Cumulant lattice Boltzmann approach: an application to hydraulic risk

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Cumulant lattice Boltzmann approach: an application to hydraulic risk

- **CumLB** method: numerical technique for the solution of the hydrodynamic problem
- **parallelization**
- The procedure includes the preparation of the input data and the visualizations of the results (Open source Qgis platform)

**Semi-automatic procedure for modelling flood events**

**GIS LB ROUTINE**

- Topographic data
- External force data
- Boundary conditions
- Initial conditions
- Thematic maps (Hydraulic Risk maps)

**interchange data**

**LB shallow water CODE**

- Manning’s coefficient
- Wind velocity
- Inlet, outlet, obstacles position
- Water height
- Velocity field
- Temperature
- Pollutants concentration

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PRESENTATION OVERVIEW

- Shallow water – lattice Boltzmann (LB) innovative models
- Wet - dry approach
- Case study: Malpasset dam-break
Cumulant lattice Boltzmann approach: an application to hydraulic risk

**SHALLOW WATER EQUATIONS**

- **continuity**
  \[
  \frac{\partial h}{\partial t} + \frac{\partial (hu_j)}{\partial x_j} = 0
  \]

- **momentum**
  \[
  \frac{\partial (hu_i)}{\partial t} + \frac{\partial (hu_i u_j)}{\partial x_j} = -g \frac{\partial}{\partial x_i} \left( \frac{h^2}{2} \right) + \nu \frac{\partial^2 (hu_i)}{\partial x_j \partial x_j} + F_i
  \]

  \[
  F_i = -gh \frac{\partial z_b}{\partial x_i} + \frac{\tau_{wi}}{\rho} + E_i
  \]

**LATTICE BOLTZMANN METHOD**

- **microscopic**
  Molecular Dynamics (MD)

- **mesoscopic**
  LATTICE BOLTZMANN METHOD
  Probability Distribution Function
  \( f(x,c,t) \)

- **macroscopic**
  Navier Stokes Equations
  Continuum fluid macroscopic properties
LATTICE BOLTZMANN MODEL

2D models

D2Q9

D2Q7

3D models

D3Q19

D3Q27

Conservation of mass and momentum in the collision

\[ f_\alpha (x + e_\alpha \Delta t, t + \Delta t) = f_\alpha (x, t) + \Omega_\alpha + F_\alpha \]

\( F_\alpha \) : external force

\( \Omega_\alpha \) : collision operator

α = 1, ..., n
LB MODELS FOR SWE

Speed of sound

\[ P = c_s^2 h \]

\[ \partial_t h + \nabla (h \mathbf{u}) = 0 \]
\[ \partial_t (h \mathbf{u}) + \nabla (PI + h \mathbf{uu} - S) = 0 \]

\[ c_s^2 \Rightarrow \frac{gh}{2} \neq \frac{1}{3} \]

Viscosity

\[ \nu = c_s^2 \left( \frac{1}{\omega} - \frac{1}{2} \right) \]

\[ \omega = \frac{1}{\tau} \]

relaxation time

Macroscopic Variables

\[ h = \sum_{\alpha=1}^{n} f_{\alpha} \]
\[ u_i = \frac{1}{h} \sum_{\alpha=1}^{n} e_{\alpha i} f_{\alpha} \]
MRT CASCADED MODEL (CaLB)

Central moments

$$\kappa_{\alpha\beta} = \sum_{i,j} (i - u)^{\alpha} (j - v)^{\beta} f_{ij} \quad i, j = -1, 0, 1$$

$$\kappa_{00} = h, \kappa_{10} = 0, \kappa_{01} = 0, \kappa_{20} = c_s^2 h, \kappa_{02} = c_s^2 h, \kappa_{11} = 0, \kappa_{12} = 0, \kappa_{22} = 0$$

$$\kappa_{\alpha\beta} = \text{central moments} \quad u, v = \text{macroscopic velocities}$$

Collision step

$$\kappa_{\alpha\beta,\text{post}} = \kappa_{\alpha\beta} - \omega_{\alpha\beta} (\kappa_{\alpha\beta} - \kappa_{\alpha\beta,\text{eq}})$$

$$\kappa_{20+02,\text{post}} = \kappa_{20+02} - \omega_{20+02} (\kappa_{20} - \kappa_{20,\text{eq}} + \kappa_{02} - \kappa_{02,\text{eq}})$$

$$\kappa_{20-02,\text{post}} = \kappa_{20-02} (1 - \omega_{20-02})$$

Relaxation rates

$$\omega_{11} = \frac{1}{3\nu + 0.5} \quad \omega_{20-02} = \omega_{11} \quad \omega_{\alpha\beta} = 1$$
MRT CUMULANT MODEL (CumLB)

**Theory of cumulant**

\[
c_{\alpha\beta\gamma} = \frac{\partial^\alpha \partial^\beta \partial^\gamma}{\partial \Xi \partial \Upsilon \partial Z} \ln \left( \frac{F(\Xi, \Upsilon, Z)}{q_0} \right) \bigg|_{\Xi, \Upsilon, Z=0}
\]

**Cumulants**: coefficients of the Taylor expansion of the logarithm of the Laplace transform of the distribution function \(f\) (PDF)

- The cumulant CO uses multiple relaxation rates (MRT CO)
- It relaxes observable quantities (cumulants) that are both Galilean invariant and statistically independent of each other by construction

**Galilean Invariance**

- Break of Galilean invariance: due to the finite number of velocities and a resulting finite number of independent moments
- not Galilean invariant model: results depend on the reference frame → presence of preferential directions

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The PDF is transformed into cumulants. The collision step is performed in terms of cumulants; after the collision, the backward transformation is applied, from cumulants to PDF.
WET-DRY APPROACH

It was necessary to develop a procedure simulating the flow propagation over a dry bed.

1. **Height Limiter**

\[ h \leq h_{\text{lim}} \rightarrow h = h_{\text{lim}} \]

2. **Velocity Limiter**

\[ Fr \geq Fr_{\text{lim}} \rightarrow |u'| = |u| \cdot \frac{Fr_{\text{lim}}}{Fr} \]

\[ \frac{Fr_{\text{lim}}}{Fr} < 1 \]

\[ |u'| = |u| \cdot (1 - \delta) \rightarrow \delta = 1 - Fr_{\text{lim}} \sqrt{\frac{gh}{|u| \cdot |u|}} \]

The velocity is limited by means of the Froude Number, multiplying its value for the ratio between the Fr-limiter and the actual Froude Number.

Threshold value: allows to avoid the error due to the division by zero.
CASE STUDY
MALPASSET DAM-BREAK (21/12/1959)

A. Valiani et al., 'Case Study: Malpasset Dam-Break Simulation using a Two-Dimensional Finite Volume Method', 2002

C. Biscarini et al., 'On the Simulation of Floods in a Narrow Bending Valley: The Malpasset Dam Break Case Study', 2016
AT at hydroelectric plants (A, B, C)
WS and velocity at points measured by the police $P_i$, $i = 1, \ldots, 17$

WS (water surface) and AT (arrival time), Physical model, EDF, $G_i \ i = 1, \ldots, 14$
**Domain – bounding box**

- **width**: 17500 m
- **height**: 10000 m

**setup**

**Model:** Cumulant model  
**Grid spacing** $\Delta x$: 10 m, 20 m  
**relaxation rate** $\tau$: 0.8  
**Manning coeff.** $n$: $0.03 \, s \, m^{1/3}$  
(Hervouet and Petitjean, 1999)

**Arrival time (AT) at Electrical Transformer**

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<th><strong>ET</strong></th>
<th><strong>At obs (s)</strong></th>
<th><strong>At sim (s)</strong></th>
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<tr>
<td>C</td>
<td></td>
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Comparison of WS at Pi - points

numerical model

\[ E_{RR_{\Delta x=20m}} \approx 5\% \]
\[ E_{RR_{\Delta x=10m}} \approx 4\% \]

Valiani et al.

\[ E_{RR} \approx 8\% \]
Satisfying accordance between the arrival time (AT) at almost all $G_i$ points
Cumulant lattice Boltzmann approach: an application to hydraulic risk

Reyran Valley

Overview

Model Analysis

Validation

Case Study

Conclusions
CONCLUSIONS

- The applicability of **CumLB** model to the propagation of **floods** has been successfully tested;

- **Innovative models** exhibit satisfying characteristics of accuracy and stability in predicting a flood wave, introducing the possible application of the **LB Cum- GIS** routine to the assessment of the **hydraulic risk**.