

# Hard to measure, hard to model: Using information theory to understand turbulent heat fluxes

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## Motivation and summary

- Simple data-driven models are often able to perform better at predicting turbulent heat fluxes at fine temporal resolutions than complex process-based models.
- To better understand how (process-based) model performance is linked to process-level interactions in simulating latent heat we compare a large ensemble of simulations
- We compare our simulations to latent heat observations across many FluxNet sites
- Instead of relying on purely descriptive performance measures we employ methods from information theory to analyze both the simulations and observations in both moisture and energy limited situations.
- We find that generally the interaction between shortwave radiation and latent heat is similar between observations and simulations, but the interaction between antecedent precipitation is not

# Study domain

We simulate and analyze 40 sites with FluxNet towers

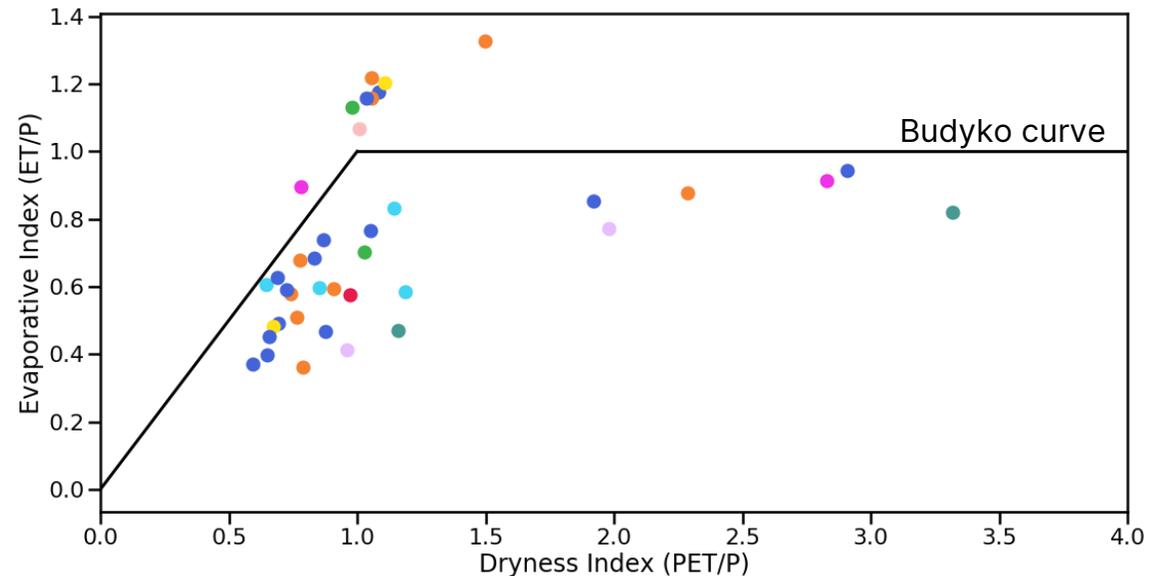
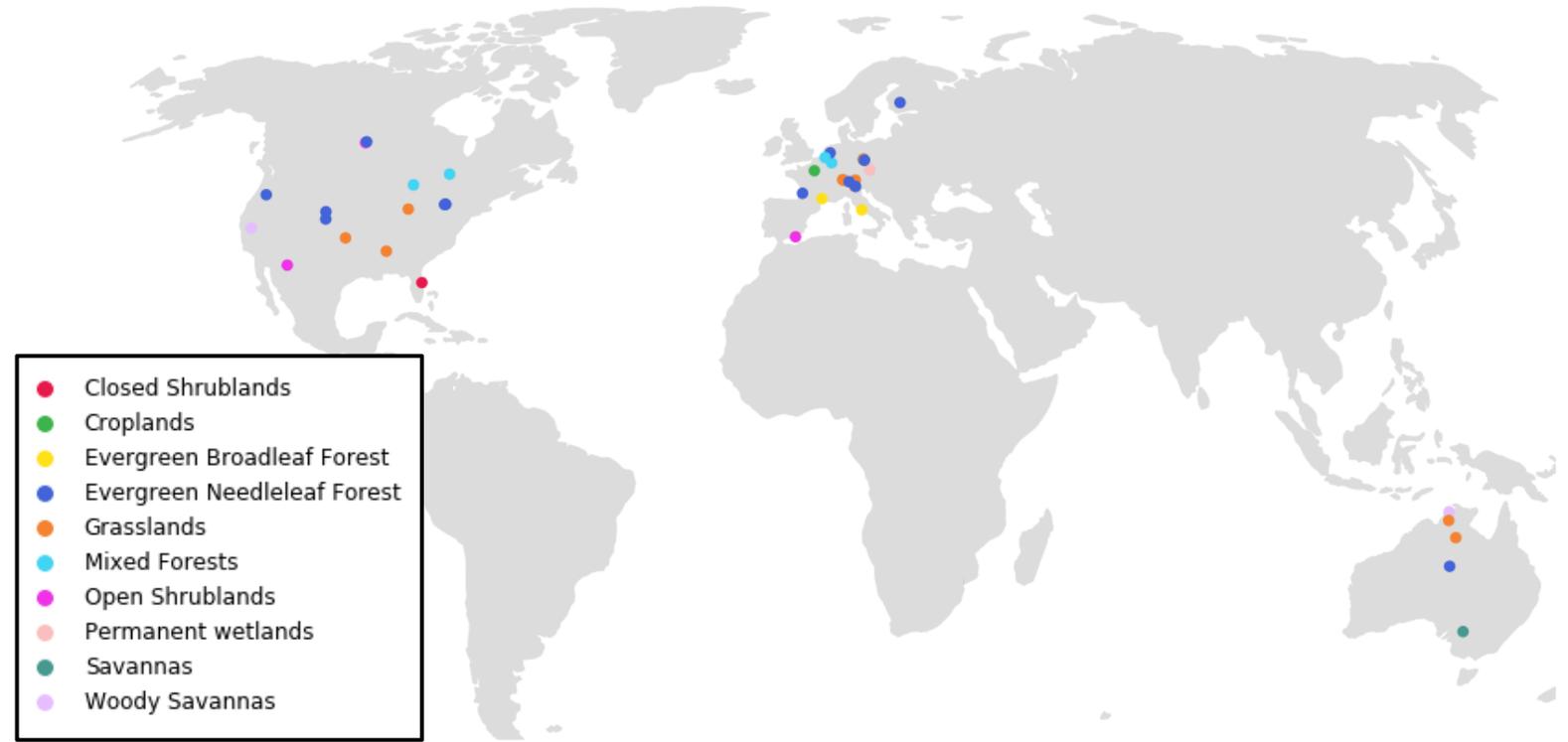
For this display we will limit the analysis to latent heat fluxes

Sites represent mid-latitude environments well, with a mix of vegetation and soil classes

All sites have at least three years of half hourly data with <20% missing data

All observed latent heat fluxes have energy balance corrections applied by FluxNet operators

All necessary forcing data for simulations is provided by observations



# Simulation details

We use SUMMA to simulate latent heat fluxes

SUMMA is a framework for implementing hydrologic models

User can choose spatial discretization and flux parameterizations allowing ensembles to be built in a controlled fashion

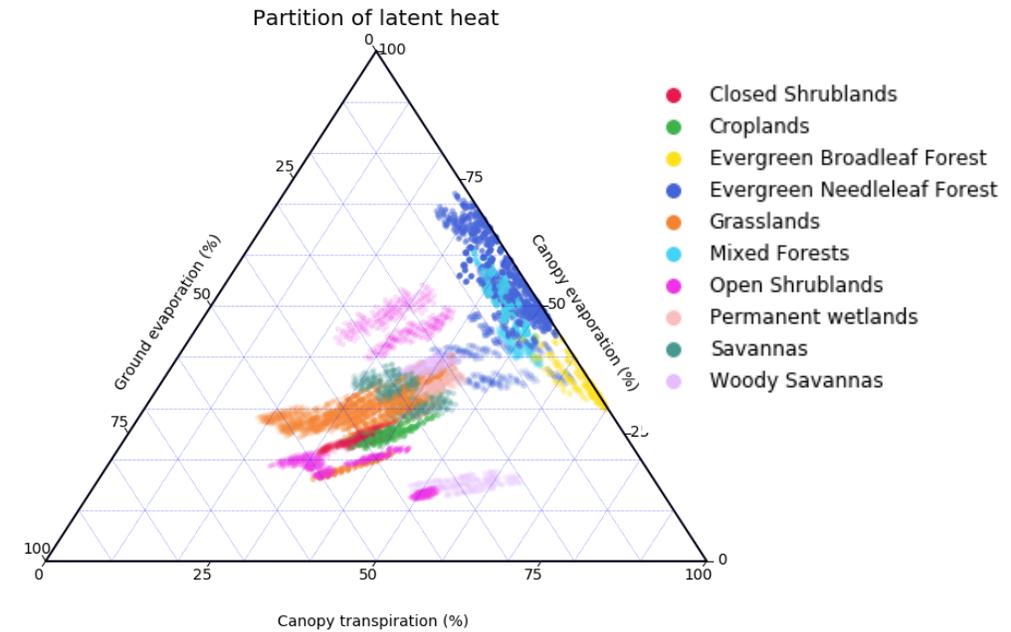
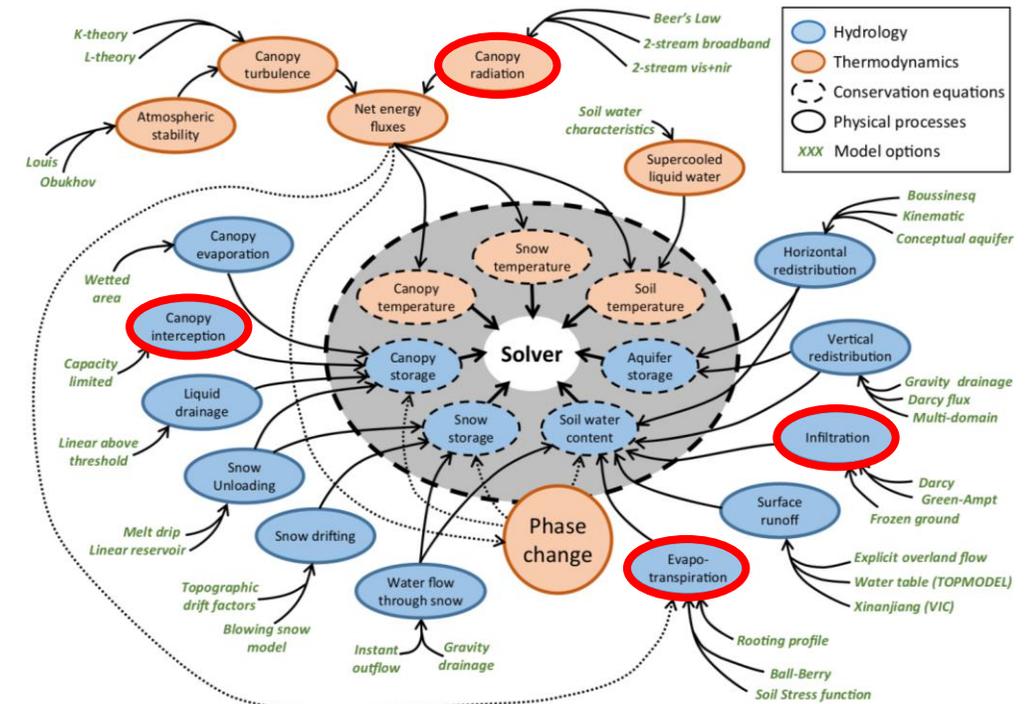
# Ensemble details

For each site we run 384 simulations varying parameters and parameterizations for vegetation and soil characteristics

The options we are modifying to build our ensembles are highlighted in red on the SUMMA diagram

This gives a good coverage of model space as shown by the ternary diagram

Ternary diagram shows relative contributions of ground evaporation, canopy evaporation, and canopy transpiration to latent heat for each simulation



# We use two performance measures

## Normalized mutual information (*NMI*):

How much does measurement of one variable reduce prediction error in another?

Alternatively, how does the joint distribution differ from the product of marginal distributions?

(higher values are better, total range in  $[0, 1]$ , common values for hydrology generally in  $[0, 0.25]$ )

## Kling-Gupta Efficiency (*KGE*):

A distance measure that equally balances errors of correlation ( $r$ ), variability ( $\sigma$ ), mean ( $\mu$ ) behavior of simulation and observations

(total range in  $[-\infty, 1]$ , reasonable values generally in  $[-0.41, 1]$ )

We refer to *NMI* as a measure functional performance and *KGE* as a measure of predictive performance

## Normalized mutual information:

$$NMI = \frac{I(X; Y)}{\min[H(X), H(Y)]}$$

Mutual information:

$$I(X; Y) = \int_{x,y} p_{XY}(x, y) \log \left( \frac{p_{XY}(x, y)}{p_X(x)p_Y(y)} \right) dydx$$

Entropy:

$$H(X) = - \int_x p(x) \log(p(x)) dx$$

## Kling-Gupta Efficiency:

$$KGE = 1 - \sqrt{(r - 1)^2 + \left( \frac{\sigma_{sim}}{\sigma_{obs}} - 1 \right)^2 + \left( \frac{\mu_{sim}}{\mu_{obs}} - 1 \right)^2}$$

# Functional vs predictive performance

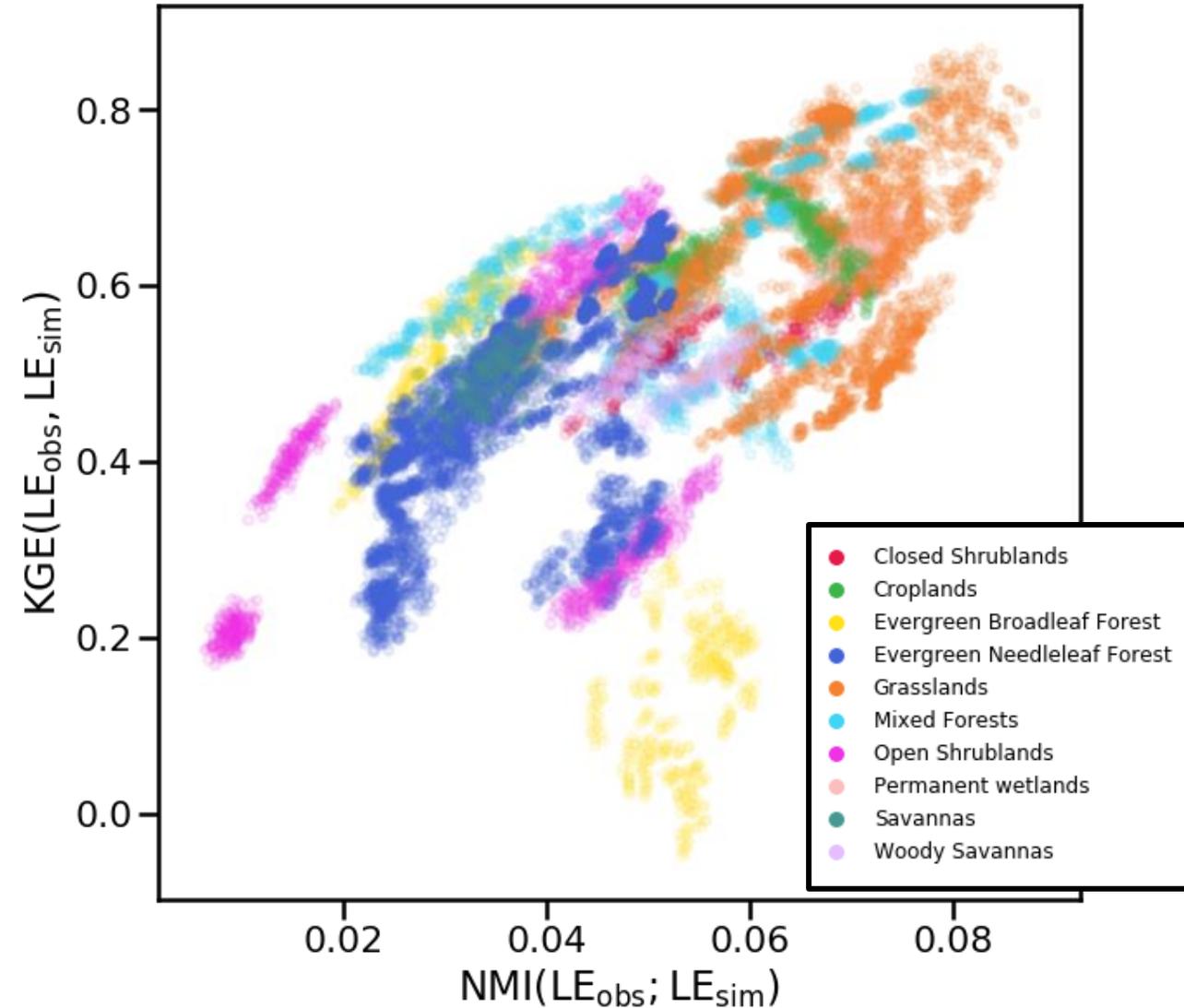
First, we explore the relationship between functional and predictive capabilities by computing the *NMI* and *KGE* for all simulations

Each dot represents a single simulation

We compute these scores at 30-minute timestep for daylight hours only, as flux measurements have high biases and low values at night

General observations:

- There are correlations between *NMI* and *KGE* both within each site's ensemble and between sites
- Sites which do not fall along the Pareto front are less likely to exhibit this correlation
- Grasslands and mixed forests tend to perform better than other vegetation classes
- There is a wide variation of performance within both site and vegetation type



# Evaluating process connections

Following the Budyko curve we explore processes related to energy and moisture limited processes

We consider that the energy-limited factor is the incoming shortwave radiation ( $SW$ ) and the moisture-limited factor is the 2-day antecedent precipitation ( $P_{ant}$ )

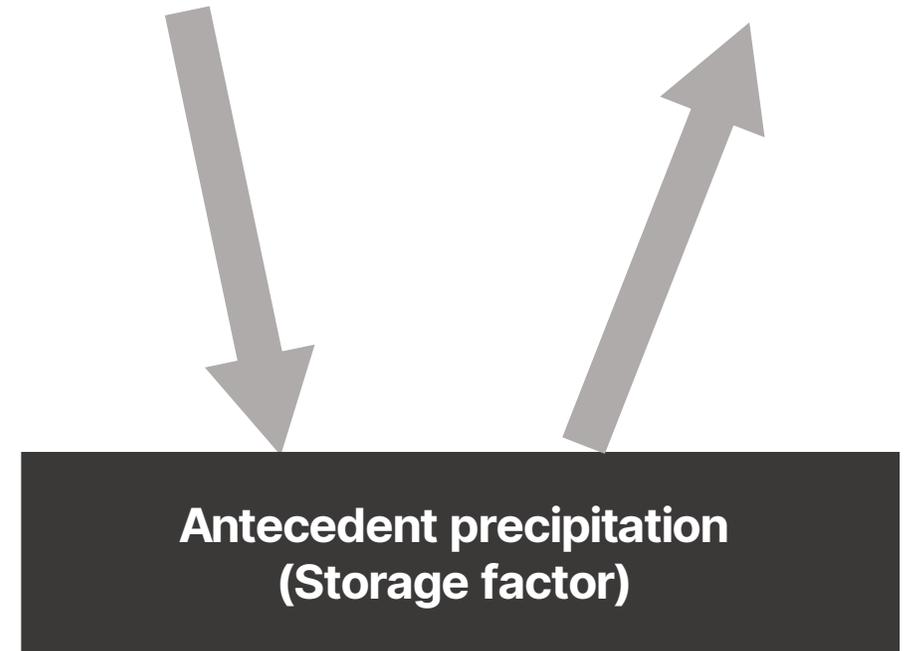
$P_{ant}$  is considered instead of soil moisture because of a lack of measurements and modeled quantities may not be commensurable with existing measurements

Then, we look at how information is shared with  $LE$  and  $SW$  and  $P_{ant}$  for both the simulations and observations

We look at wet ( $P_{ant}$  above 75<sup>th</sup> percentile) and dry ( $P_{ant}$  below 25<sup>th</sup> percentile) conditions

Shortwave Radiation  
(Energy factor)

Latent heat  
(Response)



# Evaluating process connections

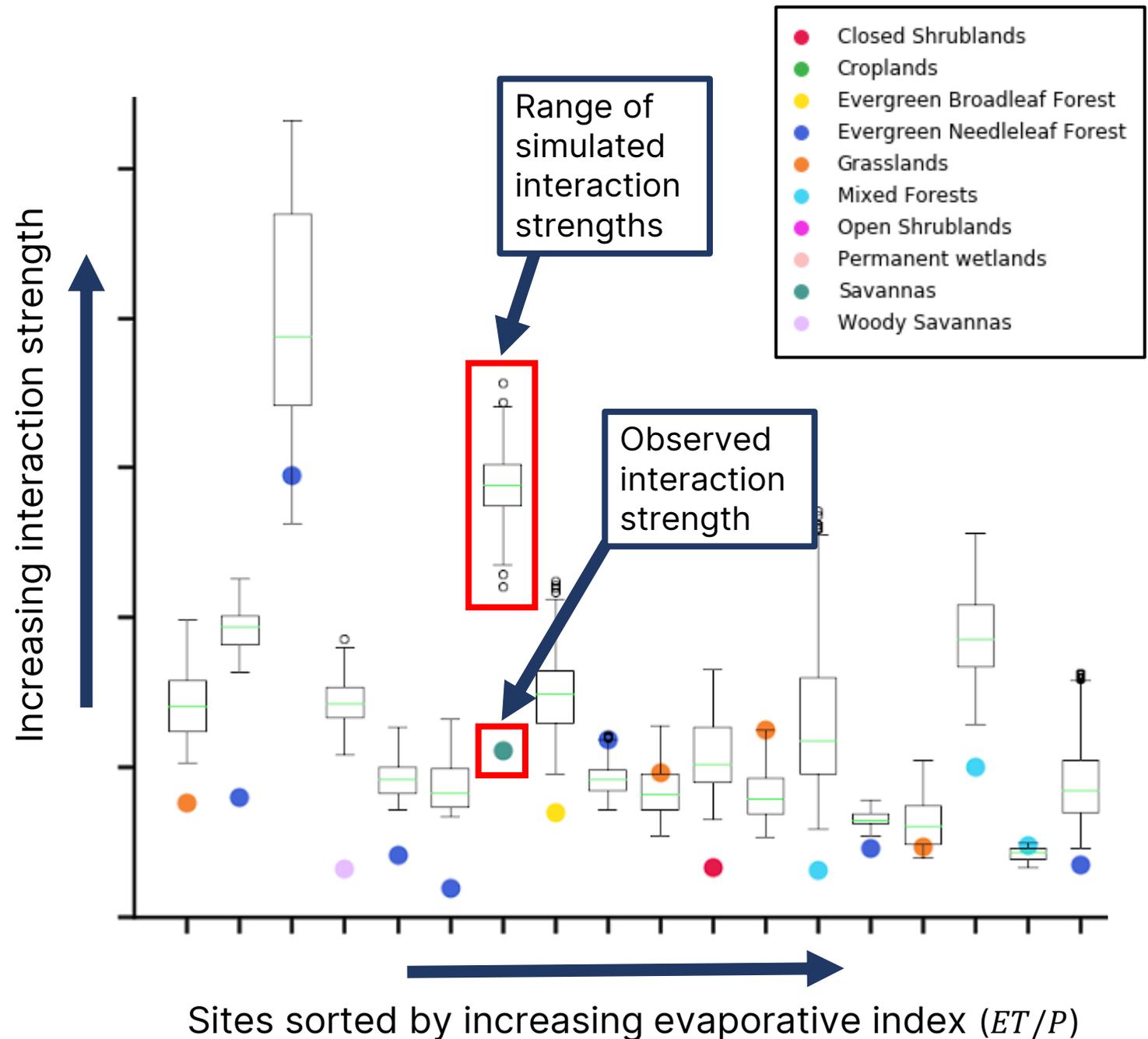
An example figure is shown to explain the details of how we visualize following results

To examine how process interactions vary we show boxplots for each site showing the range of interaction strengths computed

Additionally, we show the observed values with markers colored by their respective vegetation class

Good simulations should match the observed value

Sites are sorted by increasing evaporative index



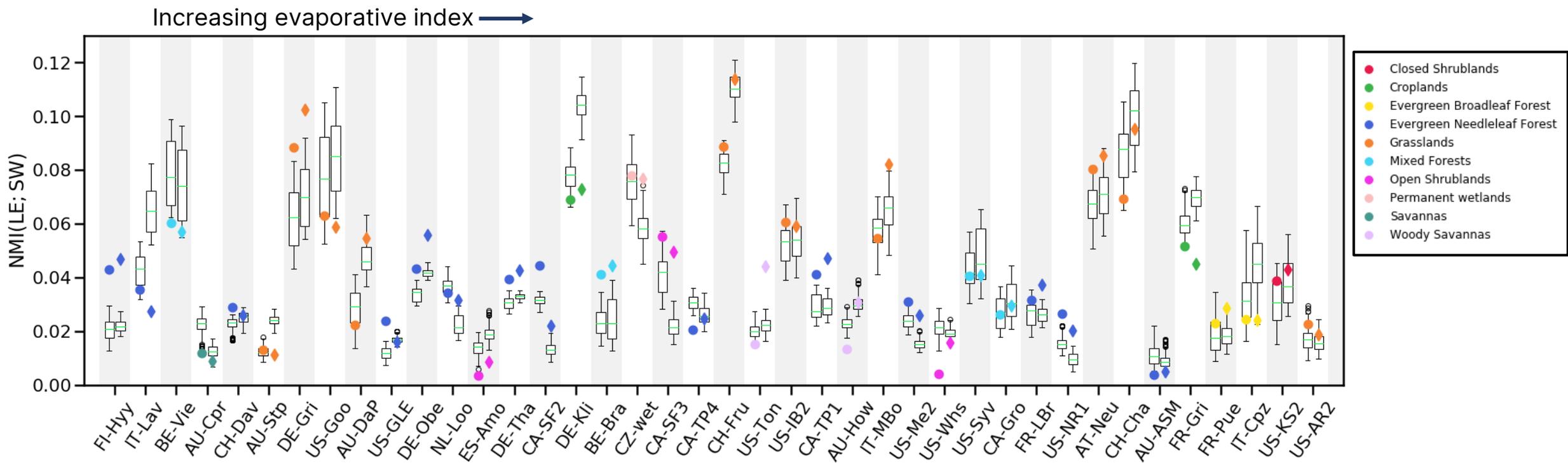
# Linkage of latent heat to shortwave radiation

Simulations capture inter-site variability quite well

Under both conditions the observed and modeled relationships are often similar

Both the dry (left boxplot, circle marker) and wet (right boxplot, diamond markers) periods share information between  $LE$  and  $SW$  similarly

There is no discernable trend with respect to either evaporative index or vegetation class

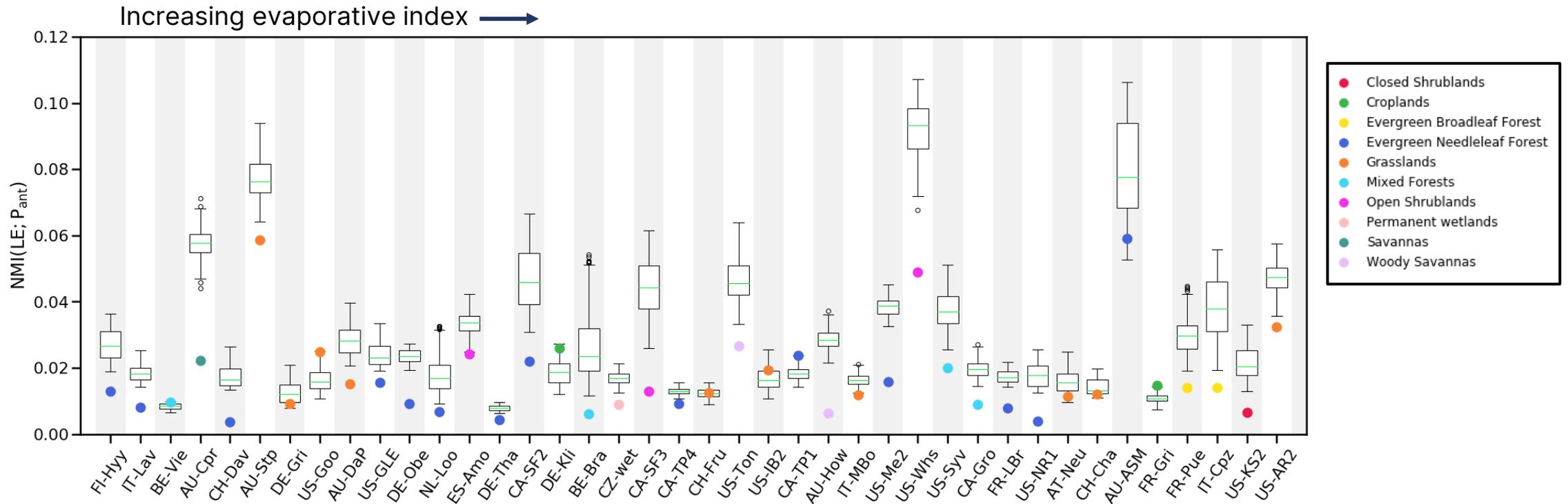


# Linkage of latent heat to antecedent precipitation

We show results for the wet period only, the dry period had no linkage for both observations and simulations (as expected)

Generally the shared information between  $LE$  and  $P_{ant}$  in the simulations is higher than the observed

This may indicate either a structural error in the simulations or a bias in the observations, but further analysis is needed



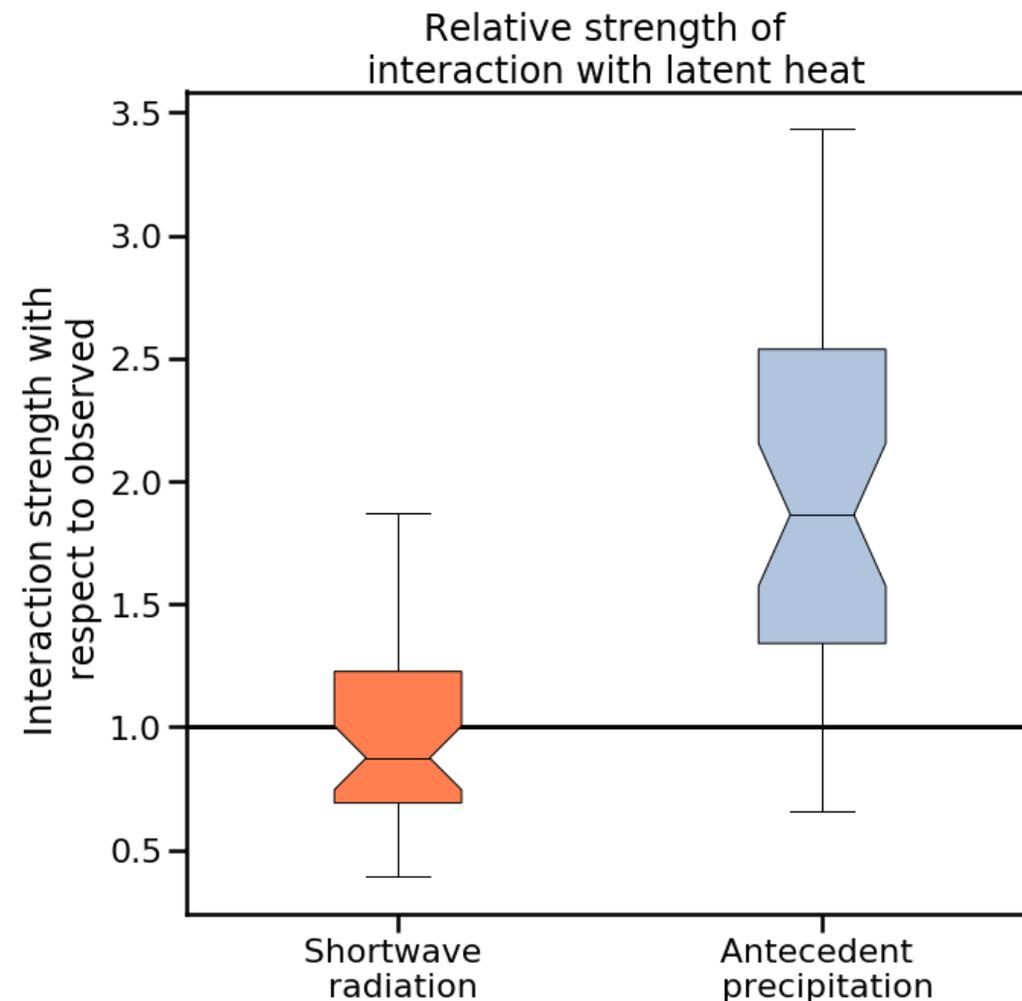
# Findings and discussion

Using a large ensemble of simulations we found that

- Predictive and functional performance tend to be well correlated both within and between sites
- the modeled relationship between shortwave radiation and latent heat is similar to the observed relationship
- the modeled relationship between latent heat and antecedent precipitation is almost twice as strong as the observed relationship

There are still many open questions and avenues for further research

- Two-day antecedent precipitation may not be a good proxy for the soil moisture
- Filtering out times when it is raining but have high antecedent precipitation may also provide better results
- We still need to explore model performance with respect to modeling options chosen



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FluxNet data provided by the FLUXNET2015 dataset as described by Pastorello et al 2017