Mountain waves produced by a stratified shear flow with a boundary layer : transition from downstream sheltering to upstream blocking

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Motivation for an academic approach of the problem

Renewed interest on the interaction between BL and mountain flows
(see discussions around TEAM-x)

Old theoretical literature on the topics is notoriously involved
(Hunt et. 1988, Belcher and Wood 1996),and not much since then(?)

Predict mountain drag but do not treat where it is deposited

Academic cases permit to control the minimal dynamical ingredients needed to produce important phenomena like the transitions :

Neutral → Stratified
Form drag → Wave drag
Downstream sheltering → Upstream blocking

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With constant kinematic (ν) and thermal (κ) difffusions and for flat terrain, the boundary layer flow with constant shears is solution, the boundary layer depth is infinite ! :

Sykes (1978)

$$u_0(z)=u_{0z}z\,;\rho_0(z)=\rho_r+\rho_{0z}z$$

A mountain height

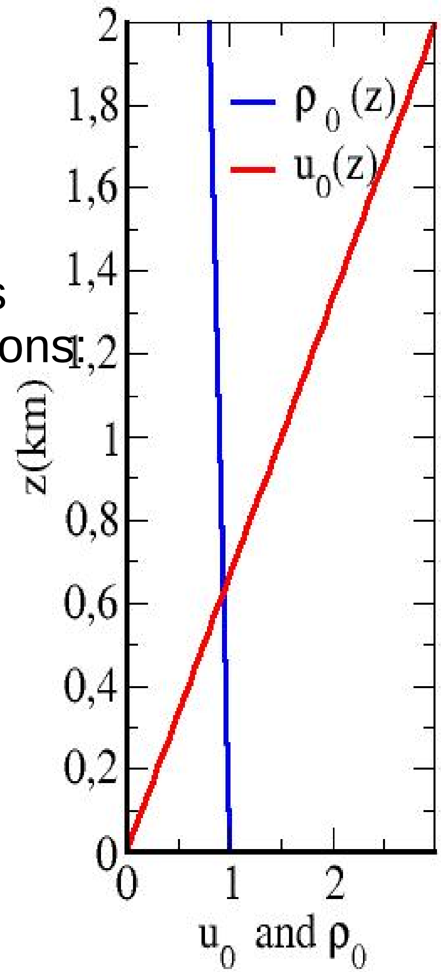
$$h(x)=H\,e^{-x^2/(2L^2)}$$

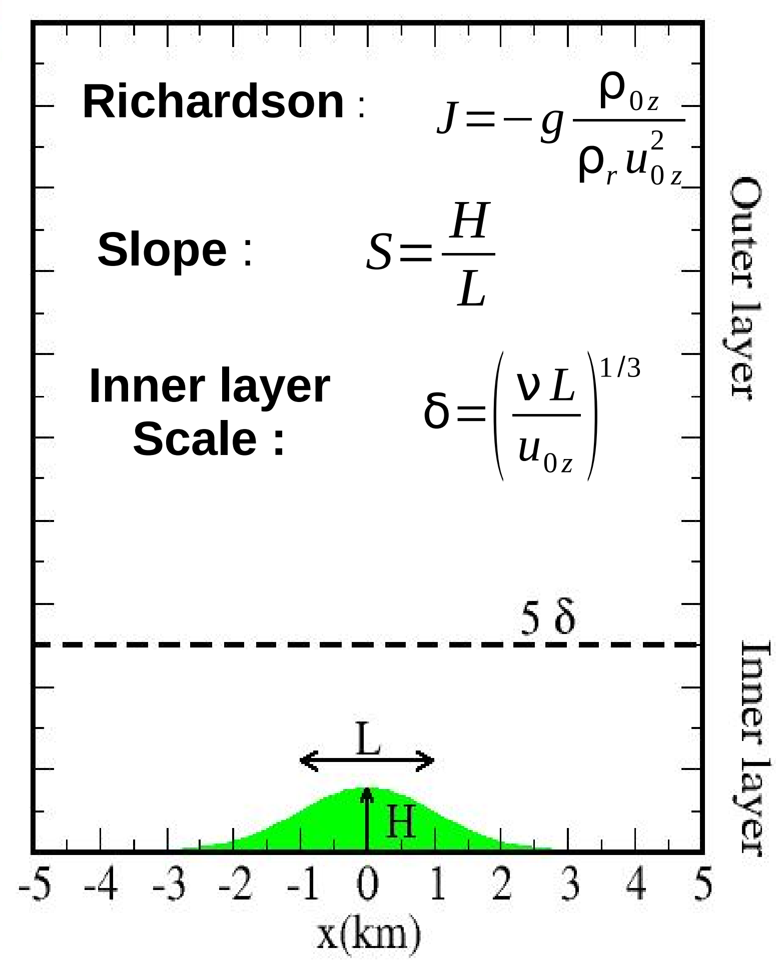
produces disturbance we express linearly and in terms of Fourier solutions:

$$w'(x,z)=\int_{-\infty}^{+\infty}\hat{w}(k,z)\,e^{ikx}\,dk$$

3 no slip boundary conditions :

$$u_{0z}h+u'(x,h)=0$$
$$w'(x,h)=0$$
$$\rho_{0z}h+\rho'(x,h)=0$$





At δ, advection of disturbance equals dissipation (here $u_0\partial_x\approx\nu\,\partial_z^2$)

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Outer layer:

Exact inviscid solution
$$\hat{w}_I(k,z) = i\sqrt{\frac{\pi k z}{2}}\, H^{(1)}_{i\mu}(ikz)$$

Evanescent when z→ ∞

$$\hat{w}_I \underset{z\to\infty}{\approx} e^{-k\ \ z}$$

All harmonics are trapped !

Matching Function when z→0:

$$\hat{w}_I \underset{z\to 0}{\approx} \hat{w}_M = \hat{a}_1 z^{1/2-i\mu} + \hat{a}_2 z^{1/2+i\mu}$$

Inner layer:
$$z = \delta\tilde{z},\quad \hat{w} = k\delta\tilde{w},\quad \delta = \left(\frac{\nu}{k u_{0z}}\right)^{1/3}$$

Six viscous solutions are tabulated, 4 are enough to satisfy the boundary conditions:

$$\tilde{w}_{12} \underset{\tilde{z}\to\infty}{\approx} \tilde{a}_1 \tilde{z}^{1/2-i\mu} + \tilde{a}_2 \tilde{z}^{1/2+i\mu}, \qquad \tilde{w}_3 \underset{\tilde{z}\to\infty}{\approx} \tilde{z}^{-5/4} e^{\frac{-2\sqrt{i}}{3}\tilde{z}^{3/2}},\quad \tilde{w}_4 \underset{\tilde{z}\to\infty}{\approx} \tilde{z}^{-9/4} e^{\frac{-2\sqrt{iP}}{3}\tilde{z}^{3/2}}$$

Matches $\hat{W}_M$ exactly

Decay exponentially fast when $\tilde{z} \to \infty$

Uniform Approximation:
$$\hat{w}(k,z) = f_{12}(k)\left(\hat{w}_I(z) - \hat{w}_M(z)\right)$$
$$+ k\,\delta\left[f_{12}(k)\tilde{w}_{12}(z/\delta) + f_3(k)\tilde{w}_3(z/\delta) + f_3(k)\tilde{w}_3(z/\delta)\right]$$

$f_{12}(k), f_3(k), f_4(k)$ evaluated via inversion of the 3 boundary conditions

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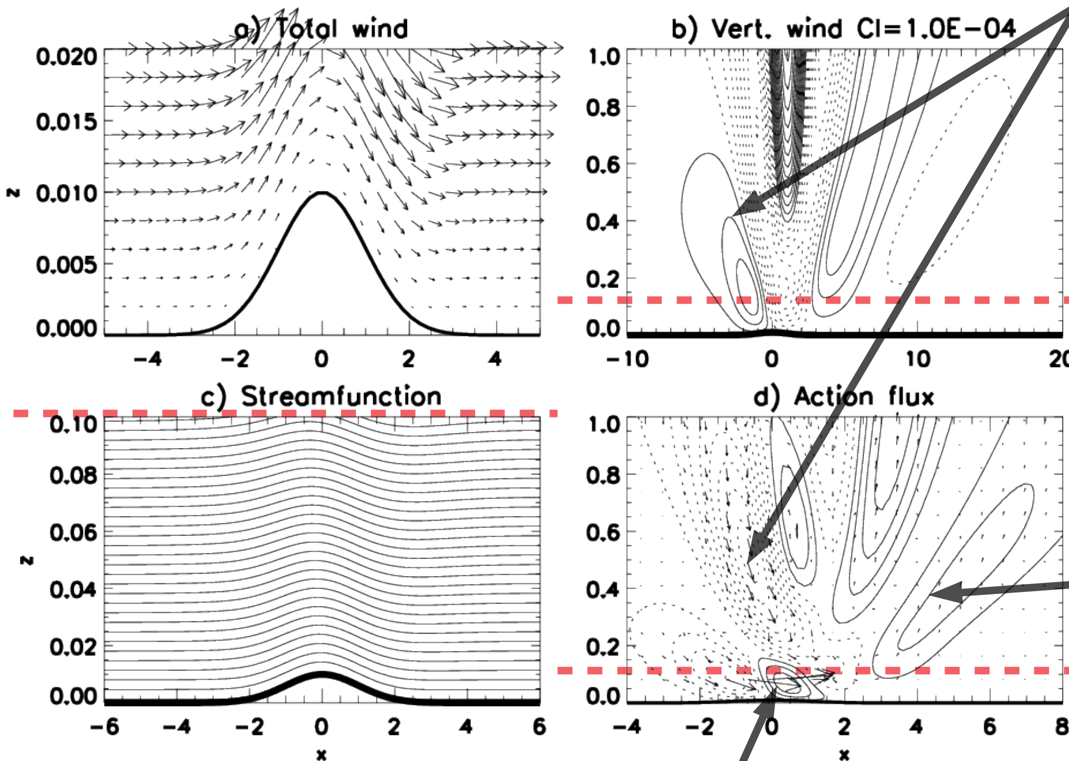
Mountain well in the inner layer stable case :

$$\delta = 0.1\,L, \quad H = 0.01\,L \ll \delta \quad (S = 0.01), J = 4$$

The flow follows well the mountain up to at least $z=\delta$



Upward waves $F^z = \rho_r u'w' < 0$

δ

Downward waves $F^z = \rho_r u'w' > 0$

δ

$z=\delta$

The action flux comes from within the boundary layer (rather than from the surface in the inviscid case, (Durran 95, Lott 98))

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Vertical Wind

Action Flux
(vector and vert. comp)

Near neutral case :
no waves propagate
aloft the BL

Action flux aloft
the BL is nul

Weakly stratified:
wave field symetric but
significant

Some action flux
aloft the BL

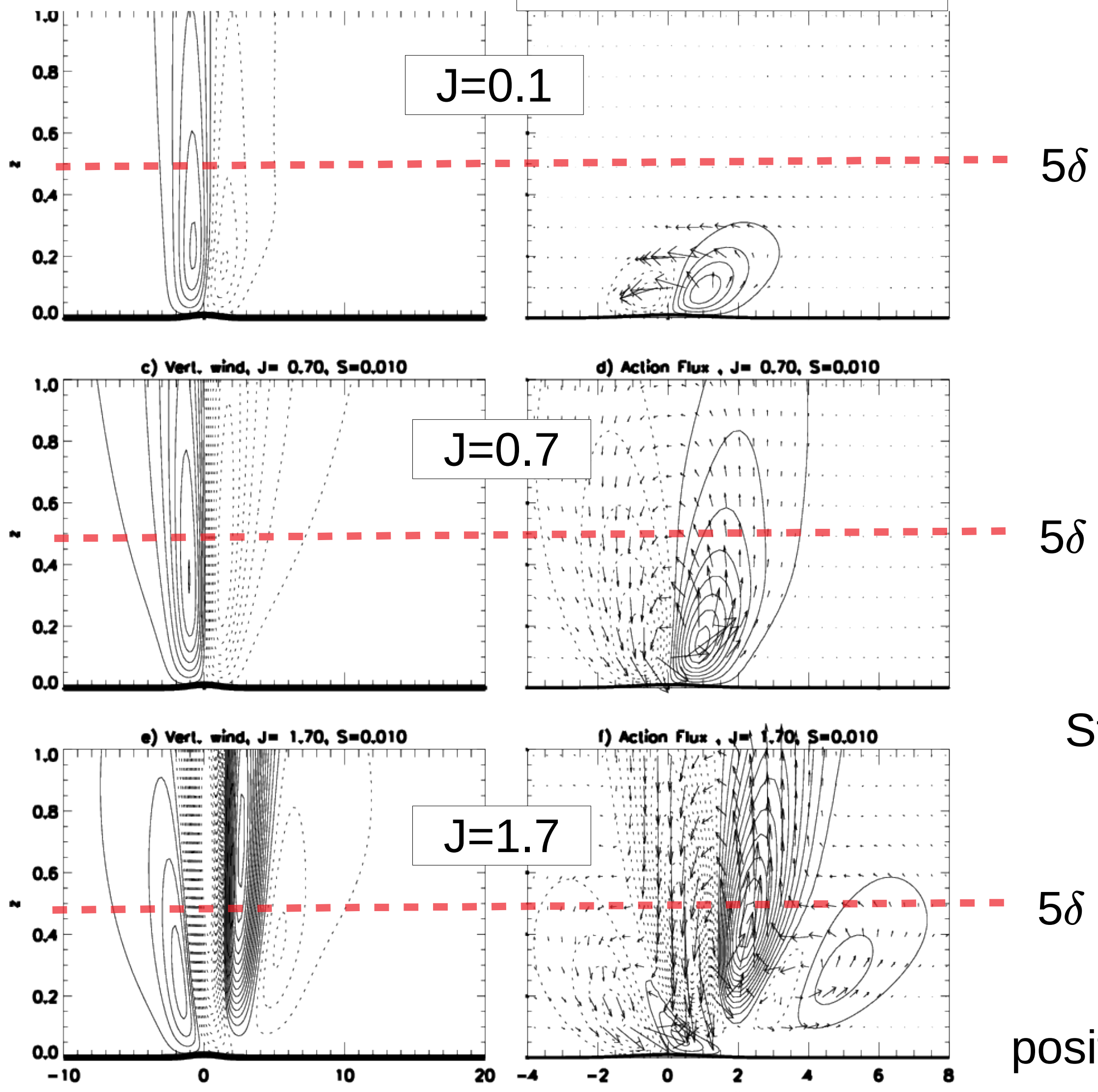
But positive F^z
balance
Negative F^z

Stratified:
Upward waves aloft
the mountain
Downward propagating
waves downstream

Strong action flux
aloft the BL

Negative F^z
over the hill

positive F^z downstream



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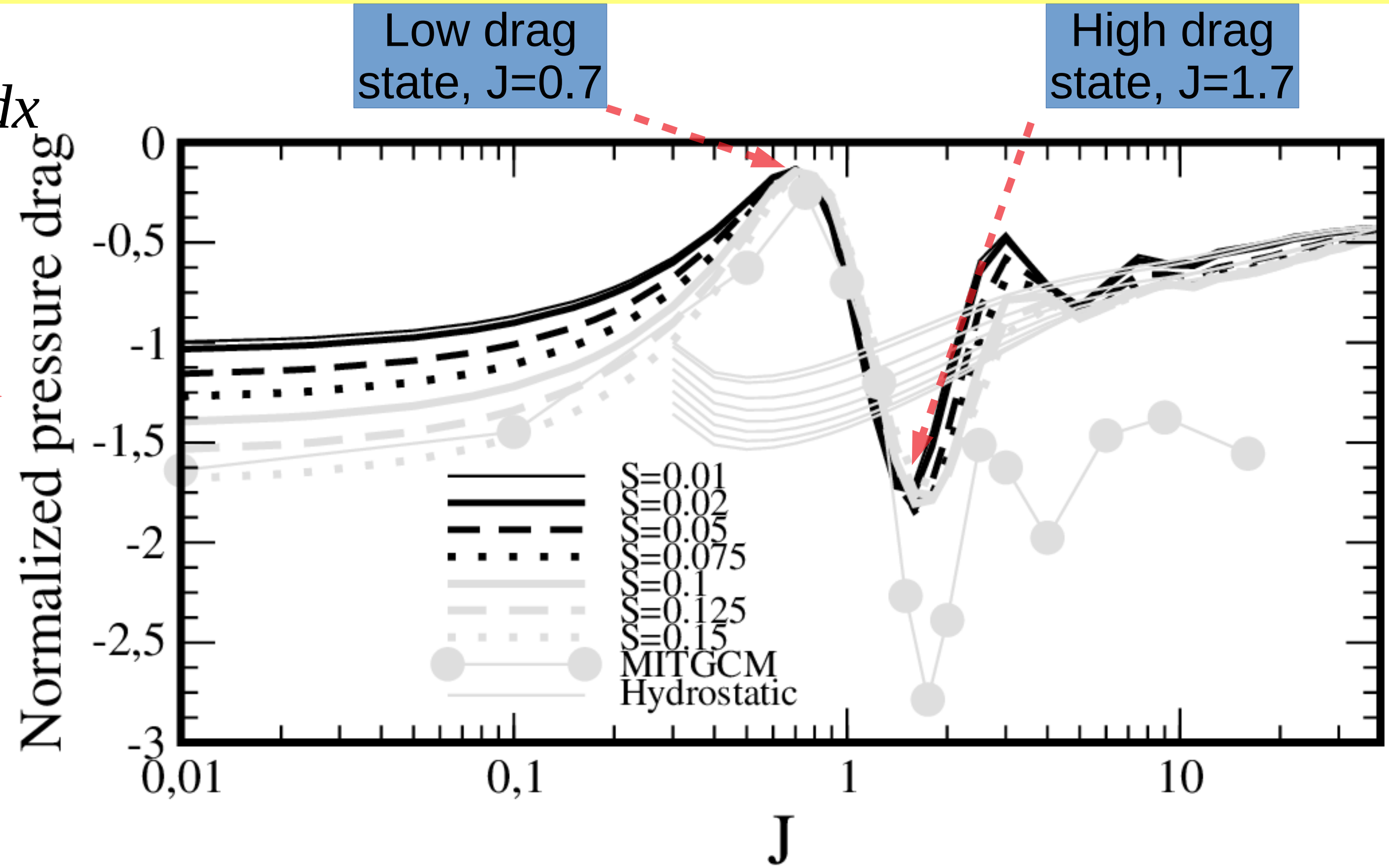
$$Drag = \int_{-\infty}^{+\infty} - p(h) \frac{\partial h}{\partial x} dx$$

$Drag$
Normalized by
Max(D_{GW},D_{FD})



Here :

$D_{GW} \gtrless D_{FD}$ when $J \gtrless 1$

The (hydrostatic) wave drag predictor $D_{GW} = \rho_r u_0(\delta) N H^2$ is adapted for J>>1

It is the linear mountain drag due to a uniform wind of intensity : $u_0(\delta)$

The form drag predictor $D_{FD} = HL \;\; \rho_r \nu u_O(H)/\delta^2$ is adapted for J<<1

It is related to the pressure horizontal variations that balances changes in Reynolds stress of amplitude $\nu u_0(H)/\delta^2$

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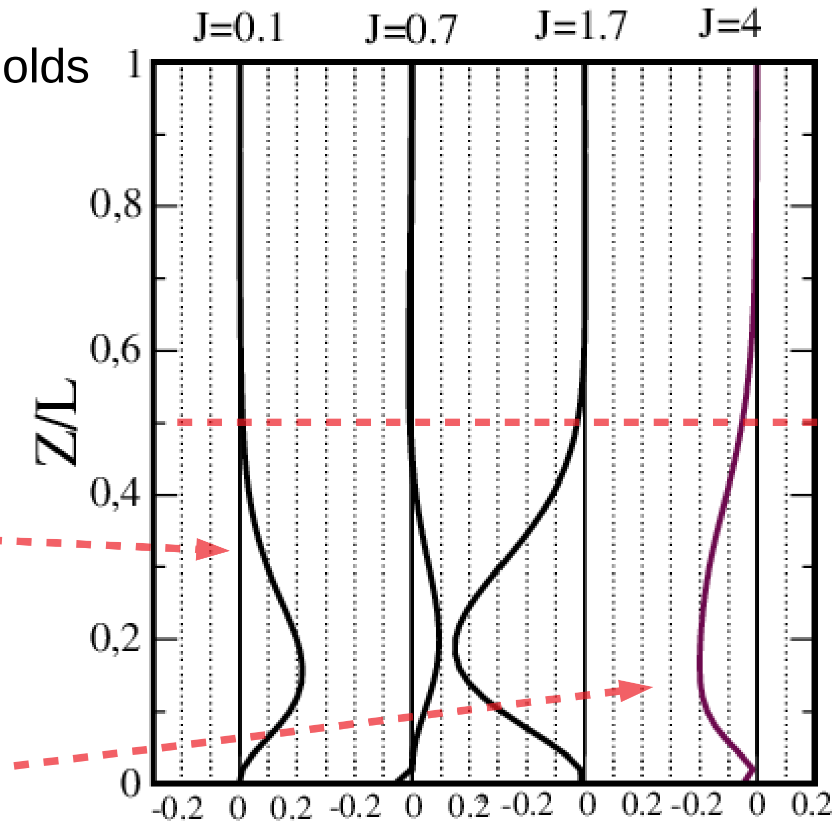
$$Reynolds\,stress = \int_{-\infty}^{\infty} \rho_r u' w' dx$$

$$H = 0.01 = 0.1\,\delta$$

As all the waves are reflected the Reynolds stress is confined to the inner layer

For small J, deceleration in the lower part of the inner layer, acceleration in the upper part (an enveloppe effect?)

For large J acceleration in the lower part of the inner layer, deceleration in the upper part



Normalized values

$5\,\delta$

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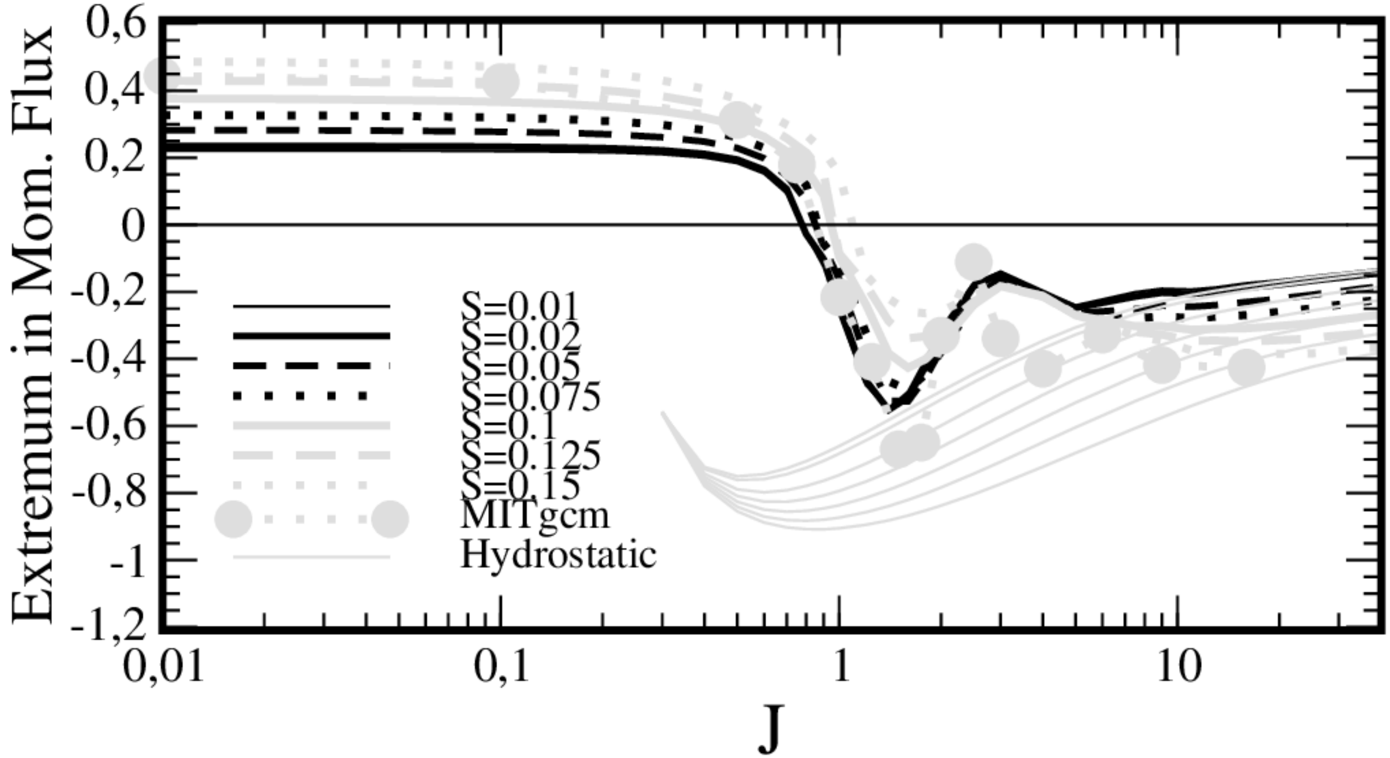
Extremes in momentum fluxes change signe when J~1.
Passage from an « enveloppe » effect to a trapped wave drag effect

$$S = H/L, \delta/L = 0.1$$



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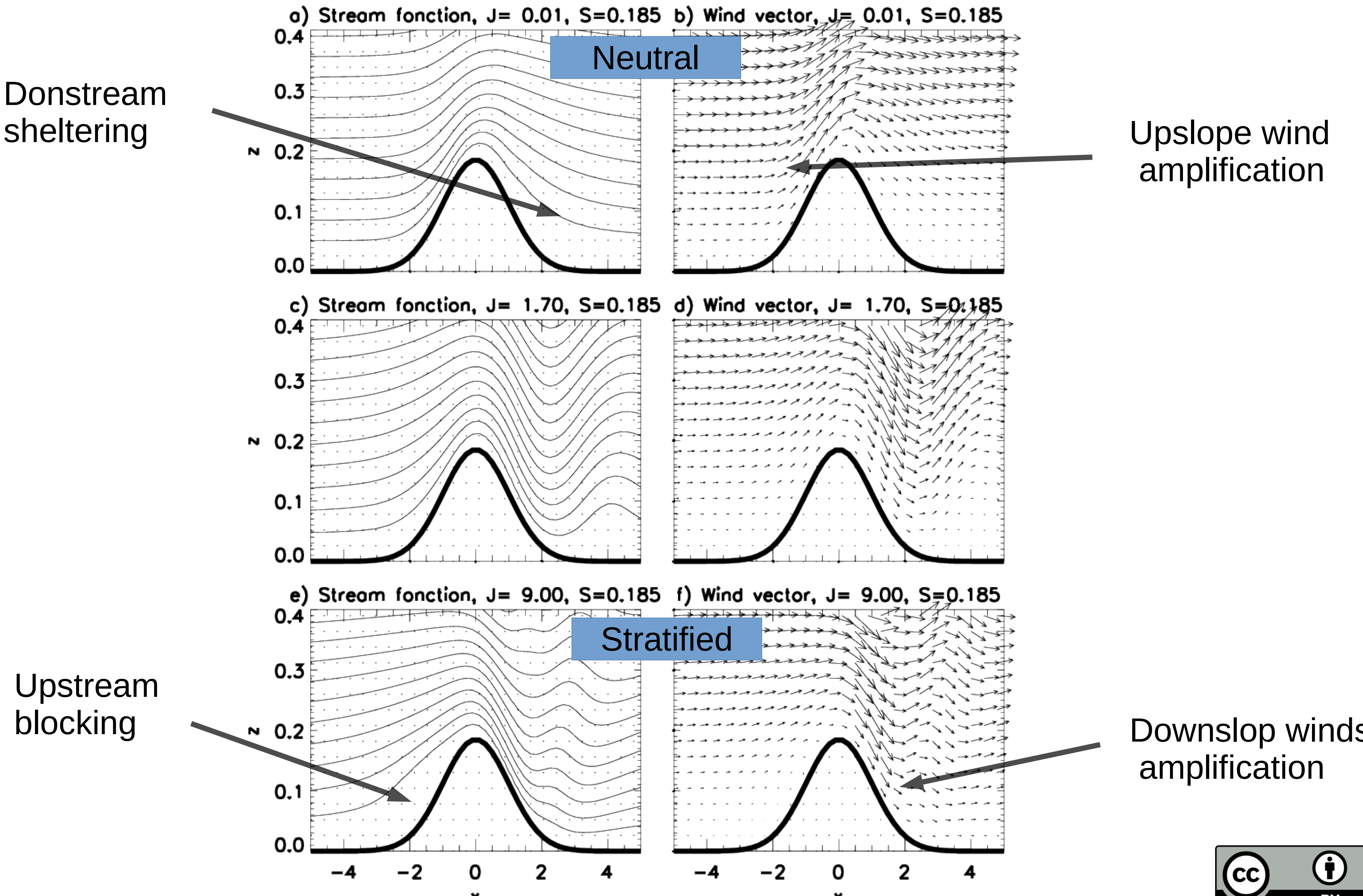

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Conclusions :

Answers to the following question (validated by fully nonlinear solutions):

Is mountain wave theory degenerated when the incident wind is nul at the surface ?

No and theory tells that the viscous critical level dynamics produces
« non separated downstream sheltering » and upslope winds in the neutral case
« non separated upstream blocking » and downslope winds in the stratified case

The drag is predictable :
A gravity wave drag in the stable case (J>>1), a form drag in the neutral case (J<<1)
During the transition (J~1) high drag and low drag states occur.

In the stable case, the wave drag decelerates the flow near the top the inner layer

In the neutral case, the form drag decelerates the flow near the surface, some acceleration occurs near the top of the inner layer (an enveloppe effect?)