Mountain waves produced by a stratified shear flow with a boundary layer : transition from downstream sheltering to upstream blocking

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Motivation for an academic approach of the problem

Renewed interest on the interaction between BL and mountain flows
(see discussions around TEAM-x)

Old theoretical literature on the topics is notoriously involved
(Hunt et. 1988, Belcher and Wood 1996),and not much since then(?)

Predict mountain drag but do not treat where it is deposited

Academic cases permit to control the minimal dynamical ingredients needed to produce important phenomena like the transitions :

Neutral → Stratified
Form drag → Wave drag
Downstream sheltering → Upstream blocking

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With constant kinematic (ν) and thermal (κ) difffusions and for flat terrain, the boundary layer flow with constant shears is solution, the boundary layer depth is infinite ! :

Sykes (1978)

$$u_0(z)=u_{0z}z\,;\,\rho_0(z)=\rho_r+\rho_{0z}z$$

A mountain height

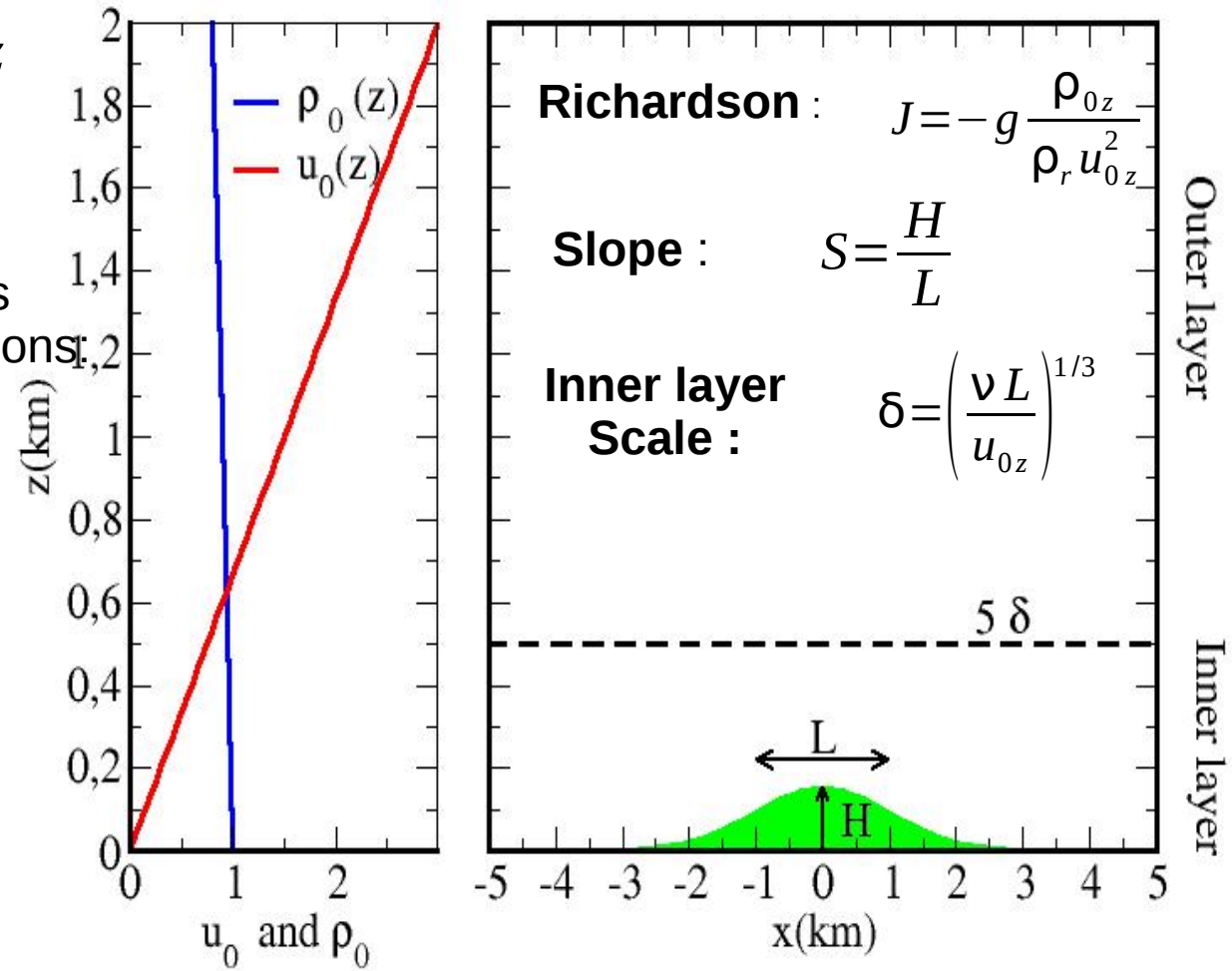
$$h(x)=H\,e^{-x^2/(2L^2)}$$

produces disturbance we express linearly and in terms of Fourier solutions :

$$w'(x,z)=\int_{-\infty}^{+\infty}\hat{w}(k,z)e^{ikx}dk$$

3 no slip boundary conditions :

$$u_{0z}h+u'(x,h)=0$$
$$w'(x,h)=0$$
$$\rho_{0z}h+\rho'(x,h)=0$$



At δ, advection of disturbance equals dissipation (here $u_0\partial_x\approx\nu\partial_z^2$)

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Outer layer:

Exact inviscid solution $\hat{w}_I(k,z) = i\sqrt{\frac{\pi k z}{2}} H^{(1)}_{i\mu}(ikz)$

Evanescent when z → ∞

$$\hat{w}_I \underset{z\to\infty}{\approx} e^{-k\ z}$$

All harmonics are trapped !

Matching Function when z → 0:

$$\hat{w}_I \underset{z\to 0}{\approx} \hat{w}_M = \hat{a}_1 z^{1/2-i\mu} + \hat{a}_2 z^{1/2+i\mu}$$

Inner layer:

$$z = \delta\tilde{z}, \quad \hat{w} = k\delta\tilde{w}, \quad \delta = \left(\frac{\nu}{k u_{0z}}\right)^{1/3}$$

Six viscous solutions are tabulated, 4 are enough to satisfy the boundary conditions:

$$\tilde{w}_{12} \underset{\tilde{z}\to\infty}{\approx} \tilde{a}_1 \tilde{z}^{1/2-i\mu} + \tilde{a}_2 \tilde{z}^{1/2+i\mu}, \qquad \tilde{w}_3 \underset{\tilde{z}\to\infty}{\approx} \tilde{z}^{-5/4} e^{\frac{-2\sqrt{i}}{3}\tilde{z}^{3/2}}, \quad \tilde{w}_4 \underset{\tilde{z}\to\infty}{\approx} \tilde{z}^{-9/4} e^{\frac{-2\sqrt{iP}}{3}\tilde{z}^{3/2}}$$

Matches $\hat{W}_M$ exactly

Decay exponentially fast when $\tilde{z} \to \infty$

Uniform Approximation:

$$\hat{w}(k,z) = f_{12}(k)\left(\hat{w}_I(z) - \hat{w}_M(z)\right)$$
$$+ k\delta\left[f_{12}(k)\tilde{w}_{12}(z/\delta) + f_3(k)\tilde{w}_3(z/\delta) + f_3(k)\tilde{w}_3(z/\delta)\right]$$

$f_{12}(k), f_3(k), f_4(k)$ evaluated via inversion of the 3 boundary conditions

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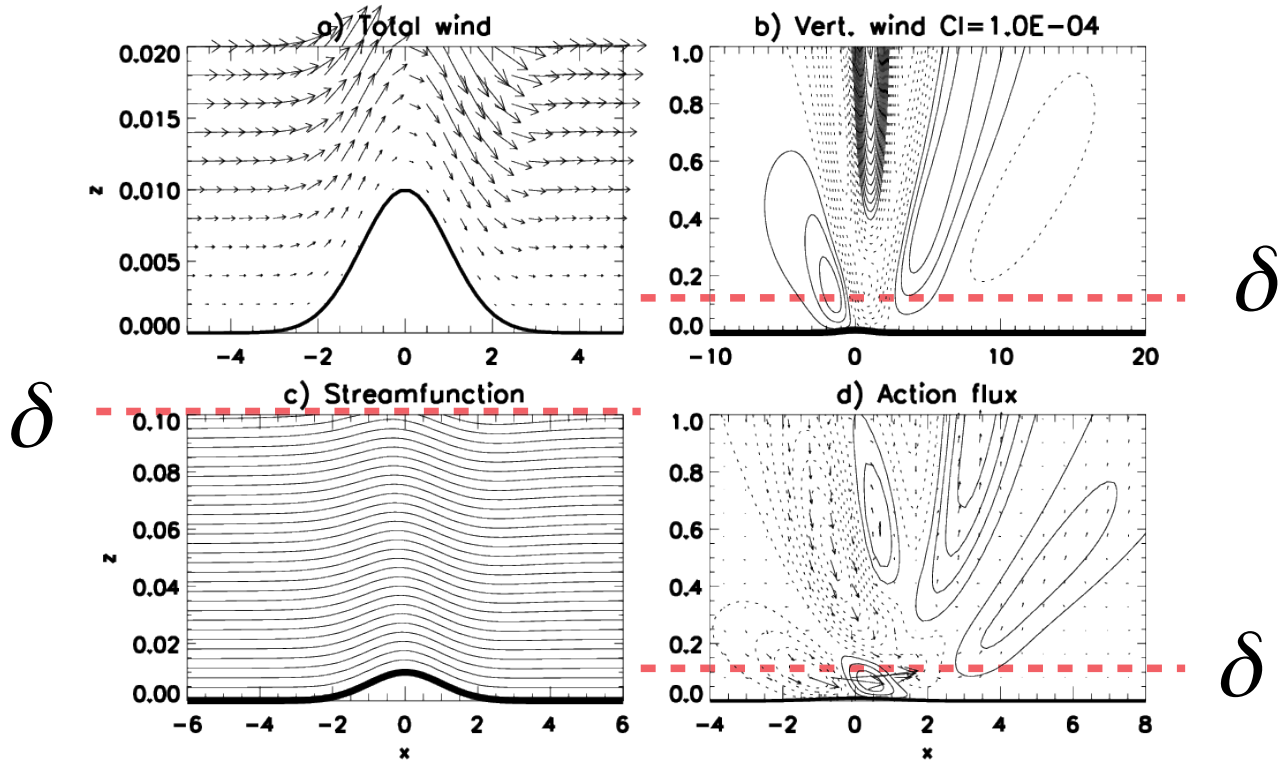
Mountain well in the inner layer :

$$\delta = 0.1\,L, \quad H = 0.01\,L \ll \delta$$

$$(S = 0.01), J = 4$$



The flow follows well the mountain up to at least $z = \delta$

Well defined system of vertically propagating gravity waves aloft $z = \delta$

Vertical component of action flux ($\rho_r u' w'$) :
The action flux comes from within the boundary layer (rather than from the surface in the inviscid case, (Durran 95, Lott 98))

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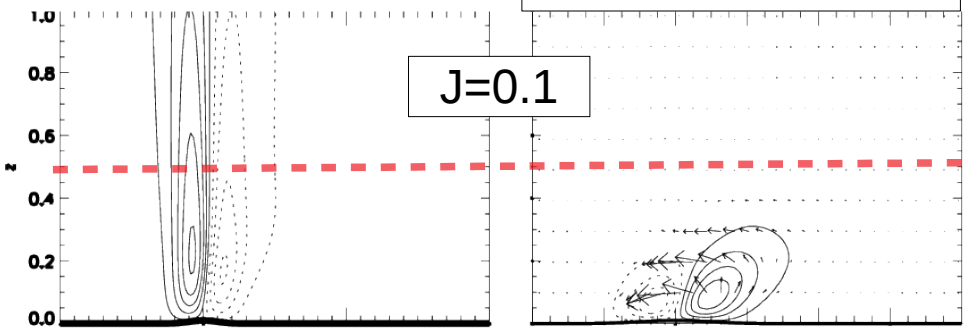
Vertical Wind

Action Flux
(vector and vert. comp)

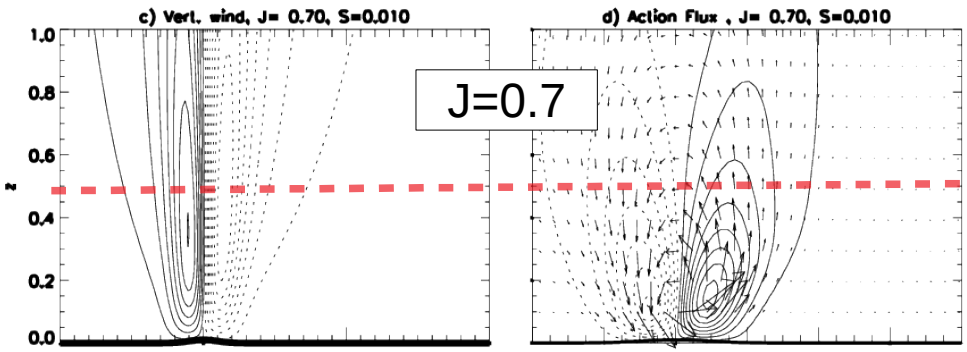
Near neutral case :
no waves propagate
aloft the BL



5δ

Action flux aloft
the BL is nul

Weakly stratified:
Wave field symetric but
significant

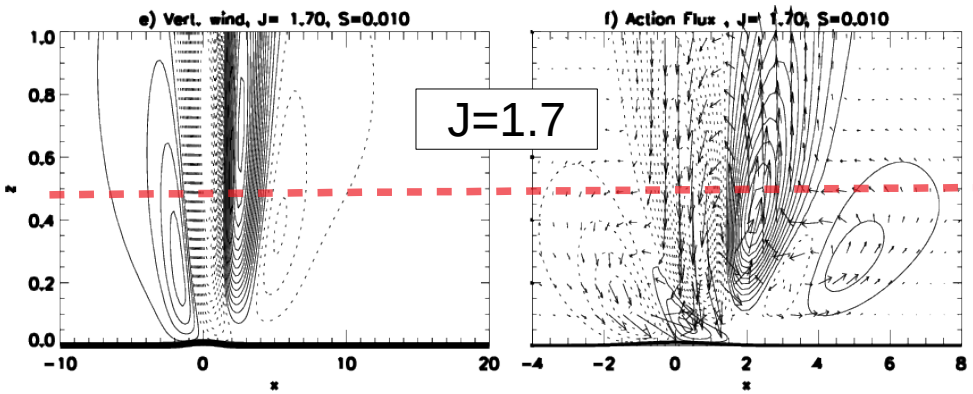


5δ

Some action flux
aloft the BL

But positive Fz
balance
Negative Fz

Stratified:
Upward waves aloft
the mountain
Downward propagating
waves downstream



5δ

Strong action flux
aloft the BL

Negative Fz over the

positive Fz downstrea

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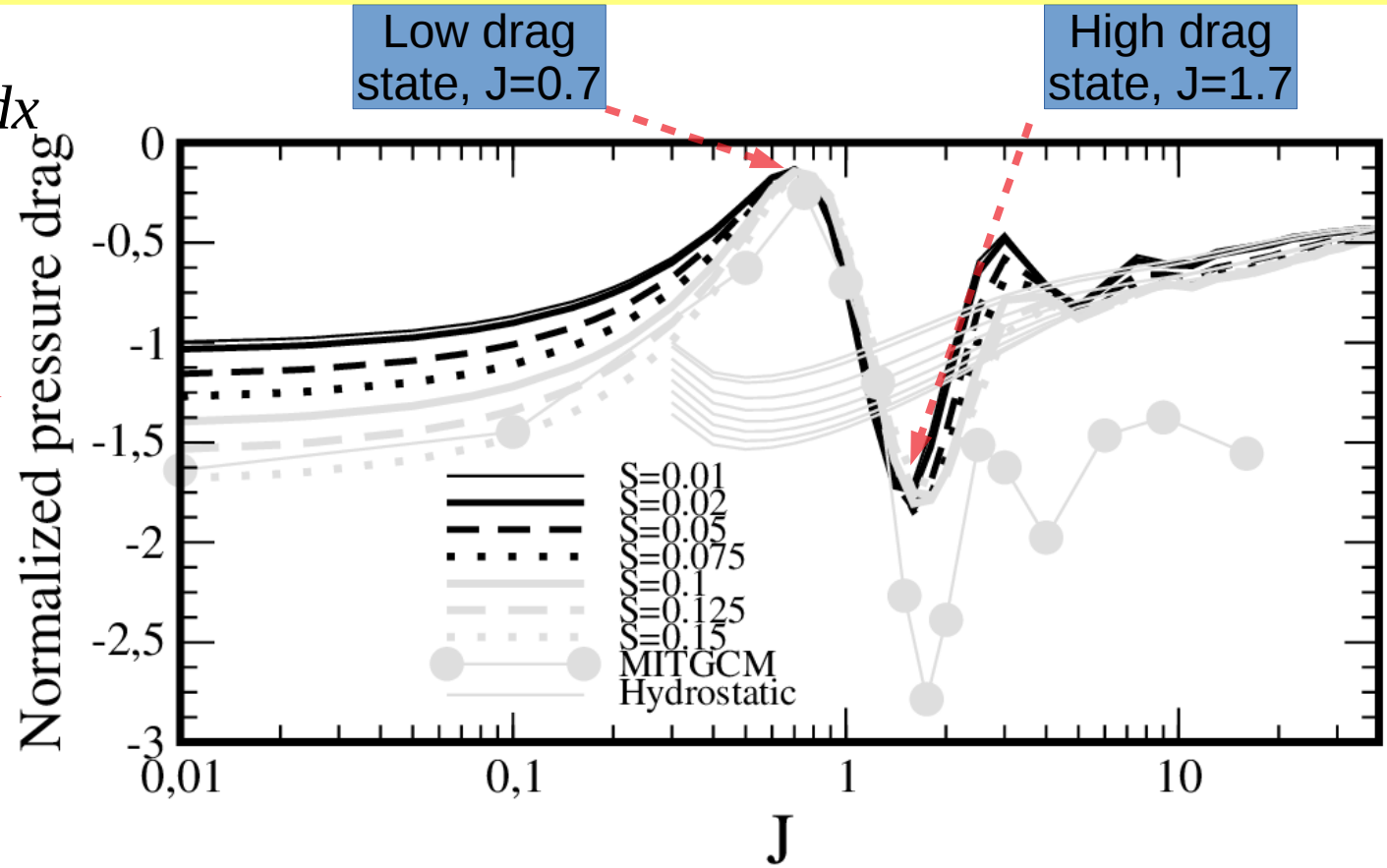
$$Drag = \int_{-\infty}^{+\infty} -p(h)\frac{\partial h}{\partial x}\,dx$$

Drag
Normalized by
Max(D_{GW},D_{FD})

Here :

$$D_{GW} \gtrless D_{FD} \quad \text{when} \quad J \gtrless 1$$



The (hydrostatic) wave drag predictor $D_{GW} = \rho_r u_0(\delta) N H^2$ is adapted for J>>1

It is the linear mountain drag due to a uniform wind of intensity : $u_0(\delta)$

The form drag predictor $D_{FD} = HL \;\; \rho_r \nu u_O(H)/\delta^2$ is adapted for J<<1

It is related to the pressure horizontal variations that balances changes in Reynolds stress of amplitude $\nu u_0(H)/\delta^2$

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$$Reynolds\ stress = \int_{-\infty}^{\infty} \rho_r u' w' dx$$

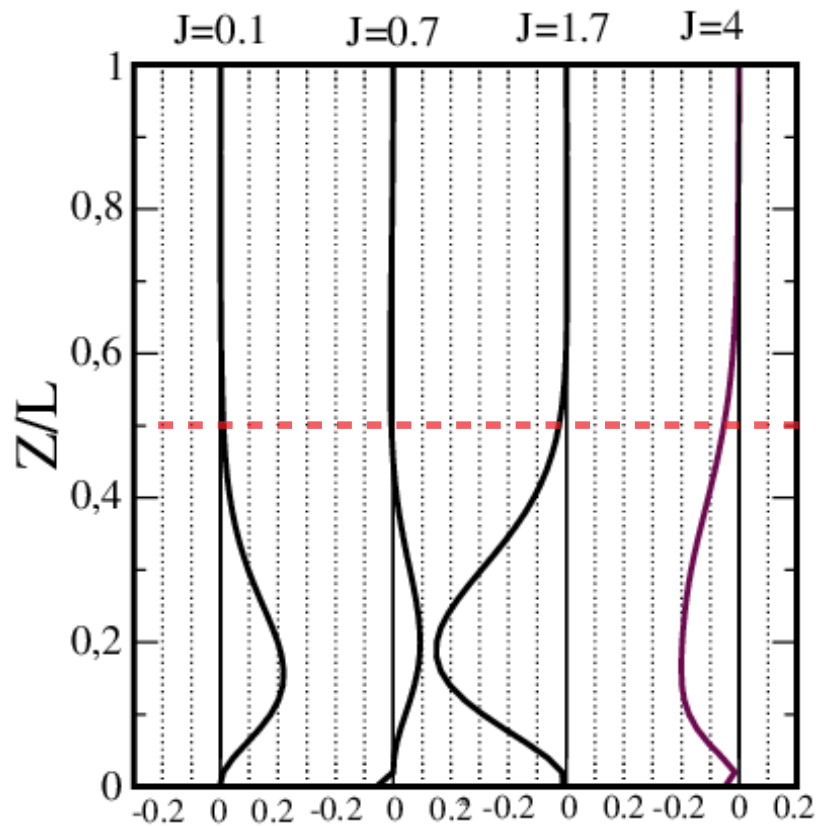
$$H = 0.01 = 0.1\,\delta$$

With small slope and large J
a good fraction of the wave drag
is not transmitted through
the boundary layer

For large J, the vertical wavelength

$$m \approx \frac{\sqrt{J}}{z}$$

increases, causing more dissipation
When travelling through the inner
layer



Deceleration zone

$5\,\delta$

acceleration zone

Normalized values

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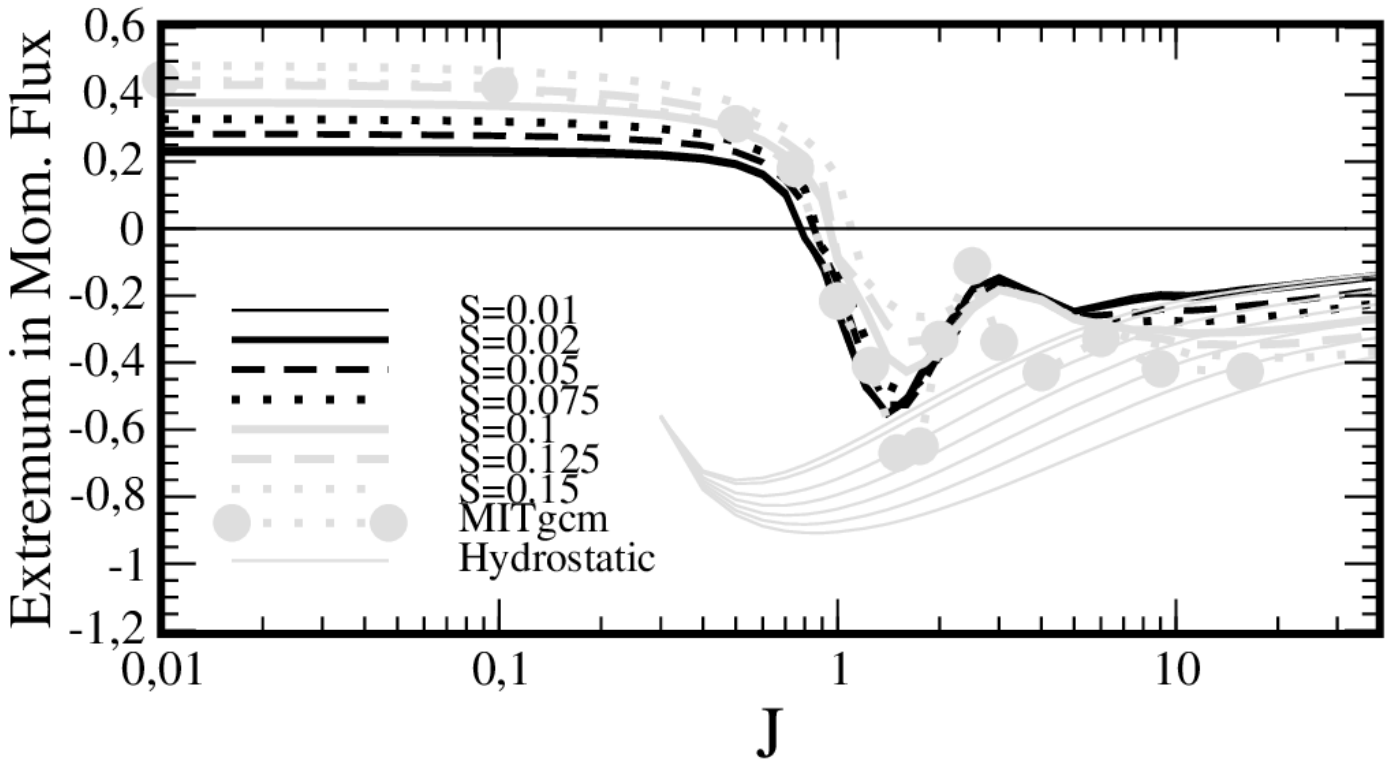
When the mountain heigh H increase compared to δ the emitted fluxes becomes closer to the drag predictor (the waves have less space to be attenuated)

$$S = H/L, \delta/L = 0.1$$



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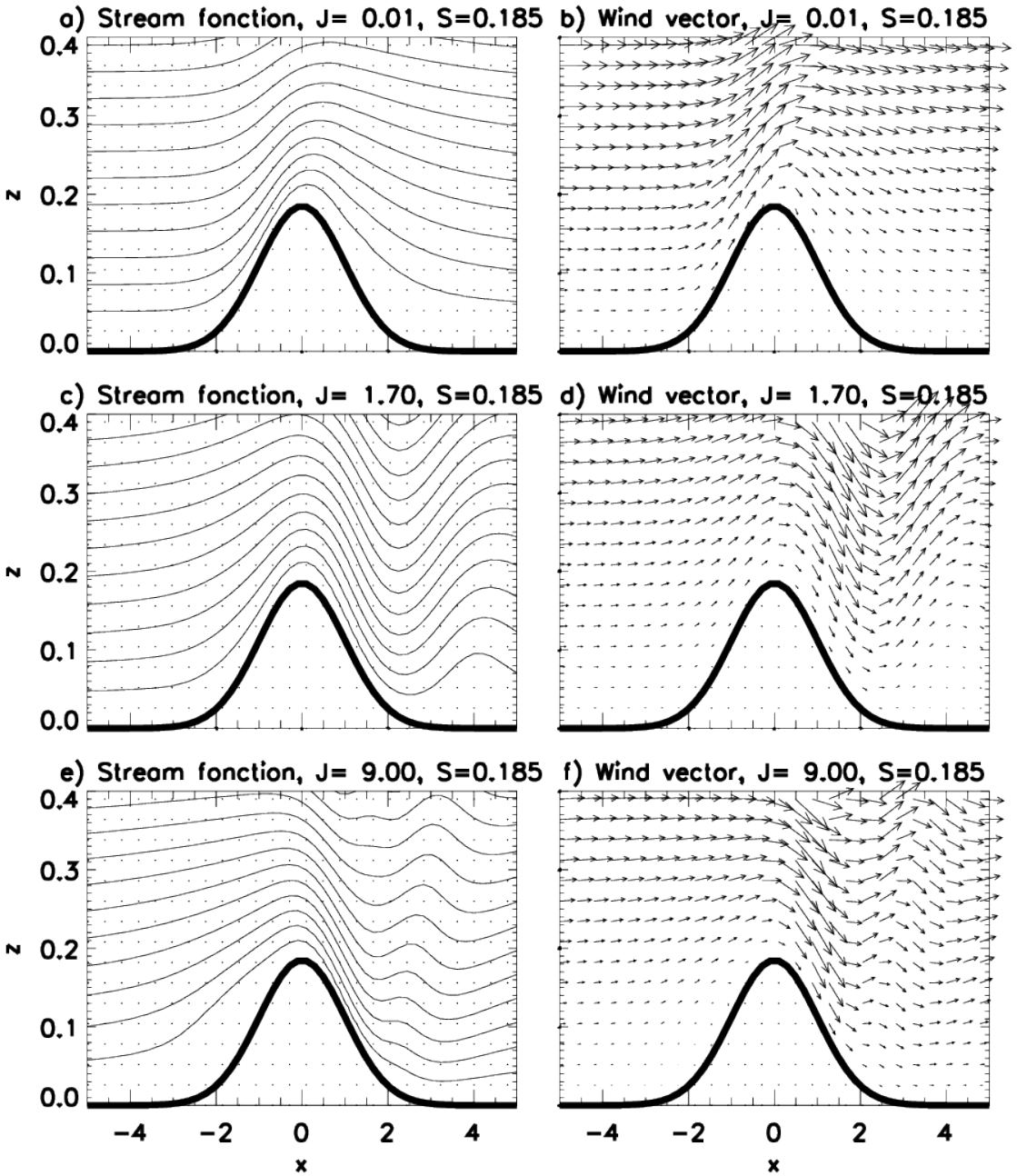

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Conclusions :

Answers to the following question (validated by fully nonlinear solutions):

Is mountain wave theory degenerated when the incident wind is nul at the surface ?

No and theory tells that the viscous critical level dynamics produces « non separated upstream blocking » and downslope winds that penetrate well in the boundary layer (in the stable case)

Can the wave drag be predicted ?

Yes and by conventionnal theories if we take for the incident flow values those at the inner layer scale (theory valid for H< δ)

Probably sensitive to the surface conditions

Probably becomes « values at H » when H> δ

Less anticipated :

When H<< δ, the wave stress is extracted from the boundary layer (funny but need to be consolidated : the waves should accelerate the low level wind in the BL!)

Remark :

Inner layer depth can easily be predicted through linearisation of more sophisticated BL conditions, it is always dependent of the horizontal scale of the mountain .