# Settling of inertial particles in turbulent Rayleigh-Bénard convection

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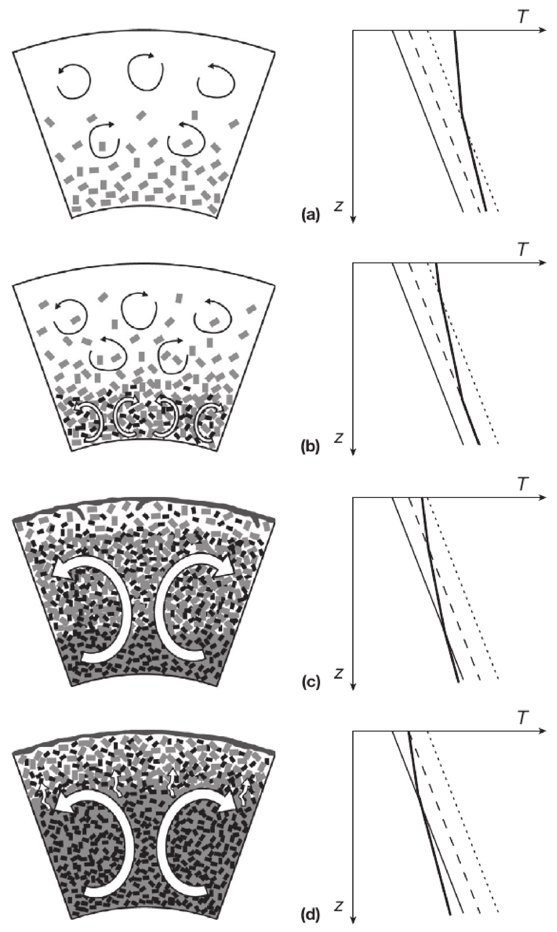
## Magma ocean

Earth and other terrestrial planets formed very hot due to

accretion: T up to thousands of K
core segregation: T 1700 K
Moon forming impact: T 7000 K
short lived isotopes: $Al^{26}$

Rayleigh number ~ $10^{27}$
is outside the reach of numerical simulations



## Motivation (Figures are from a review by Solomatov, 2015 [1])

Early differentiation of planetary mantles takes place upon crystallization of magma oceans. The convection of large-scale magma reservoirs is highly vigorous and turbulent. Motion of small crystals in such flow is non-trivial and can be captured by the Maxey-Riley equation [2]. To what extent turbulent convection affects the fate of suspended crystals is a matter of debate. Scaling laws for settling velocities of solid particles exist [3], but are not confirmed by direct numerical simulations in turbulent regimes at high Rayleigh numbers. Understanding the crystal behavior during magma ocean solidification is important for understanding the differentiation and initial compositional distribution of planetary mantles.

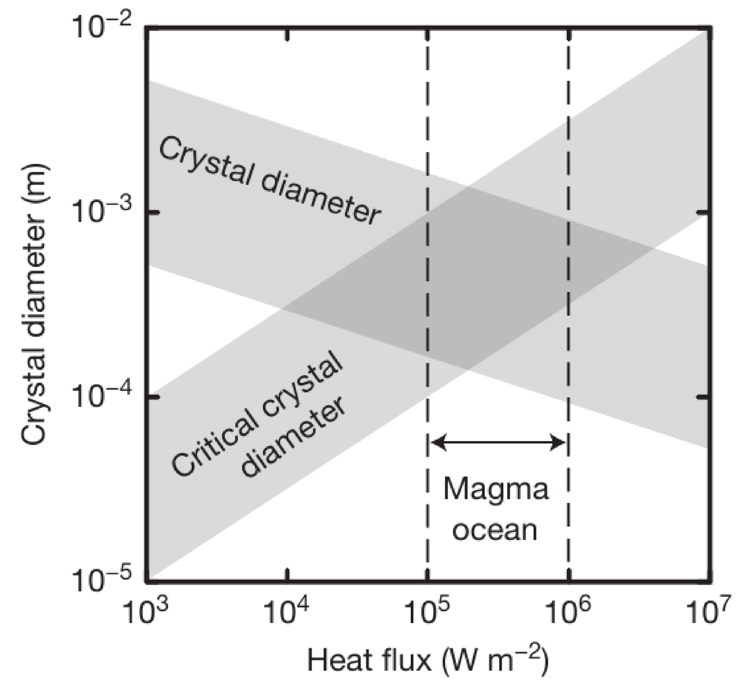



**Figure 8** The crystal size during the early stages of crystallization of the magma ocean (assuming that it is controlled by nucleation in the downwelling flow) and the critical crystal size for suspension are shown as functions of the heat flux. The width of the curves (gray bands) represents the uncertainty range of one order of magnitude.

## This work

Relative density difference between forming crystals and residual liquid depends on pressure, temperature, and bulk composition of magma. Depending on the stage of solidification, both light crystals in a denser fluid and dense crystals in a lighter fluid can be expected. In this study we perform simulations of highly vigorous (Rayleigh number $10^8$ - $10^{12}$) and turbulent (Prandtl number 10 - 50) convection with suspended particles. We vary the density and size of the particles and investigate the rate at which crystals settle and accumulate near the top and bottom boundaries of the model domain. The numerical experiments are performed in 2D Cartesian geometry using a freely available lattice Boltzmann code (https://github.com/ecalzavarini/ch4-project).

## Numerical model

Navier-Stokes equations in non-dimensional form

$$\begin{aligned}
\partial_\tau \mathbf{U} + (\mathbf{U}\cdot\nabla)\mathbf{U} &= -\nabla P + \sqrt{\frac{Pr}{Ra}}\,\nabla^2\mathbf{U} + \theta\hat{\mathbf{z}} \\
\nabla\cdot\mathbf{U} &= 0 \\
\partial_\tau \theta + (\mathbf{U}\cdot\nabla)\theta &= \frac{1}{\sqrt{Pr\,Ra}}\,\nabla^2\theta
\end{aligned}$$

characteristic time is based on $U_f$, relating $Ra, Pr, Re$ as

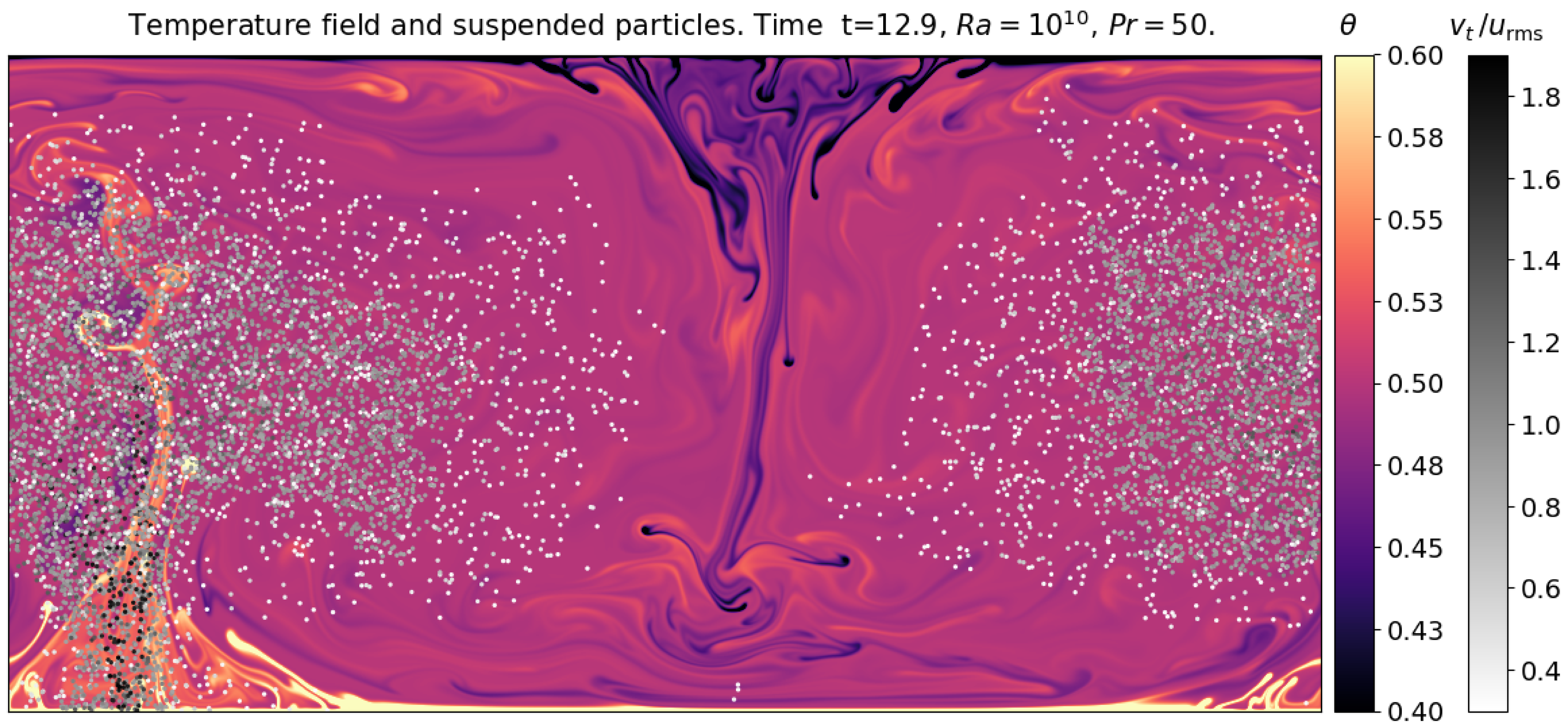
$$\sqrt{\frac{Pr}{Ra}} = \sqrt{\frac{\nu\nu\kappa}{\kappa\alpha g\Delta T H^3}} = \frac{\nu}{\sqrt{\alpha g \Delta T H}H} = \frac{\nu}{U_f H} = \frac{1}{Re}$$

Particle dynamics are described by Maxey-Riley equation

$$\frac{d\mathbf{V}}{d\tau} = \beta\frac{D\mathbf{U}}{D\tau} + \frac{1}{St}(\mathbf{U}-\mathbf{V}) + \Lambda\hat{\mathbf{z}}$$

with control parameters $\Lambda = (\beta-1)/(\alpha\Delta T)$ and

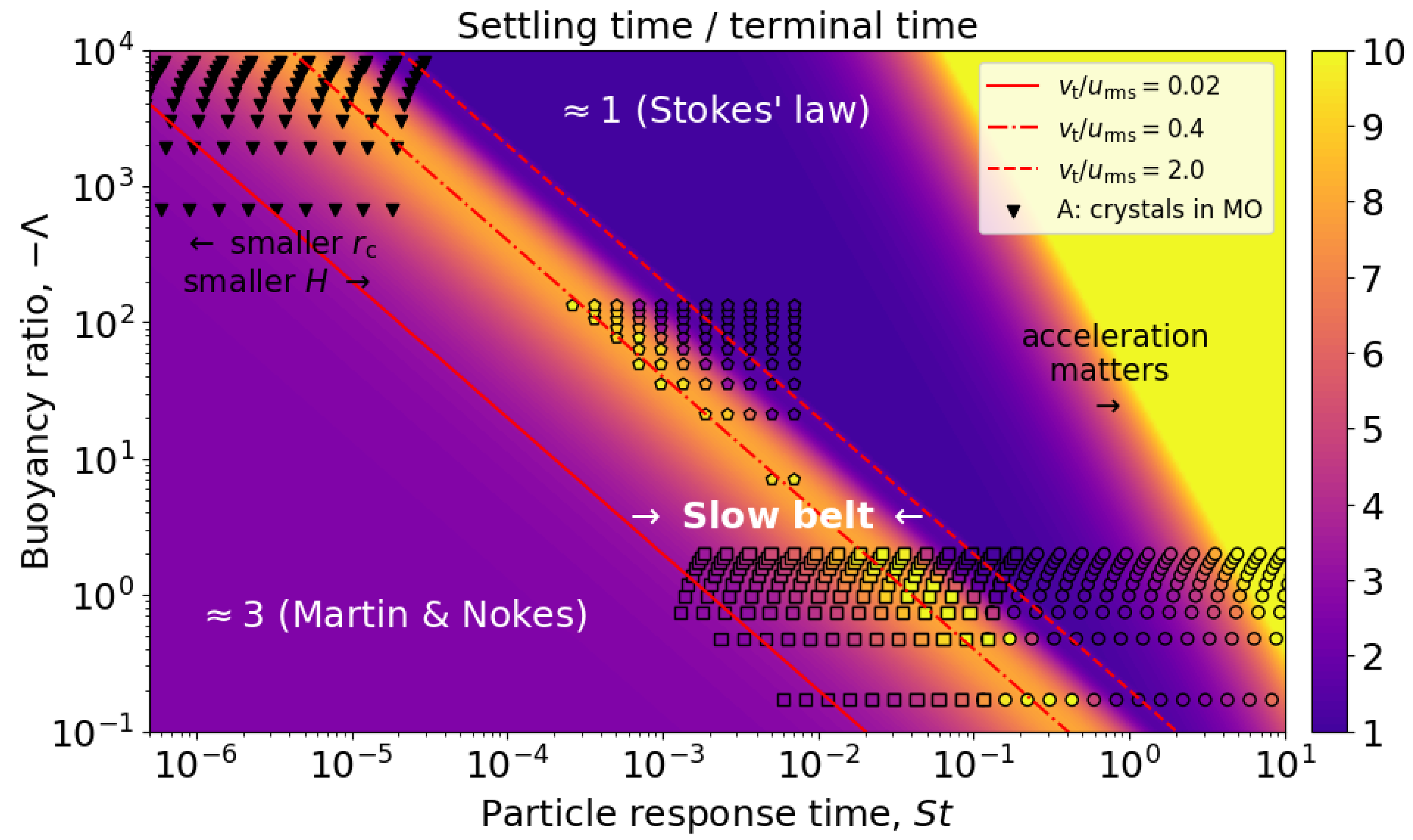
$$\beta = 3\rho_f/(\rho_f + 2\rho_p), \quad St = R^2\sqrt{\alpha g \Delta T}/(3\nu\beta\sqrt{H}).$$




Temperature field in the simulation with Ra=$10^{10}$ and Pr=50 at the time t = 12.9. The temperature range is clipped for a better visibility of the up- and downwellings. Dots show particles with 0.3 < vt /urms < 2.0 that have not settled by the respective time. Most of these heavy particles settle in the large cluster of upwellings located in the left bottom part of the model domain. Characteristic velocity of the flow is $u_{rms}$ = 0.1.

## Results

The key parameter that controls crystal settling is the ratio $v_s/u_{rms}$, where $v_s = \Lambda\,St$ is the Stokes velocity and $u_{rms}$ is the characteristic velocity of the flow. In the figure below we show the ratio of observed settling time with respect to the time it would take to sink through the model domain in a motionless fluid (terminal time, H / $v_s$). The observed settling time never exceeds the terminal time by more than one order of magnitude, and the resulting ratio does not depend strongly on Rayleigh number [4].




## References

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## Acknowledgement

VP and NT acknowledge support from the Helmholtz Association (project VH-NG-1017) and the DLR-DAAD research fellowship program.