Uncertainties in $\chi$ analysis
Implications for drainage network and divide stability

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How big is big?

• Uncertainty in $\chi$ not quantified
• No way to tell whether a difference across the divide is significant!
• Uncertainties in drainage area control uncertainties in $\chi$

$$\chi(x) = \int_{x}^{x_b} \left( \frac{A_0}{A(\hat{x})} \right)^\theta d\hat{x}$$
Sources of error of drainage area

• Divide follows a line, pixels are area elements
• Pixels are fully assigned to one basin, but contain area belonging to another basin
Sources of error of drainage area

• Pixels may be assigned to the wrong basin
• This can happen if the pixel elevation error exceeds the difference in elevation in the adjacent pixels at the divide
Analytical solution

• Gaussian error propagation
  • Uncorrelated errors
• Hack’s law for drainage basin shape
• Simple quantification of error sources
• Arrive at analytical solution dependent on
  • Pixel size
  • Concavity index (set to 0.5)
  • Basin shape (Hack exponent, geometric shape factor)
Dependence on drainage area

- Uncertainty largest close to the divide
- Depending on input parameters, 10% to >100% of the value of $\chi$!
Dependence on pixel size

• For constant absolute elevation error, uncertainty decreases with pixel size

• For elevation errors that scale with pixel size (constant relative errors), positive quadratic dependence
Dependence on concavity

- Strong negative exponential dependence on concavity index

Normalized error in $\chi$ at channel head

$\chi$ error evaluated at the channel head

Concavity index $\theta$

Pixel size 90m
Pixel size 25m
Pixel size 10m
Alternative approach: Multiple flow directions

- Different method to obtain errors in drainage area
  - Relies on interpretation of fractions of multiple flow directions as probabilities
  - Multiple possible flow directions allow different flow paths
  - Probabilistic assignment of divide pixels (and adjacent pixels) to individual outlets
  - Monte-Carlo method to select a flow path realization
- Drainage area and divide location varies with the specific flow path
- Allows calculation of variance of drainage area and \( \chi \)
Alternative approach: Multiple flow directions

Example calculation for the Big Tujunga basin.
Alternative approach: Multiple flow directions

Solid lines give mean $\chi$-value.

Red shading gives uncertainty.
Comparison

• Consistency of analytical and MC derived errors
Covariance?

• Errors in $\chi$ depend on pixels at the divide
• Although drainage area is serially correlated, divide pixels are not, at least not to the same extent
• Example to the right
  • Blue basin shares $\sim\frac{1}{2}$ of boundary with orange basin
  • Orange basin shares $\sim\frac{1}{3}$ of boundary with blue basin
  • Yellow basin does not share any boundary pixels with the blue or orange basins