Evolution of extreme wave statistics in surface elevation and velocity field over a non-uniform depth

Christopher Lawrence¹, Karsten Trulsen¹ Odin Gramstad²
¹Department of Mathematics, University of Oslo, chrislaw@math.uio.no, karstent@math.uio.no
²Group Technology and Research, DNV GL, Høvik, Norway, Odin.Gramstad@dnvgl.com

It was shown experimentally in Trulsen et al. (2012) that irregular water waves propagating over a slope may have a local maximum of kurtosis and skewness in surface elevation near the shallower side of the slope. Later on, Raustøl (2014) did laboratory experiments for irregular water waves propagating over a shoal and found the surface elevation could have a local maximum of kurtosis and skewness on top of the shoal, and a local minimum of skewness after the shoal for sufficiently shallow water. Numerical results by Sergeeva et al. (2011), Zeng & Trulsen (2012), Gramstad et al. (2013) and Viotti & Dias (2014) support the experimental results mentioned above. Just recently, Jorde (2018) did new experiment with the same shoal as in Raustøl (2014) but with additional measurement of the interior horizontal velocity. The experimental results from Raustøl (2014) and Jorde (2018) were reported in Trulsen et al. (2020) and it was found the evolution of skewness for surface elevation and horizontal velocity have the same behaviour but the kurtosis of horizontal velocity has local maximum in downslope area which is different with the kurtosis of surface elevation.

In present work, we utilize numerical simulation to study the effects of incoming significant wave height, peak wave frequency on evolution of wave statistics for both surface elevation and velocity field with more general bathymetry. Numerical simulations are based on High Order Spectral Method (HOSM) for variable depth Gouin et al. (2016) for wave evolution and Variational Boussinesq model (VBM) Lawrence et al. (2018) for velocity field calculation.

References


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Christopher Lawrence\textsuperscript{1}
Karsten Trulsen\textsuperscript{1}
Odin Gramstad\textsuperscript{2}

\textsuperscript{1} Department of Mathematics, University of Oslo, Norway
\textsuperscript{2} Group Technology and Research, DNV GL, Høvik, Norway
Lab experiments at UiO

Anne Raustøl (2014)

Stian Jorde (2018)

<table>
<thead>
<tr>
<th>Run</th>
<th>$T_p$ [s]</th>
<th>$h_1$ [cm]</th>
<th>$k_p h$</th>
<th>$H_s$ [cm]</th>
<th>$\epsilon$</th>
<th>$Ur$</th>
<th>$h_2$ [cm]</th>
<th>$k_p h$</th>
<th>$H_s$ [cm]</th>
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Table 1. Key parameters for all runs. The runs are numbered according to increasing dimensionless depth over the shoal. Run 3 belongs to the second campaign (Jorde 2018), the rest are from the first campaign (Raustøl 2014). The values of significant wave height $H_s$, steepness $\epsilon$ and Ursell number $Ur$ are averages over all probes in front of or above the shoal, not including the probes at the edges of the sloping bottom.
Numerical model
• High Order Spectral Method for wave evolution over variable depth


• Variational Boussinesq model for kinematics calculation


High Order Spectral Method

Dynamic equations

\[
\frac{\partial \eta}{\partial t} = W \left( 1 + \left| \frac{\partial \eta}{\partial x} \right|^2 \right) - \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x}
\]

\[
\frac{\partial \phi}{\partial t} = -g \eta - \frac{1}{2} \left| \frac{\partial \phi}{\partial x} \right|^2 + \frac{1}{2} W^2 \left( 1 + \left| \frac{\partial \eta}{\partial x} \right|^2 \right)
\]

Bottom boundary condition

\[
\frac{\partial \Phi}{\partial x} \frac{\partial B}{\partial x} - \frac{\partial \Phi}{\partial z} = 0 \quad \text{at} \quad z = -h_0 + B(x)
\]

\[\nabla \cdot \nabla \Phi = 0\]

\[\phi = \Phi(x, z = \eta, t)\]

\[W = \frac{\partial \Phi}{\partial z} \bigg|_{z=\eta}\]
High Order Spectral Method

The velocity potential is truncated as power series

\[ \Phi(x, z, t) = \sum_{m=1}^{M} \Phi^{(m)}(x, z, t) \]

\[ \Phi^{(m)} = \Phi_0^{(m)} + \Phi_B^{(m)} \]

\[ \Phi_0^{(m)} = \sum_j A_j \frac{\cosh (k_j (z + h_0))}{\cosh (k_j h_0)} e^{i k_j x} \]

\[ \Phi_B^{(m)} = \sum_j B_j \frac{\sinh (k_j z)}{\cosh (k_j h_0)} e^{i k_j x} \]
Variational Boussinesq model (VBM)

\[ \Phi(x, z) \approx \phi(x) + \sum_{m=1}^{N} \left( \frac{\cosh \kappa_m(z + h)}{\cosh \kappa_m(\eta + h)} - 1 \right) \psi_m(x) \]

Solve the Laplace via Dirichlet principle (minimize the kinetic energy)

\[ K(\phi, \eta) = \text{Min}(D(\Phi)| \Phi = \phi \text{ at } z = \eta) \]

\[ D(\Phi) = \int \int \frac{1}{2} |\nabla \Phi|^2 dxdz \]
Statistics of irregular waves propagating over a shoal
Simulation setup

Incoming waves with JONSWAP spectrum: $H_s=2.5\text{cm}$, $T_p=1.1\text{s}$, $\gamma=3.3$

100 different realizations with time series of $200T_p$ are used to calculate statistical quantities (kurtosis and skewness).

The bathymetry is the same with laboratory experiments.

$\begin{align*}
  x_1 &= 30\ \text{m} \\
  x_2 &= 31.6\ \text{m} \\
  x_3 &= 33.2\ \text{m} \\
  x_4 &= 34.8\ \text{m} \\
  h_1 &= 0.53\ \text{m} \\
  h_2 &= 0.11\ \text{m}
\end{align*}$
Simulation

Experiment
Effect of $H_s$

Incoming waves with JONSWAP spectrum: $H_s=2, 2.5, 3.0$ cm, $T_p=1.1$ s, $\gamma=3.3$

- $x_1 = 30$ m
- $x_2 = 31.6$ m
- $x_3 = 33.2$ m
- $x_4 = 34.8$ m
- $h_1 = 0.53$ m
- $h_2 = 0.11$ m
$H_s = 2\text{cm}$

$H_s = 2.5\text{cm}$

$H_s = 3\text{cm}$
Effect of $T_p$

Incoming waves with JONSWAP spectrum: $H_s=2.5$ cm, $T_p=1, 1.1, 1.2$ s, $\gamma = 3.3$

$x_1 = 30$ m
$x_2 = 31.6$ m
$x_3 = 33.2$ m
$x_4 = 34.8$ m

$h_1 = 0.53$ m
$h_2 = 0.11$ m
$T_p = 1.0s$

$T_p = 1.1s$

$T_p = 1.2s$
Effects of upslope

Incoming waves with JONSWAP spectrum: Hs=2.5cm, Tp=1.1s, $\gamma =3.3$

Three different upslope were investigated with numerical simulations

\[
\begin{align*}
x_1 &= 30 \text{ m} \\
x_2 &= 31.6, 35, 40 \text{ m} \\
x_3 &= 100 \text{ m} \\
x_4 &= 110 \text{ m} \\
h_1 &= 0.53 \text{ m} \\
h_2 &= 0.11 \text{ m}
\end{align*}
\]
Surface elevation

$u$ at $z = -0.04$ m

$u$ at $z = -0.06$ m

$u$ at $z = -0.08$ m
Effects of downslope

Incoming waves with JONSWAP spectrum: $H_s=2.5\text{cm}$, $T_p=1.1\text{s}$, $\gamma=3.3$

Three different downslope were investigated with numerical simulations

$x_1 = 30 \text{ m}$
$x_2 = 31.6 \text{ m}$
$x_3 = 33.2 \text{ m}$
$x_4 = 34.8, 36.2, 38.2 \text{ m}$

$h_1 = 0.53 \text{ m}$
$h_2 = 0.11 \text{ m}$
Surface elevation $u$ at $z = -0.04\ m$
$u$ at $z = -0.06\ m$
$u$ at $z = -0.08\ m$
Conclusion

• For the first time according to the authors knowledge, the numerical simulations are able to reproduce statistical properties of wave kinematics of irregular waves propagating over a shoal as in the laboratory experiments [1].

• For irregular waves propagating over a shoal in sufficiently shallow water, the surface elevation has local maximum of skewness and kurtosis near the edge of the upslope on shallower side and a local minimum of skewness on the downslope of the shoal. Meanwhile, the horizontal velocity has local maximum on downslope of the shoal and local minimum at the same location with local maximum of kurtosis of surface elevation.

• The local effects on kurtosis and skewness may disappear if the length of slope is sufficiently long.
References


