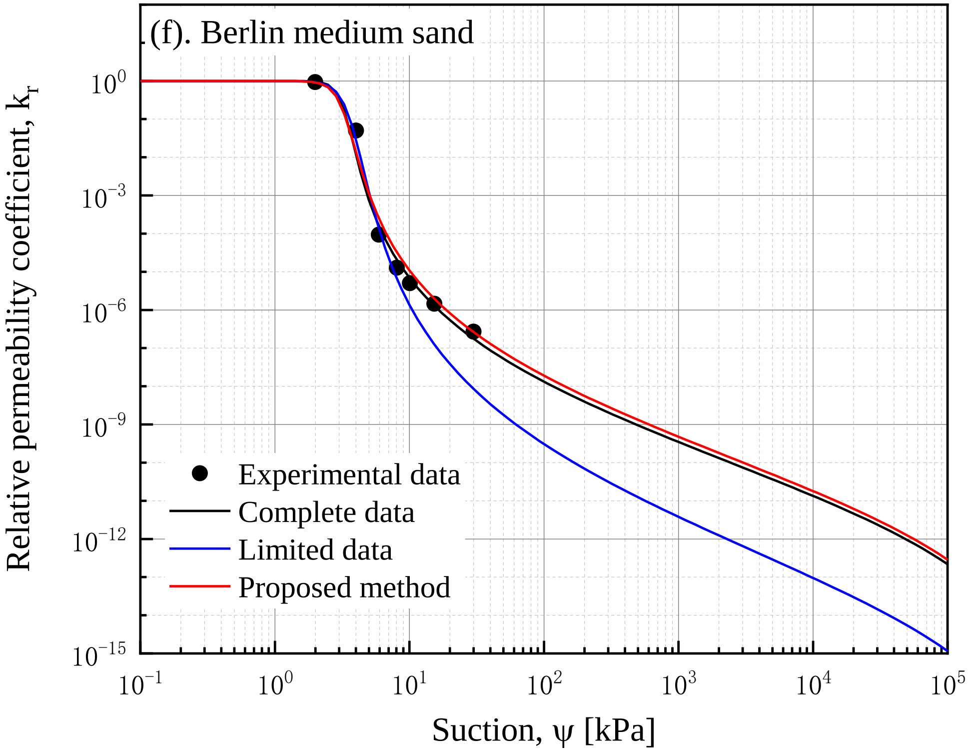




➢Hydraulic Conductivity Curve (K)

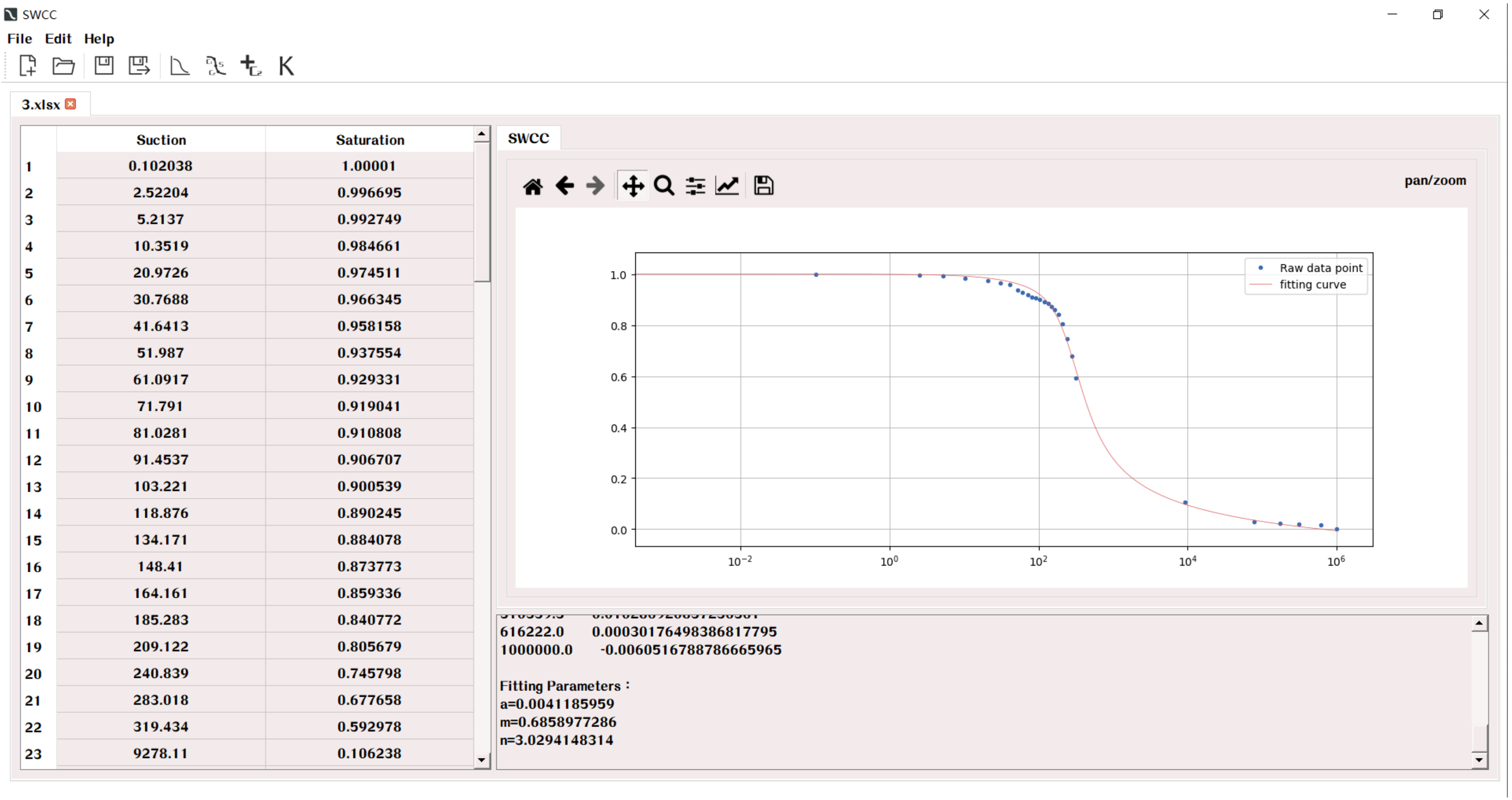
艰苦樸素 求真务實





➢Application

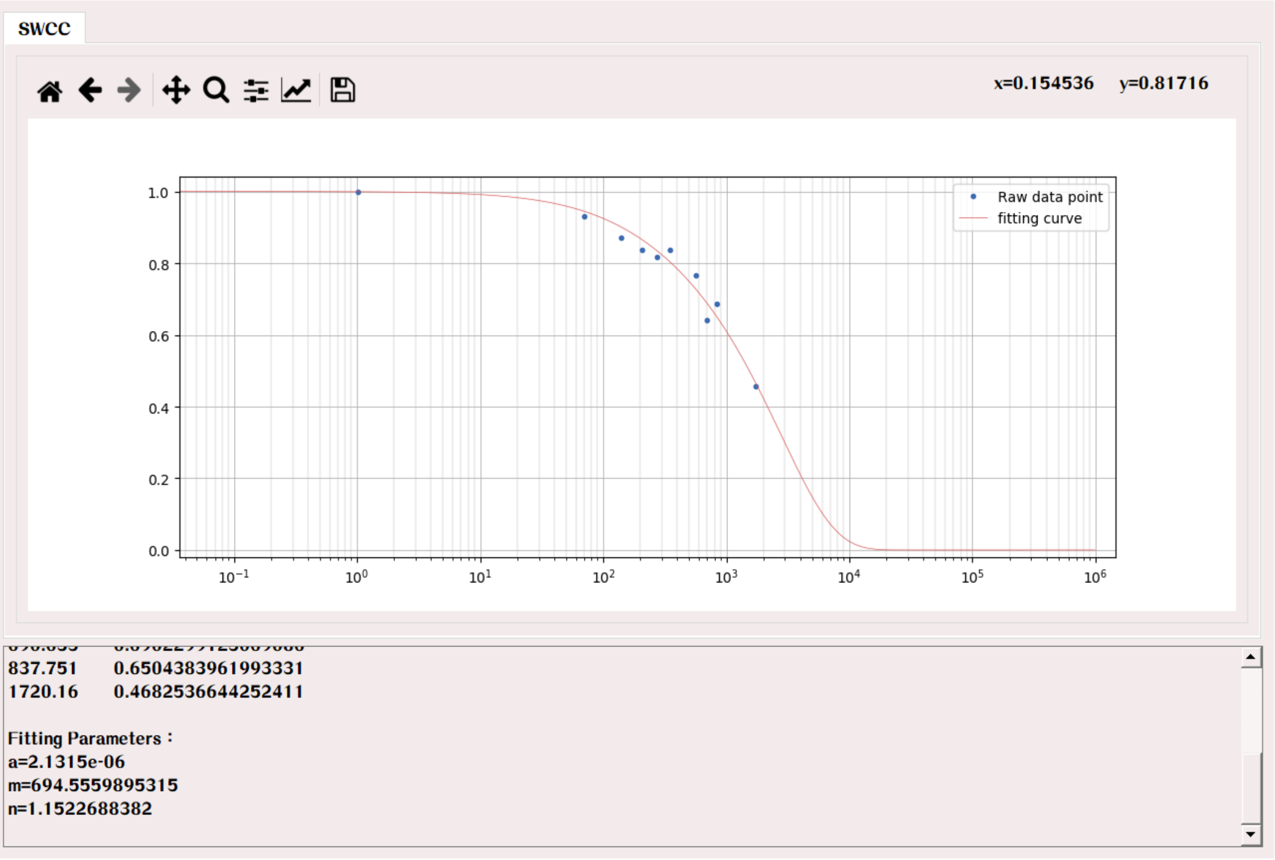
SWCC



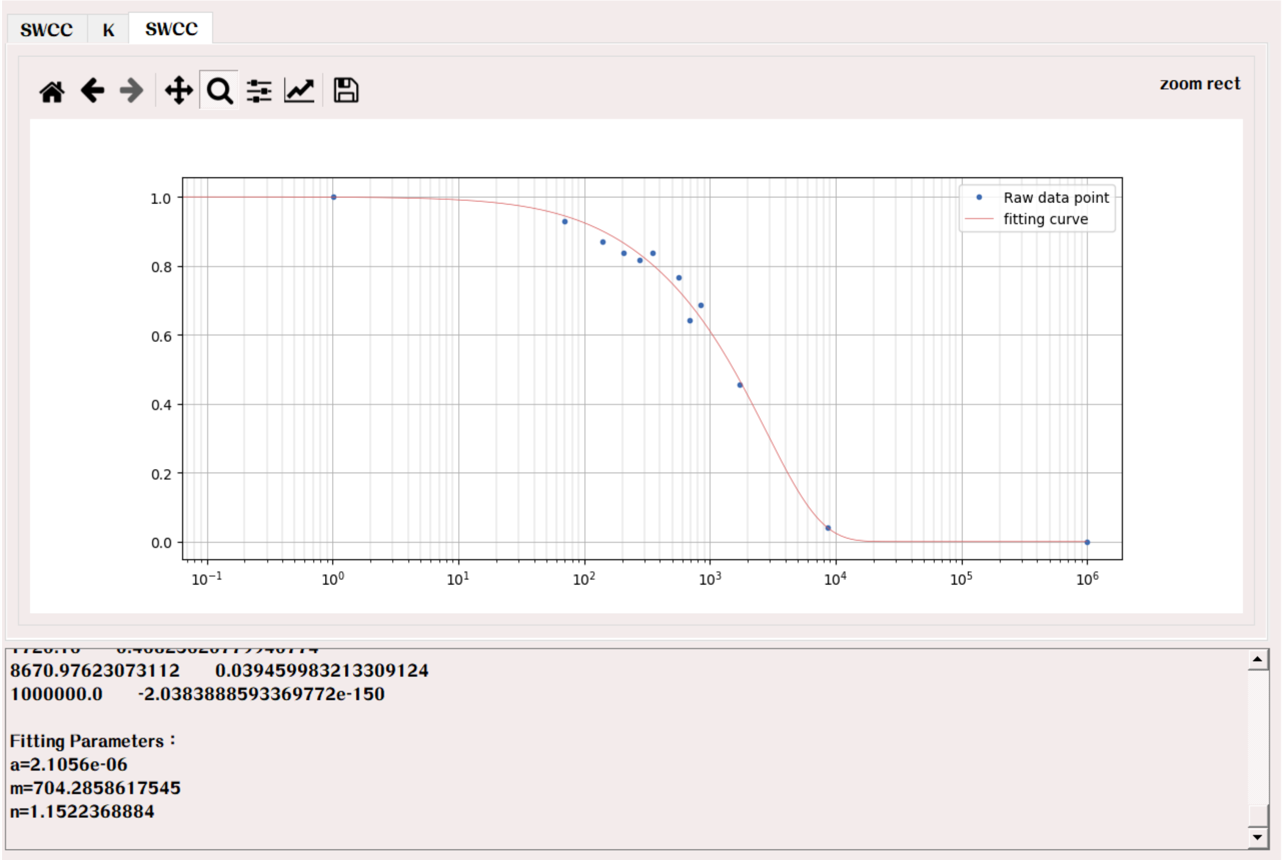
Self-developed application software

➢Self – developed application software

Soil Water Characteristic Curve Fitting



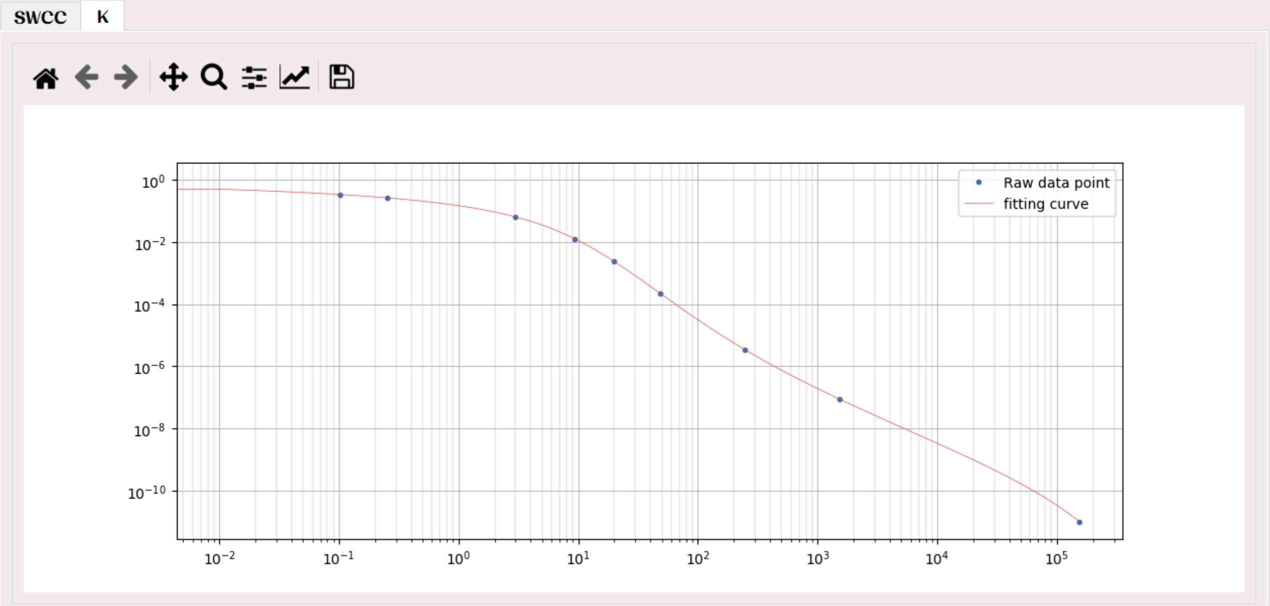
Limited data



Proposed method

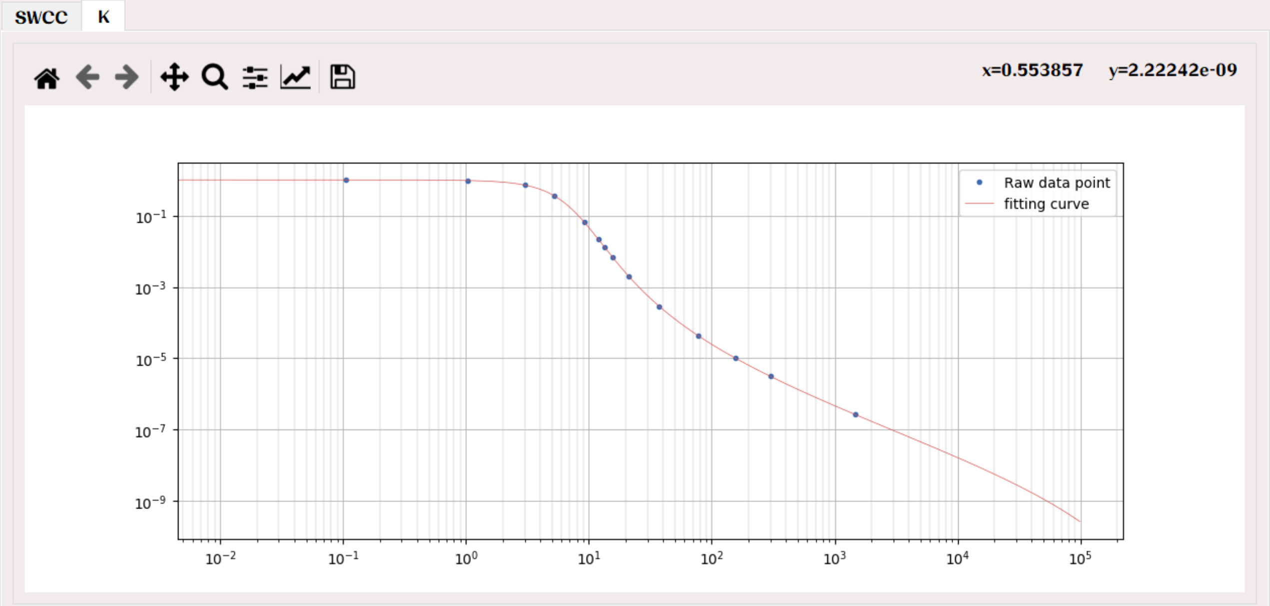
➤Self – developed application software

Hydraulic Conductivity Curve fitting with completed data



0.25486	0.262787438242387
2.98054	0.0644813663933114
9.40732	0.0125104466032872
19.8842	0.00239268181263337
48.038799999999995	0.000226943800949706
245.31199999999998	3.47819310282938e-6
1510.44	8.88846532994774e-8
153955.9	9.93644495187496e-12

a) sandy loam



13.361	0.0132828344168695
15.7148	0.00659618404486639
21.159399999999998	0.00197566455504642
37.3377	0.000288130740769960
77.49289999999999	4.31749399978894e-5
156.542	1.00183242443306e-5
299.57800000000003	3.13354002462430e-6
1477.28	2.64693011656872e-7

b) Pachapa fine sandy clay

SWCC

File Edit Help

Watch a tutorial video of the software of SWCC.

Please don't hesitate to contact us if you need the software. It's free.

录屏开始