

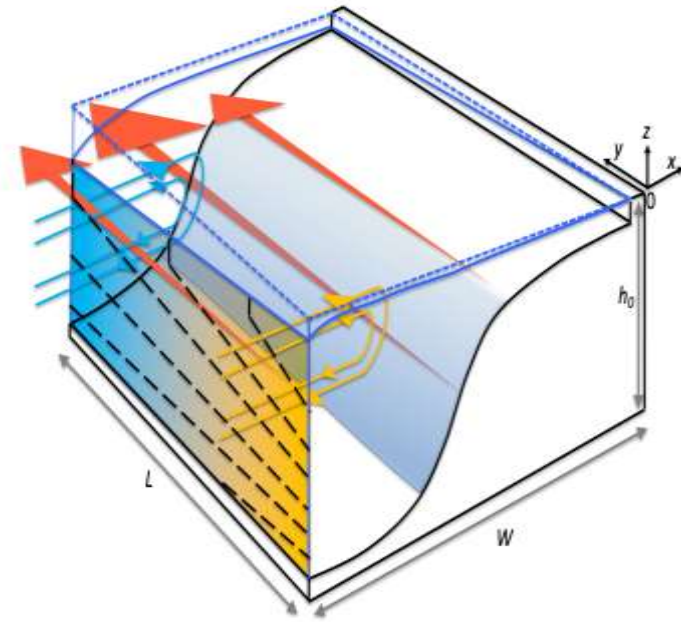
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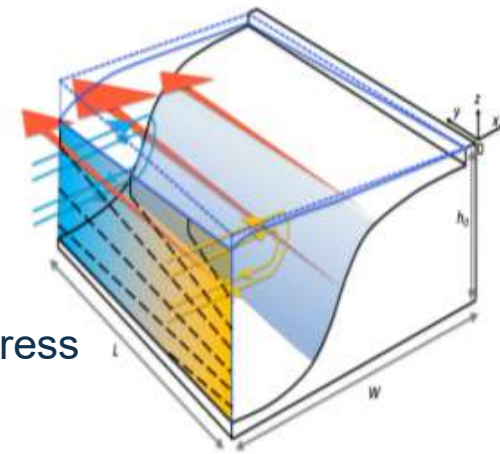
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# Oceanic density/pressure gradients and slope currents



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# Summary and conclusions



- **Analytic form of ocean slope current**

\* equilibrated with oceanic density gradient “JEBAR” & wind stress

\* steady, along-slope uniformity

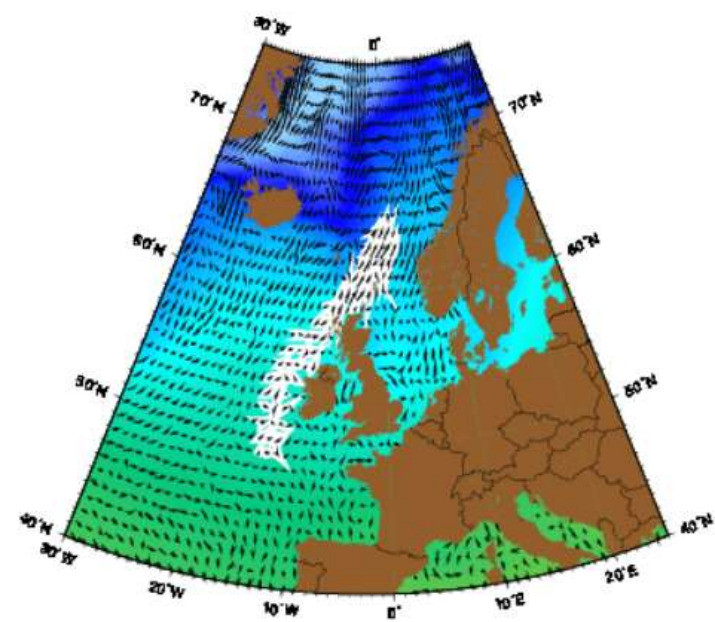
(flow, depth, wind; gradients of density, surface elevation, pressure)

Simplified by equilibrium, along-slope uniformity, assumed linearity (JPO paper justifies)

- **Direct relation between slope current strength, friction and along-slope forcing** (e.g. wind)  
also between the total along-slope forcing and bottom Ekman transport
  - **any forcing implies bottom stress (not “slippery”) and Ekman transport**
  - should not / cannot assume zero along-slope pressure gradient
- Boundary currents are energetic → significant meridional transports (water, heat, salt)  
but poorly resolved in models, sparsely measured, sparse literature on dynamics
- **Analysis hopefully contributes to dynamical understanding, model testing and basis for further study** of roles of friction, wind, changing oceanic density

# Introduction

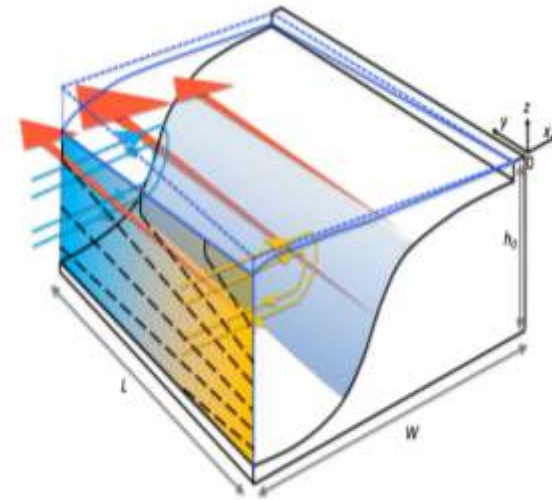
- **Eastern ocean-boundary currents are common**  
(e.g. W Europe, W USA, W Australia)  
**and important,**  
e.g.  $\sim \frac{1}{4}$  of Atlantic inflow to the Nordic Seas



The Slope Current as represented by the Mariano Global Surface Velocity Analysis (MGSVA). The Slope Current is just off-shore of the British Isles and transports Northeast Atlantic water into the Norwegian Sea. Click here for <https://oceancurrents.rsmas.miami.edu/atlantic/slope.html>

- **No rest state if  $\partial\rho/\partial y$**  (along-slope  $\nabla$  density) **and slope  $\partial h/\partial x$**
- Literature analysis very restricted regarding form of density  $\rho$
- **Analysis here allows any  $\rho = \rho_y(z)y + \rho_2(x, z)$**  (y-uniform  $\partial\rho/\partial y$  but wide scope to vary  $\rho$ )
- **Wind stress also added,** flow constrained by balancing bottom friction

# Analysis (see JPO on-line for details)



- **Assumptions:**

Hydrostatic pressure  $p$ , free surface  $z = \eta(x, y)$

**Linear** (justified in JPO paper;

non-linear effects of eddies or tides are another forcing)

Lateral viscosity = 0 (discussed separately)

Surface wind stress, (turbulent) frictional stress  $(\tau^x, \tau^y)(x, z)$  through depth to bottom

**Steady, along-slope (y-) uniform** context and flow

**No/small y-transport in adjacent deep ocean**, sets  $\frac{\partial \eta}{\partial y}$  there *and* over slope and shelf

$$\left( \frac{\partial^2 \eta}{\partial x \partial y} = \frac{\partial^2 \eta}{\partial y \partial x} = 0 \right)$$

- Resulting **explicit formula for along-slope flow** (from along-slope momentum)

$$\rho_s r v = \left( g \frac{h}{h_0} \int_{-h_0}^0 \int_z^0 \rho_y(z') dz' dz - g \int_{-h}^0 \int_z^0 \rho_y(z') dz' dz - \frac{h}{h_0} \tau_{s0}^y + \tau_s^y \right) - \frac{gr}{f} \int_{-h}^z \frac{\partial \rho_2}{\partial x} dz'$$

bottom  
friction

JEBAR

relative to deep ocean

wind stress

geostrophic  
shear

# Physical description

- E.g. density increases with  $y$  / poleward
- If no other forcing and deep-sea transport  $\sim 0$

$$\text{then } \int_{-h_0}^0 \frac{\partial p}{\partial y} dz = 0$$

(raised surface where less dense  
 $\rightarrow$  upper  $\frac{\partial p}{\partial y} < 0$  but lower  $\frac{\partial p}{\partial y} > 0$ )

- $\frac{\partial p}{\partial y}$  geostrophically balances zonal flow  $u$

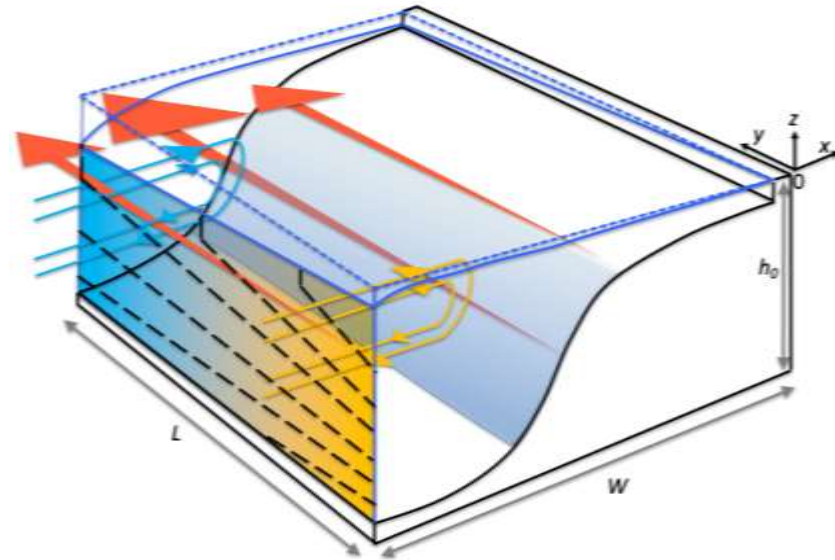
$$\text{with deep-sea } \int_{-h_0}^0 u dz = 0$$

- Over **shallower** slope, less “lower”

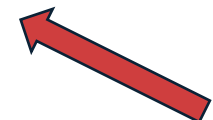
$$\rightarrow \int_{-h}^0 \frac{\partial p}{\partial y} dz < 0, \quad \int_{-h}^n u_{geostrophic} dz > 0$$

“excess”  $u$  returns in bottom Ekman layer

$$\leftrightarrow \text{friction on along-slope } v \leftrightarrow \int_{-h}^0 \frac{\partial p}{\partial y} dz < 0$$



-----  $\Delta$ Pressure contours  
 ( $\Delta p$  increases up)

 Along-slope flow

 Cross-slope flow

 cool warm

# Bottom stress and Ekman transport

- From depth-integrated along-slope momentum

with  $\frac{\partial}{\partial y}$  (along-shore transport) = 0 and coast  $\implies$  zero onshore transport

$$0 = \rho_r f \int_{-h}^{\eta} u dz = -gh\rho_s \frac{\partial \eta}{\partial y} - g \int_{-h}^0 \int_z^0 \frac{\partial \rho}{\partial y} dz' dz + \tau_s^y - \tau_b^y$$

onshore  
transport

pressure gradient (z)  
generally  $\neq 0$  !

wind bottom  
stresses

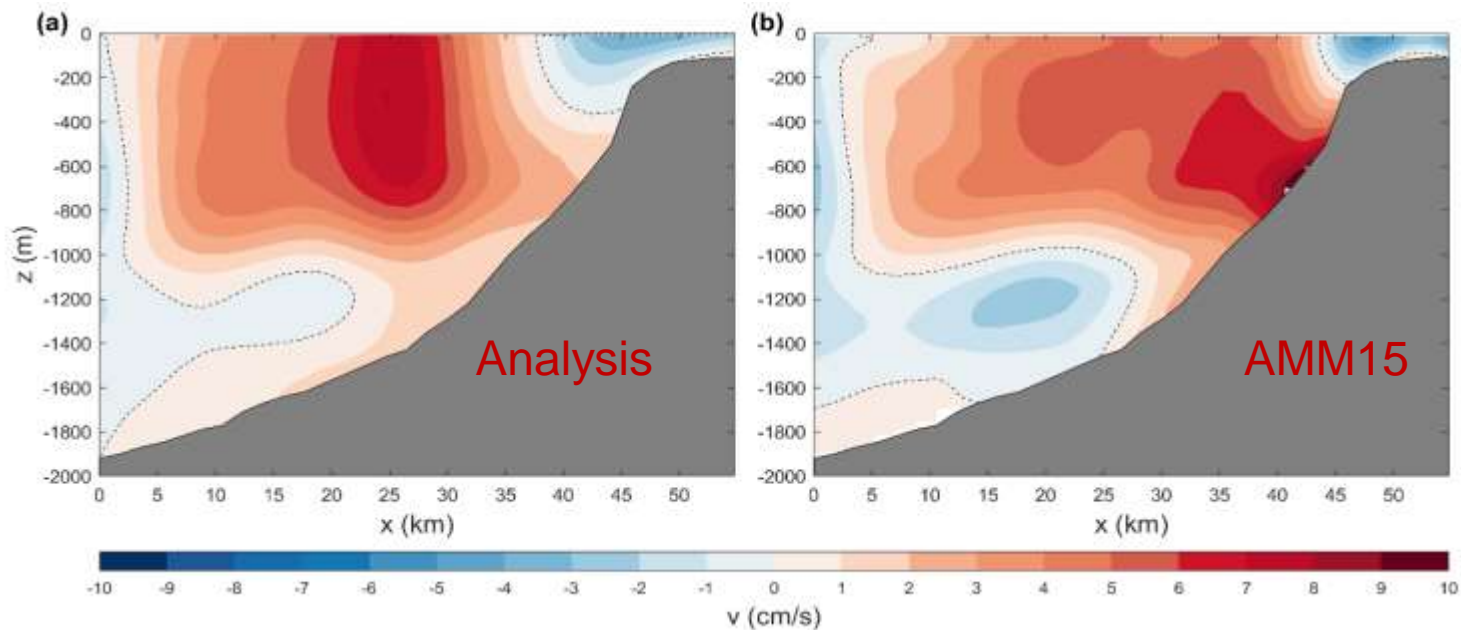
Bottom stress **almost certainly** not zero if there is any forcing or density y-gradient  
 $\rightarrow$  not “slippery”, Ekman transport  $\neq 0$

# Along-slope evolution

- Velocity evolves along slope (and in time) towards equilibrium
- Coastal-trapped waves (CTW) carry information about initial/"upstream" conditions
  - evolution scale set by decay distance of CTWs: days / 100s – 1000s km
    - depends strongly on context & forcing pattern
    - forcing match to higher-mode CTWs → shorter scales
    - strong friction or narrow shelf with weak stratification → shorter scales
- Density fields approximating  $\rho = \rho_y(z)y + \rho_2(x, z)$  over distance  $>$  evolution scale
  - expected to give near- equilibrated velocity
- Oceanic eddies impinging on the slope, and storms,
  - likely to cause departure from analysis owing to short scales.

# Comparison with AMM15 (numerical model; Graham et al. 2018)

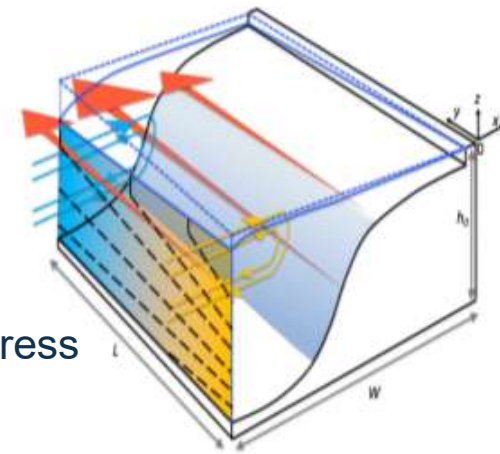
- $\rho = \rho_y(z)y + \rho_2(x, z)$  fitted to February 2018 in AMM15 (1.5 km NEMO)  
over 100 km along-slope  $x$  (200 – 1700 m cross-slope) at 56°N west of Scotland
- Analysis with bed friction factor  $r = 0.01 \text{ ms}^{-1}$ ,  $\tau_s = 0.1 \text{ Pa}$  (mean February wind stress)  
**Plots show along-slope flow:** along-slope (100 km) and February **average**  
“Analysis” laterally smoothed (15 km window) for estimated lateral “diffusion”



- Northward flow mostly from JEBAR and “thermal wind”
- Discrepancies! Factors might be **uniform  $r$  and wind stress (analysis)**; on/off-shelf **tidal currents** and tidal rectification, irregular **topography, variability** in time and **space (AMM15)**



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Thank-you!

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