Mountain waves produced by a stratified shear flow with a boundary layer: transition from downstream sheltering to upstream blocking

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Motivation for an academic approach of the problem

Renewed interest on the interaction between BL and mountain flows (see discussions around TEAM-x)

Old theoretical literature on the topics is notoriously involved (Hunt et. 1988, Belcher and Wood 1996), and not much since then(?)

Predict mountain drag but do not treat where it is deposited

Academic cases permit to control the minimal dynamical ingredients needed to produce important phenomena like the transitions:

Neutral → Stratified
Form drag → Wave drag
Downstream sheltering → Upstream blocking
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With constant kinematic ($\nu$) and thermal ($\kappa$) diffusions and for flat terrain, the boundary layer flow with constant shears is solution, the boundary layer depth is infinite!

$$u_0(z) = u_0 z; \rho_0(z) = \rho_r + \rho_0 z$$

A mountain height

$$h(x) = H e^{-x^2/(2L^2)}$$

produces disturbance we express linearly and in terms of Fourier solutions:

$$w'(x, z) = \int_{-\infty}^{+\infty} \hat{w}(k, z) e^{ikx} dk$$

3 no slip boundary conditions:

$$u_{0z} h + u'(x, h) = 0$$
$$w'(x, h) = 0$$
$$\rho_{0z} h + \rho'(x, h) = 0$$

At $\delta$, advection of disturbance equals dissipation (here $u_0 \partial_x \approx \nu \partial_z^2$)
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Outer layer:

Exact inviscid solution

\[ \hat{w}_I(k, z) = i \sqrt{\frac{\pi k z}{2}} H_{i\mu}^{(1)}(ikz) \]

Inner layer:

\[ z = \delta \tilde{z}, \quad \hat{w} = k \delta \tilde{w}, \quad \delta = \left( \frac{v}{ku_0} \right)^{1/3} \]

Six viscous solutions are tabulated, 4 are enough to satisfy the boundary conditions:

Evanescent when \( z \to \infty \)

\[ \hat{w}_I \approx e^{-k z} \]

All harmonics are trapped!

Matching Function when \( z \to 0 \):

\[ \hat{w}_I \approx \hat{w}_M = \hat{a}_1 z^{1/2-i\mu} + \hat{a}_2 z^{1/2+i\mu} \]

Uniform Approximation:

\[ \hat{w}(k, z) = f_{12}(k)(\hat{w}_I(z) - \hat{w}_M(z)) + k \delta [f_{12}(k) \tilde{w}_{12}(z/\delta) + f_3(k) \tilde{w}_3(z/\delta) + f_3(k) \tilde{w}_3(z/\delta)] \]

\[ f_{12}(k), f_3(k), f_4(k) \] evaluated via inversion of the 3 boundary conditions
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Mountain well in the inner layer
stable case:
\[ \delta = 0.1 L, \quad H = 0.01 L \ll \delta \quad (S = 0.01), J = 4 \]

The flow follows well the mountain up to at least \[ z = \delta \]

The action flux comes from within the boundary layer
(rather than from the surface in the inviscid case, (Durran 95, Lott 98))
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Near neutral case: no waves propagate aloft the BL

Weakly stratified: wave field symmetric but significant

Stratified: Upward waves aloft the mountain
Downward propagating waves downstream

Action flux aloft the BL is null

Some action flux aloft the BL
But positive $F_z$ balance
Negative $F_z$

Strong action flux aloft the BL
Negative $F_z$ over the hill
Positive $F_z$ downstream
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\[
\text{Drag} = \int_{-\infty}^{+\infty} -p(h) \frac{\partial h}{\partial x} \, dx
\]

The (hydrostatic) wave drag predictor \( D_{GW} \) is adapted for \( J \gg 1 \)

\[
D_{GW} = \rho \, u_0(\delta) \, N \, H^2
\]

Normalized by \( \text{Max}(D_{GW}, D_{FD}) \)

Here:

\( D_{GW} > D_{FD} \) when \( J > 1 \)

\( D_{GW} < D_{FD} \) when \( J < 1 \)

The form drag predictor \( D_{FD} = HL \, \rho \, u_0(\delta) \, H^2 / \delta^2 \) is adapted for \( J \ll 1 \)

It is related to the pressure horizontal variations that balances changes in Reynolds stress of amplitude \( \nu \, u_0(\delta) \, H / \delta^2 \).
Reynolds stress \( = \int_{-\infty}^{\infty} \rho_r u' w' \, dx \)

As all the waves are reflected the Reynolds stress is confined to the inner layer.

For small J, deceleration in the lower part of the inner layer, acceleration in the upper part (an enveloppe effect?)

For large J acceleration in the lower part of the inner layer, deceleration in the upper part

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Extremes in momentum fluxes change signe when $J \sim 1$. Passage from an « enveloppe » effect to a trapped wave drag effect

$$S = \frac{H}{L}, \frac{\delta}{L} = 0.1$$
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Conclusions:

Answers to the following question (validated by fully nonlinear solutions):

Is mountain wave theory degenerated when the incident wind is nul at the surface?

No and theory tells that the viscous critical level dynamics produces « non separated downstream sheltering » and upslope winds in the neutral case « non separated upstream blocking » and downslope winds in the stratified case.

The drag is predictable:
A gravity wave drag in the stable case (J>>1), a form drag in the neutral case (J<<1)
During the transition (J~1) high drag and low drag states occur.

In the stable case, the wave drag decelerates the flow near the top the inner layer

In the neutral case, the form drag decelerates the flow near the surface, some acceleration occurs near the top of the inner layer (an enveloppe effect?)