UNIVERSITY of HOUSTON

COLLEGE of NATURAL SCIENCES & MATHEMATICS

Combining tomographic images and geodynamic modeling of past mantle flow:

from simple analytical solutions to numerical inverse methods

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EGU 2020 – Sharing Geoscience Online

Mantle flow in a nutshell

Thermal convection of an extremely viscous fluid in a spherical shell: hot and light material rises outward while cold and dense material sinks inward.

It is governed by well-understood conservation equations of fluid mechanics, which are based on **physical principles**:

- Conservation of mass
- Conservation of momentum
- Conservation of energy

Mantle flow has far-reaching implications

- Tectonic stresses on the lithosphere
  - Normal stresses: dynamic topography, falcogenic events, variations of accommodation space
  - Shear stresses: tectonic force balance (with plate boundary forces), intraplate seismicity
- Advection of mantle material
  - Provenance of geochemical fingerprints
  - Existence and evolution of reservoirs
  - Fate of slabs, plumes
    - Interpretation of seismic structure
    - Implications for kinematic models of past plate motions

Modeling mantle flow

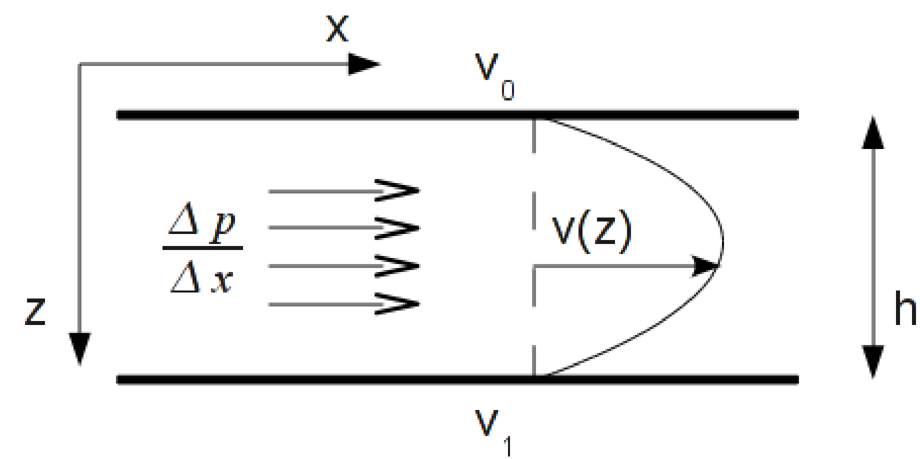
- The governing equations can be solved analytically only for special cases under rather strong simplifying assumptions
- Computational geodynamics aims at solving the governing equations accurately and efficiently using numerical methods
- Both approaches have strength and weaknesses which must be taken into account

Part one:

Analytical solution

Pressure-driven channel flow

- A thin and low-viscosity asthenosphere can be modelled as a viscous fluid sandwiched between two infinite parallel plates
- The fluid is driven by a pressure gradient
- The pressure gradient implies lateral variations in the normal stress on the overlying lithosphere, i.e. dynamic topography

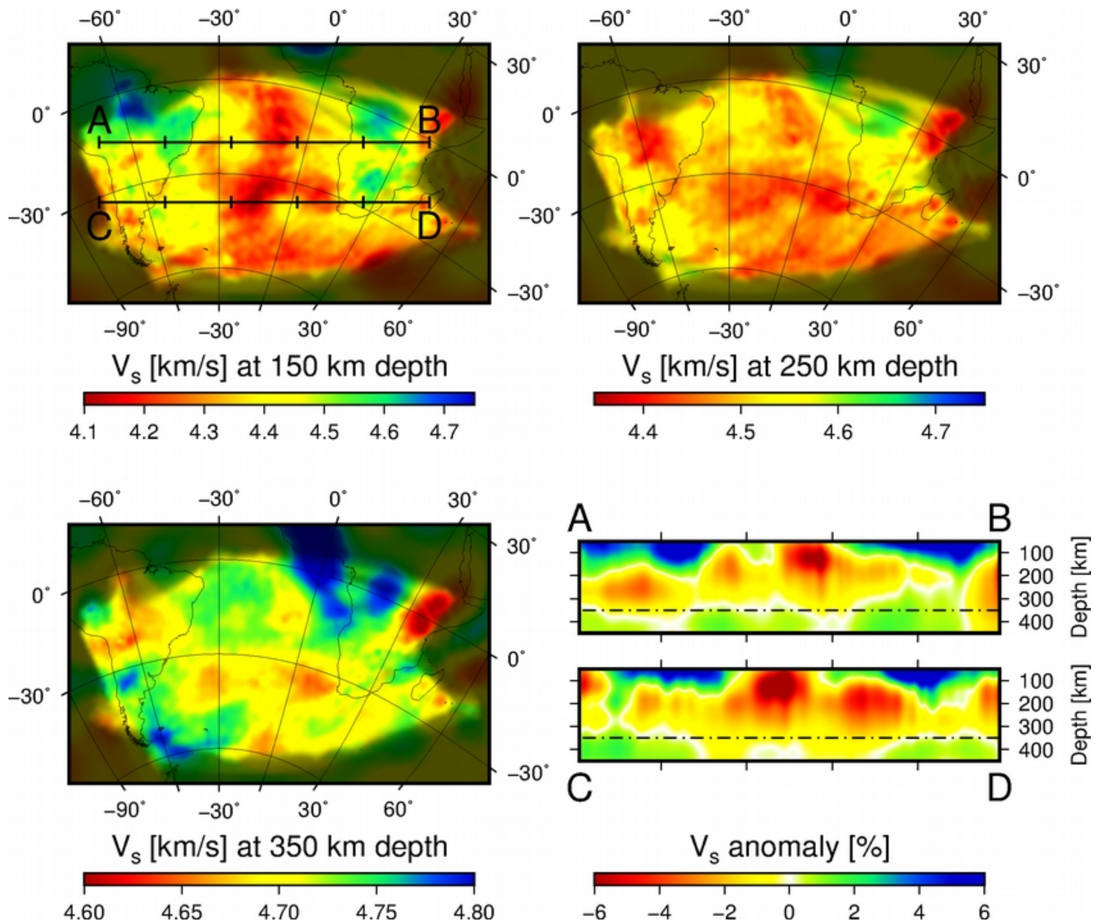


$$v(z)=\frac{1}{\eta}\frac{\Delta p}{\Delta x}z(h-z)$$

$$\sigma_{xz}(z=0)=\frac{\Delta p}{\Delta x}h$$

Application I: South Atlantic Ocean

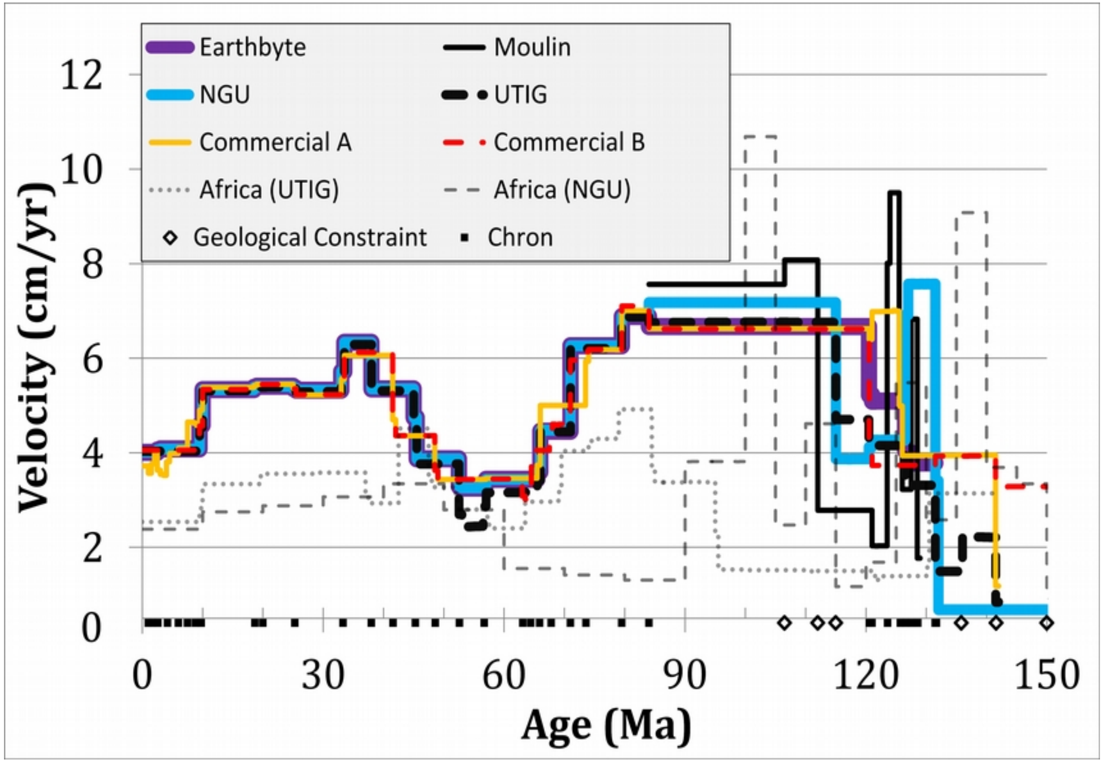
- Tomographic imaging suggests that the asthenosphere in the South Atlantic Ocean is ~200 km thick (Colli et al. 2013)
- Similar results in the North Atlantic (Rickers et al. 2013), in the Pacific (French, Lekić and Romanowicz 2013) and in the Caribbean (Zhu et al. 2020)



Colli et al. 2014

Application I: South Atlantic Ocean

- The South Atlantic experienced big variations (2x–3x) in spreading rate over short timescales (~10 Ma)



Colli et al. 2014

Application I: South Atlantic Ocean

- The South Atlantic experienced big variations (2x–3x) in spreading rate over short timescales (~10 Ma)
- The main plate-driving forces come from large-scale buoyancy anomalies mediated by viscous stresses in a convecting mantle (Forsyth & Uyeda, 1975; Lithgow-Bertelloni & Richards, 1998)
- but they evolve over longer time scales (a transit time, ≈100 Ma). As such we need:
  - A mechanism to decouple the lithosphere from the lower mantle
  - A tectonic force that can change rapidly

Colli et al. 2014

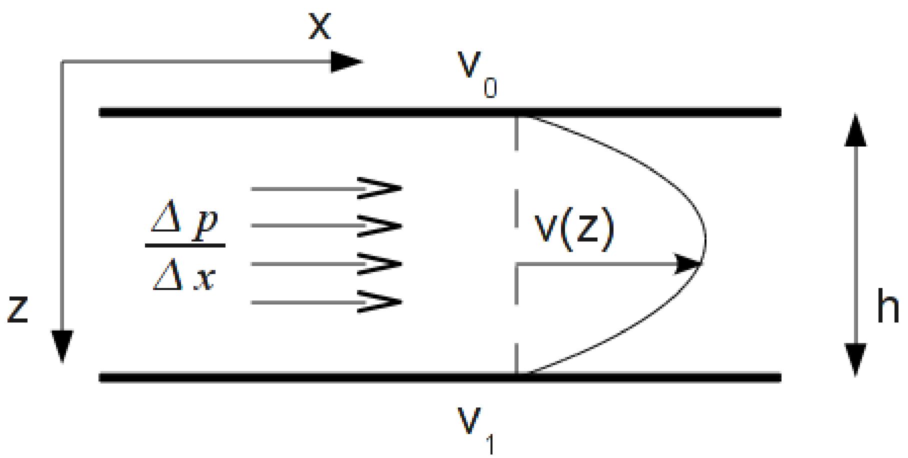
Application I: South Atlantic Ocean

- The growth of the Andes has been linked to the recent slowdown since Oligocene-Miocene (Iaffaldano et al., 2006, 2007), but it can't explain the Late Cretaceous to Eocene slowdown and speedup
- Hypothesis: it was caused by time variations in viscous shear stresses at the base of the lithosphere

Colli et al. 2014

Application I: South Atlantic Ocean

- Consequence: times of faster/slower spreading should correspond with higher/lower overpressure on the African side of the Atlantic basin
- Testable prediction: high/low dynamic topography in Africa coeval with periods of fast/slow spreading



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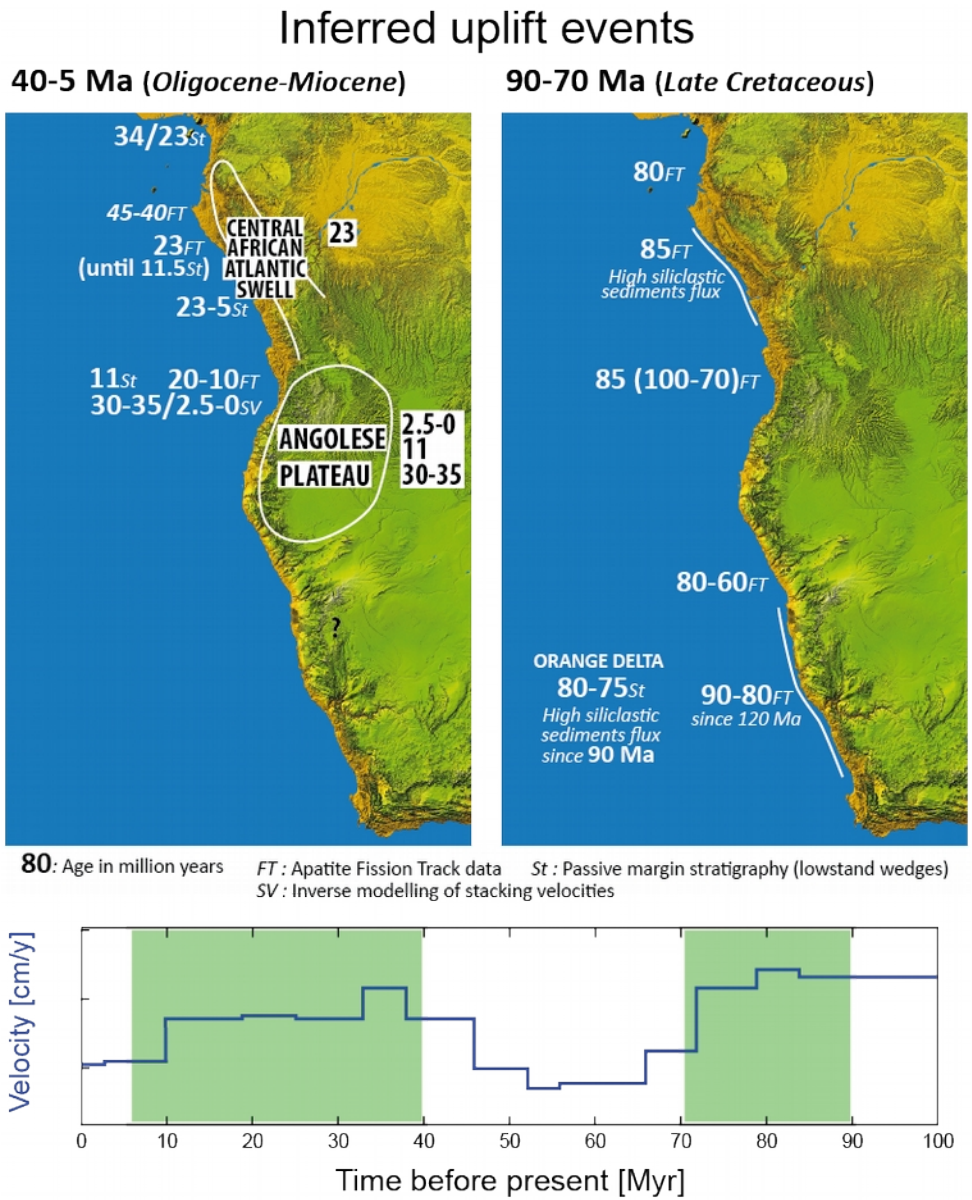
Colli et al. 2014

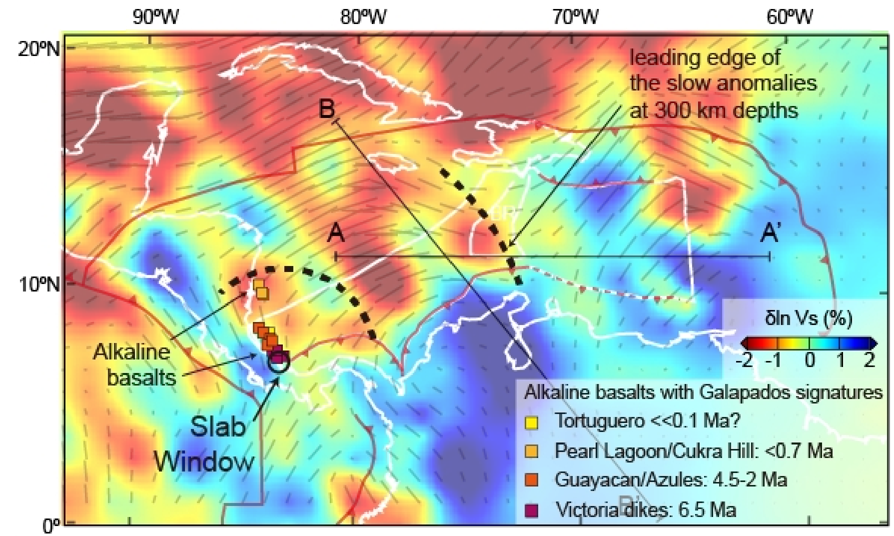


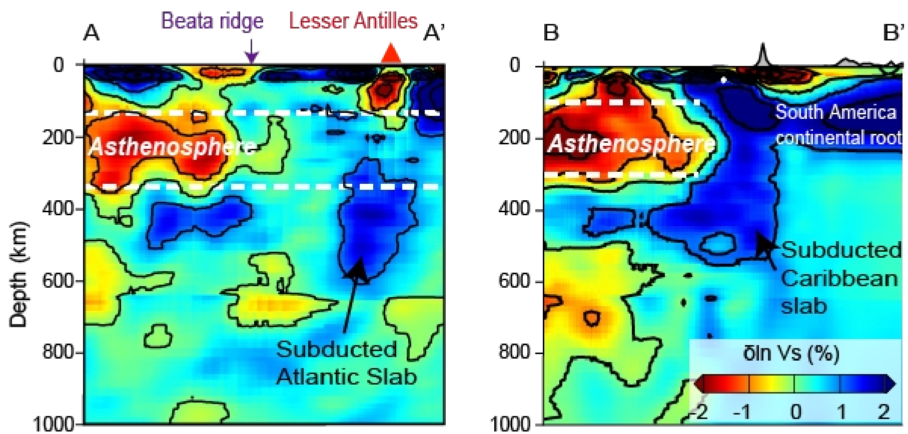
- Two phases of uplift in Oligocene-Miocene and in Late Cretaceous
- No signs of uplift in the intervening period
- Correlation of horizontal plate motions and vertical deflections of the surface

Colli et al. 2014

Application II: Caribbean basin

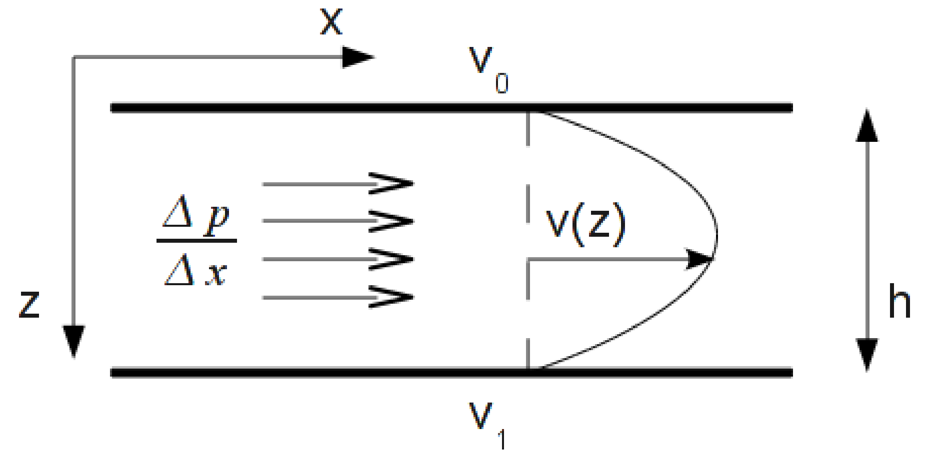
- Seismic tomography (Zhu et al. 2020) suggests thin asthenosphere
- Panama slab window opened at 8 Ma
- Material from Galapagos hotspot started intruding, funnelled by slabs and continental keels towards Antilles
- We can estimate flow speed from leading edge of slow anomalies and timing of slab window
- Additional velocity constraints from propagation of magmatism





Application II: Caribbean basin

- We have flow velocity and channel thickness
- Careful removal of isostatic topography allows us to quantify dynamic topography
  - This gives us the pressure gradient across the Caribbean basin
- We can constrain the absolute value of the viscosity!
- For all the details see Yi-Wei Chen's poster D1421|EGU2020-12682 in this session



$$v(z)=\frac{1}{\eta}\frac{\Delta p}{\Delta x}z(h-z)$$

Part two:

Sequential assimilation

Assimilation of kinematic plate motions

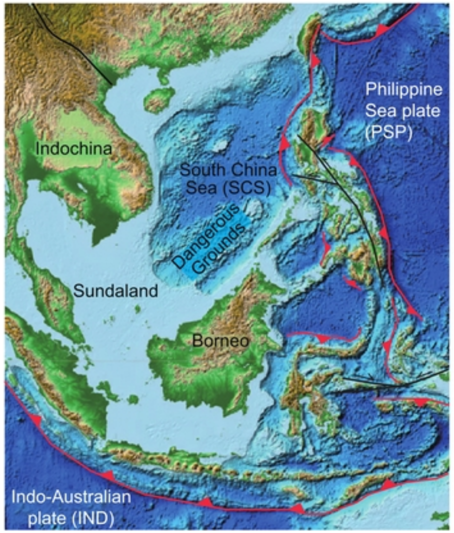
- Mantle convection is an initial condition problem: models are initialized and run forward in time
- Use present day state to predict future evolution?
  - Testing of future states impractical
- Start in the past and make prediction-in-the-past?
  - Lack suitable initial condition!
- Start in the distant past with arbitrary initial condition and assimilate past plate motions (e.g., Bunge et al. 1998)
  - Directly conditions flow field (Hager & O'Connell 1979)
  - Injects slabs at the right places and times (if plate model is correct), conditioning buoyancy field

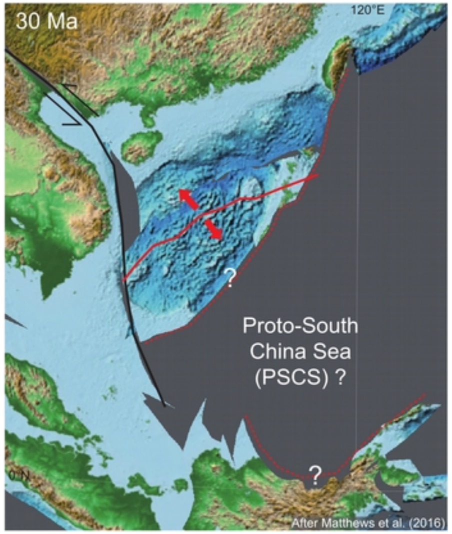
Assimilation of kinematic plate motions

- If assimilation time is long enough memory of arbitrary initial condition is lost (Colli et al. 2015)
- Modeled present-day state of the mantle depends on geodynamic parameters and kinematic history
- Can be tested against seismic imaging
- It’s important to account for finite resolution of seismic tomography and mineralogical effects

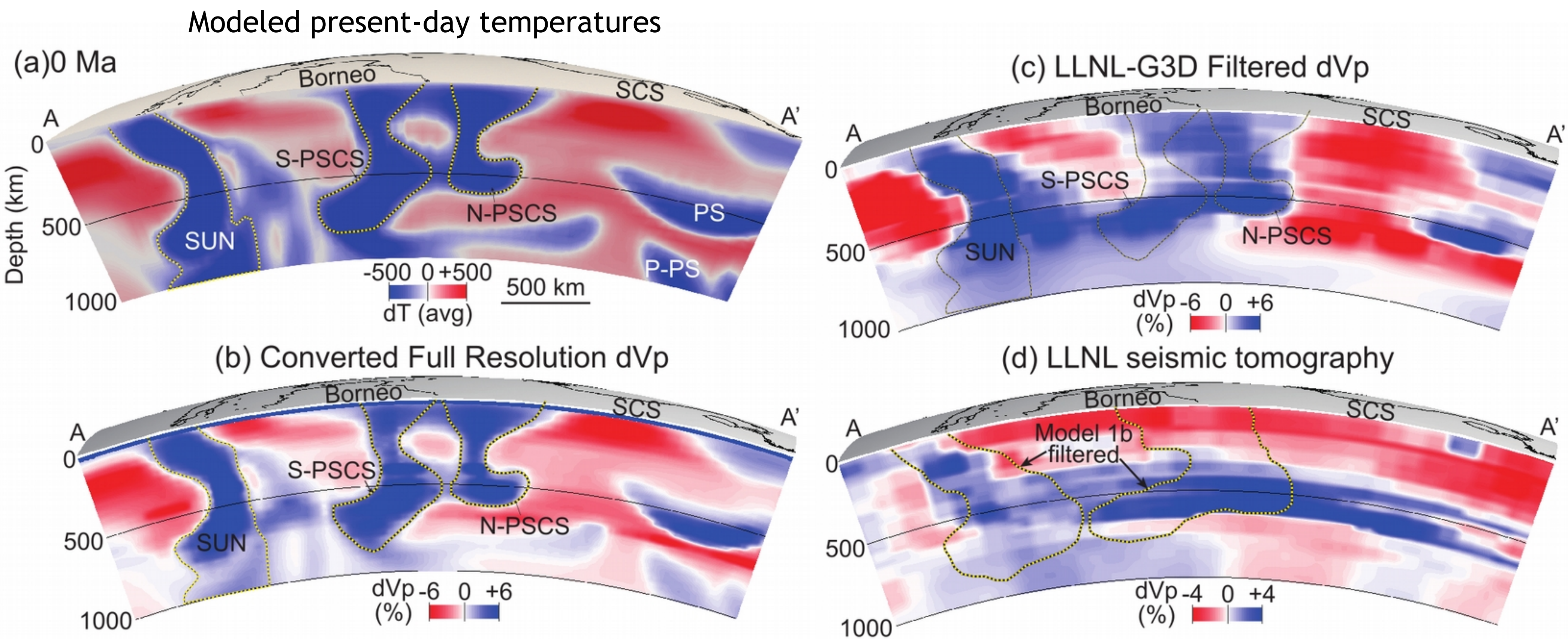
Application: proto-South China Sea

- Southeast Asia is tectonically complex and dominated by history of subduction
- Past kinematic motions uncertain and highly debated
- Different scenarios imply different positions and morphologies of subducted material
- Assimilation into geodynamic model computes them explicitly
- Comparison against tomographic images can help constrain best model



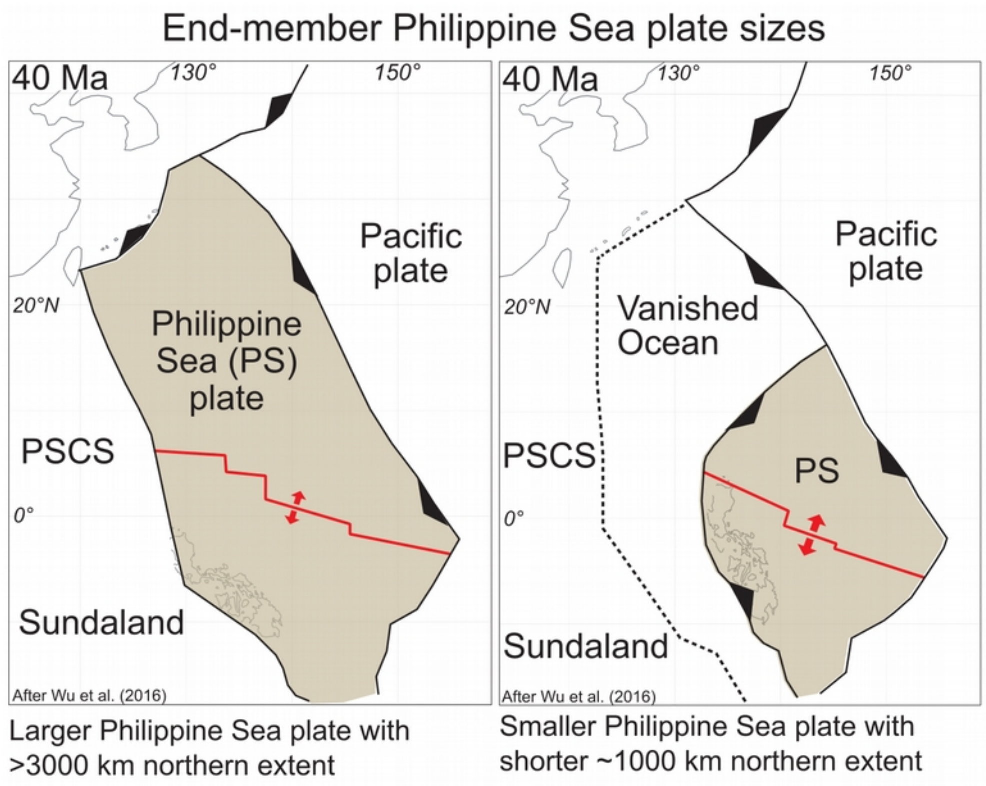


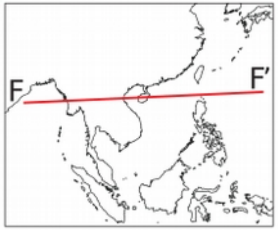
Application: proto-South China Sea

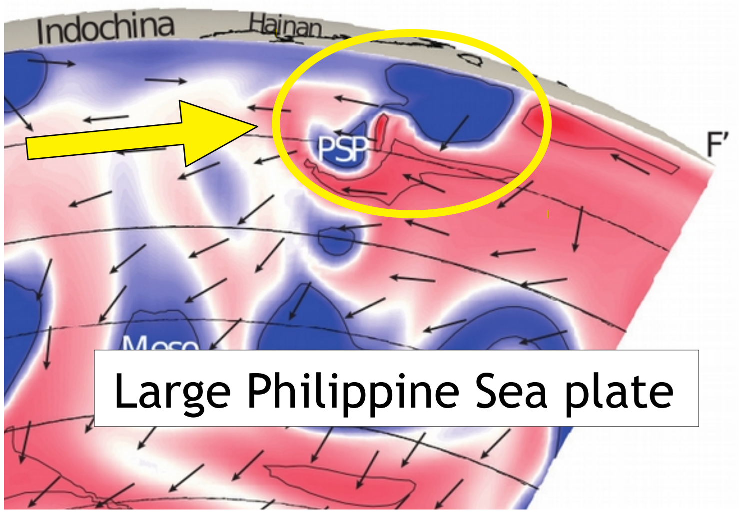


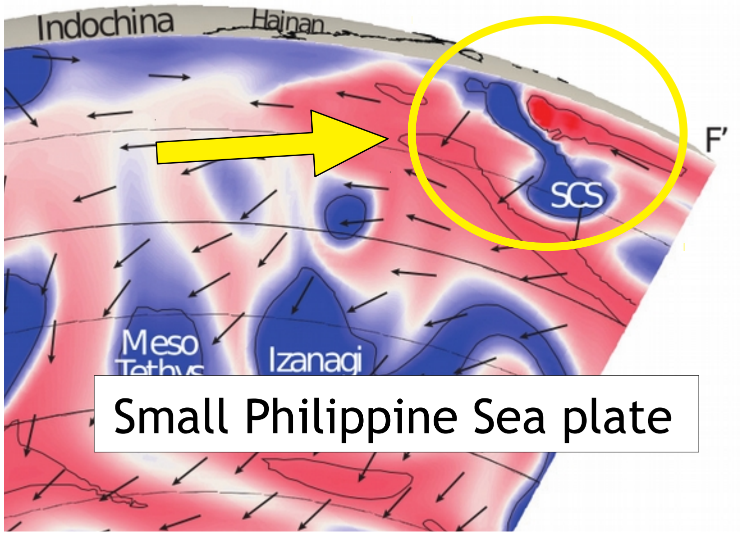
Need to account for tomographic resolution if possible!

Application: proto-South China Sea









- Smaller PS plate yields right apparent dip of subducted SCS slab
- For full details see Yi-An Lin's poster D1420|EGU2020-12407 in this session

Part three:

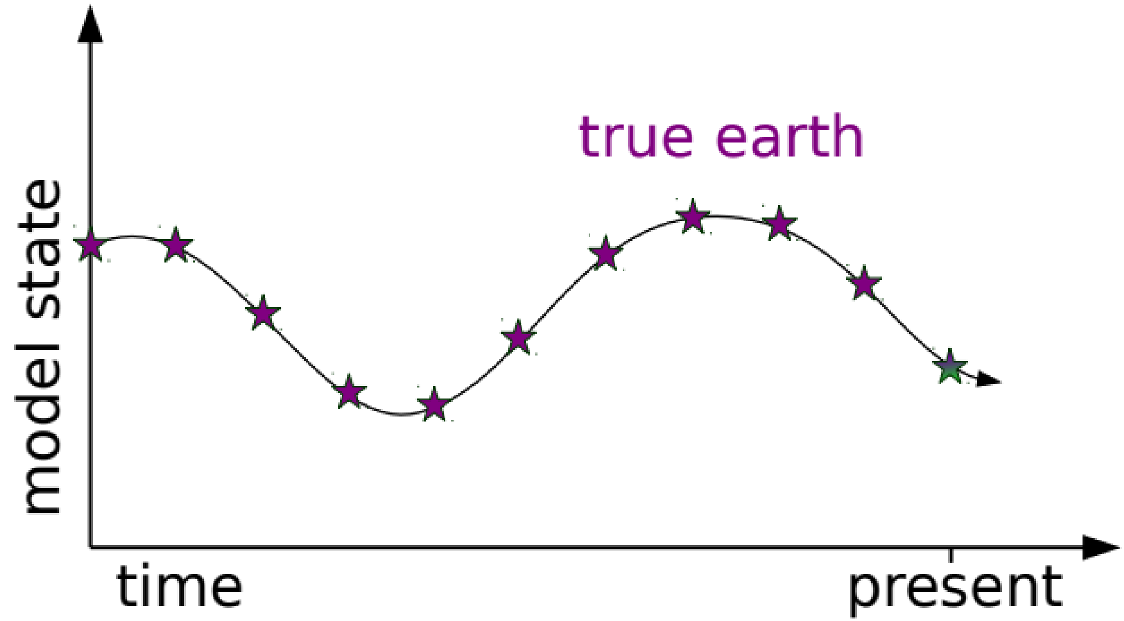
Adjoint method

Geodynamic inverse problem

- Mantle convection is an initial condition problem: models are initialized and run forward in time
- Use present day state to predict future evolution?
  - Testing of future states impractical
- Start in the past and make prediction-in-the-past?
  - Lack suitable initial condition!
- Start in the distant past with arbitrary initial condition and assimilate past plate motions (e.g., Bunge et al. 1998)
- Pose a formal inverse problem: find initial condition that evolves into known present-day state

Setting up an inverse problem

- True Earth trajectory is largely unknown



Setting up an inverse problem

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Setting up an inverse problem

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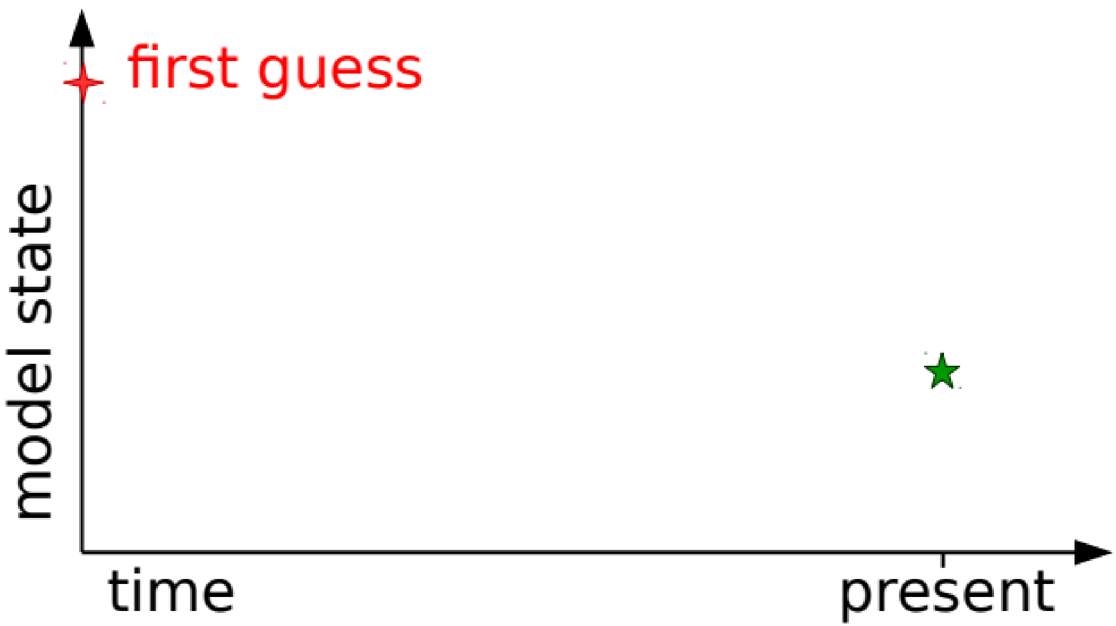
- Unknown initial condition. Must guess



Setting up an inverse problem

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- First guess trajectory doesn’t arrive at known present-day state

Setting up an inverse problem

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Setting up an inverse problem

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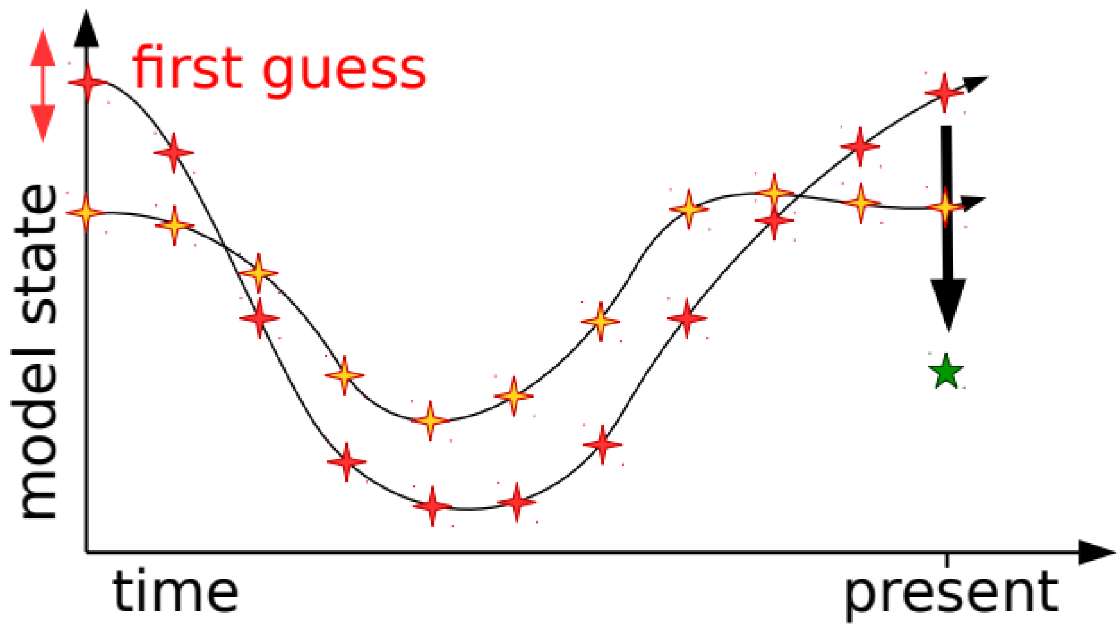
  - Adjoint method
- Update iteratively



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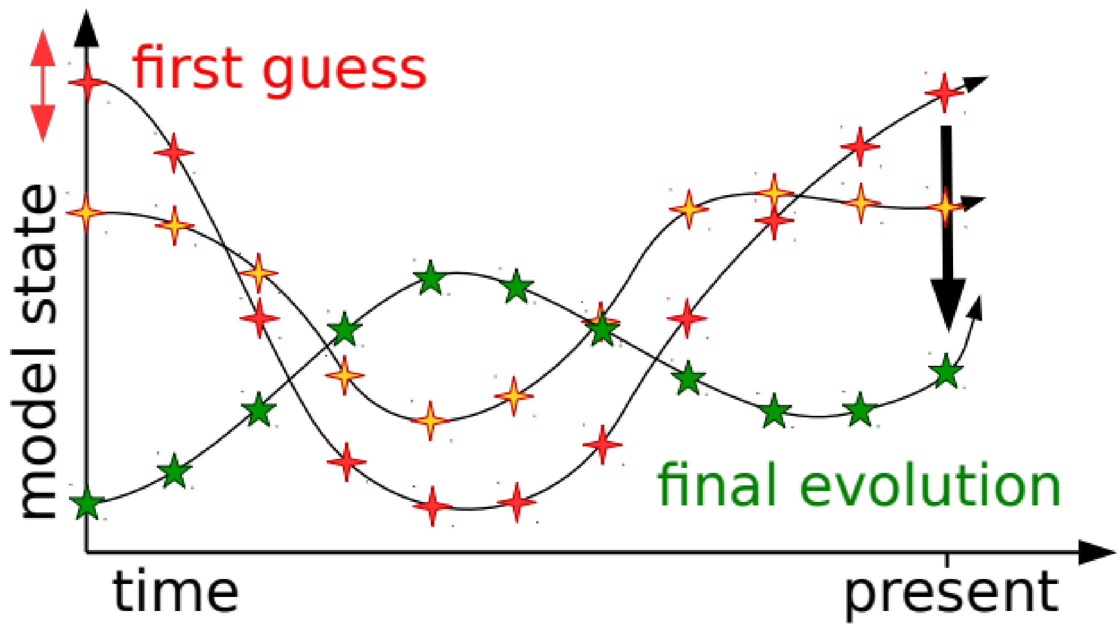

Setting up an inverse problem

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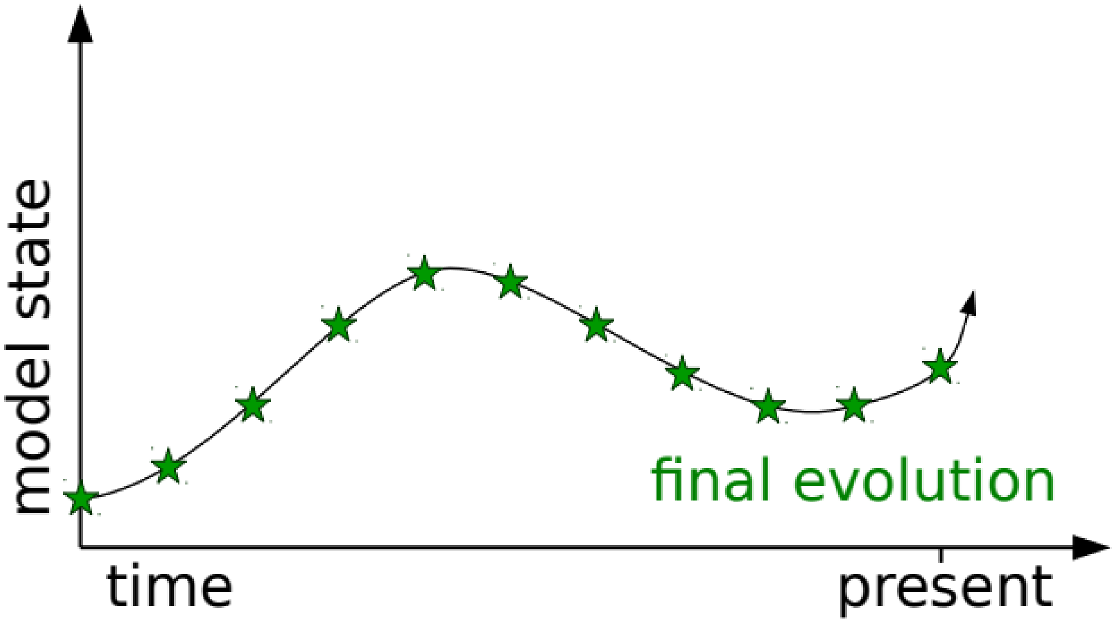
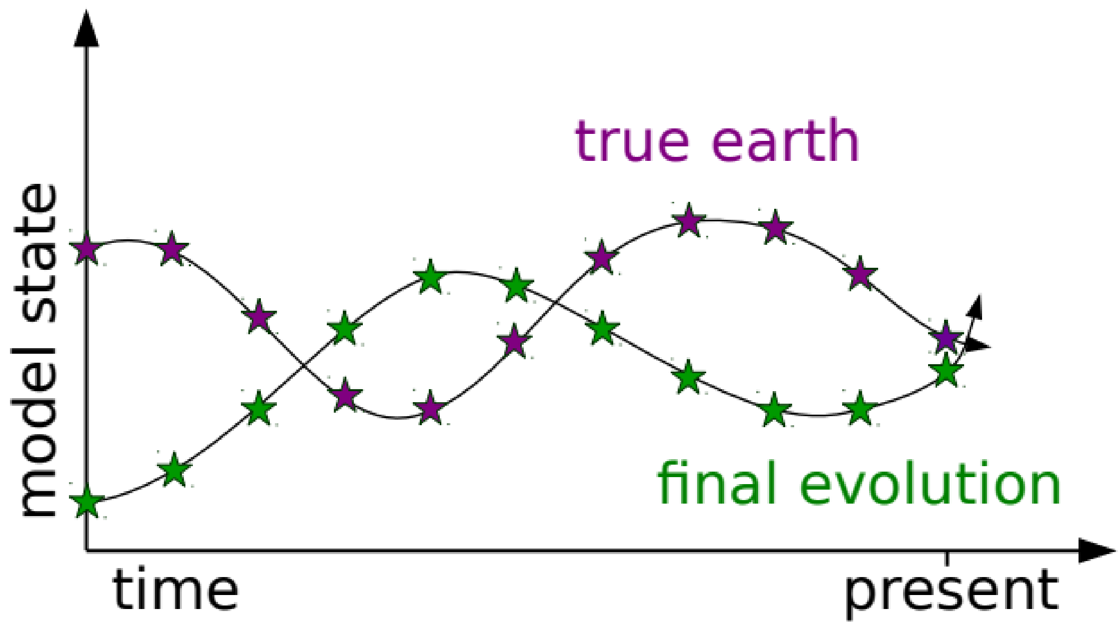
  - Adjoint method
- Update iteratively
- Optimize history



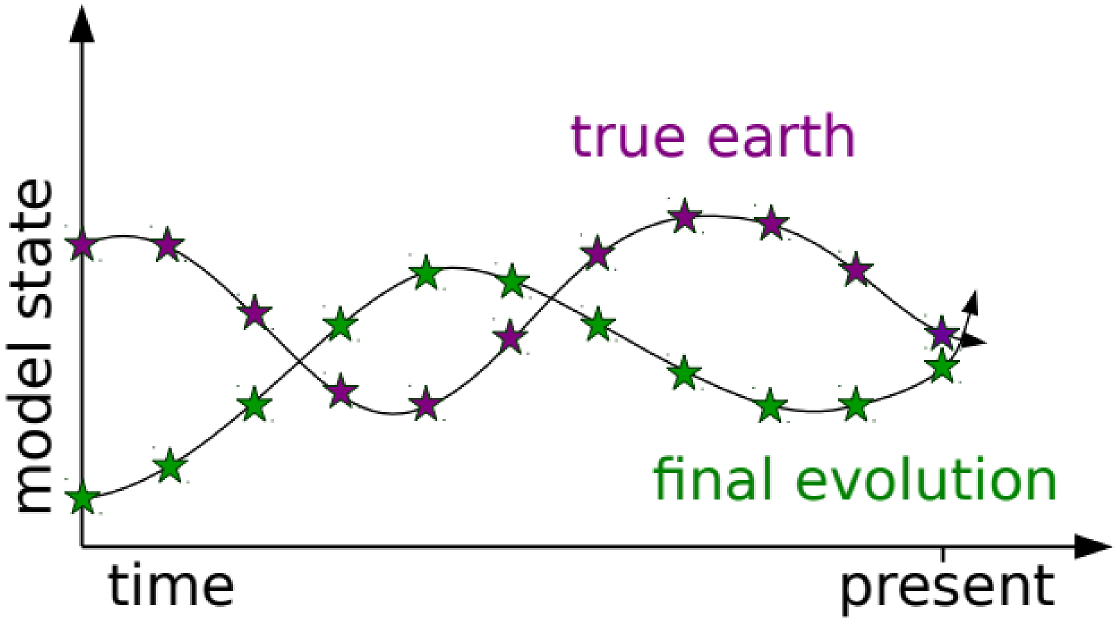
Setting up an inverse problem

- Optimized history is subject to geophysical working hypothesis (e.g. thermal vs thermochemical), choice of parameters (e.g. viscosity layering) and various uncertainties/errors
- Given a certain set of choices, the optimized history is characterized by a small null space
- Can be tested against geological and geophysical observations



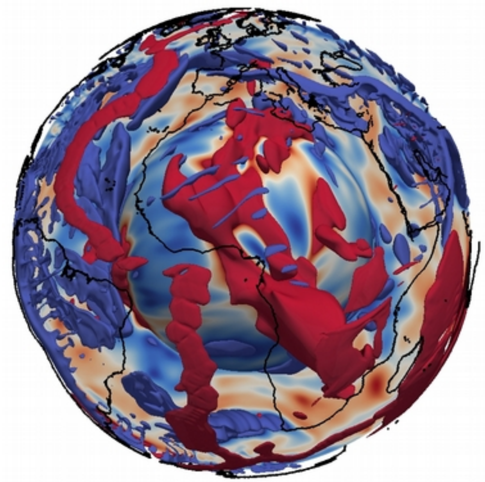
Setting up an inverse problem

- One source of uncertainty is given by our incomplete knowledge of the true present-day state of the Earth
- In part due to the finite resolution of seismic tomography, in particular at global scale
- Structures down to a few 10s of km and possibly smaller may contribute significantly to mantle dynamics but are either severely smeared or missed completely
- What are the implications for the optimized history?

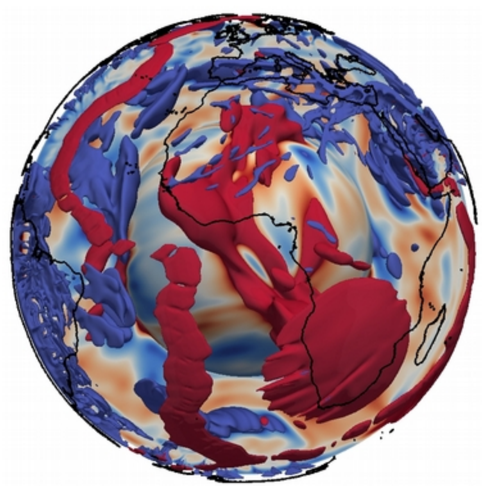


Synthetic study

- We can investigate this using a synthetic test
- Compute some reference evolution
- Assume only final condition at present day and history of surface motions are known
- Invert for initial condition
- Compare true initial condition against reconstructed initial condition
- Change inversion parameters (e.g., how much is known about the true final condition) and repeat



True initial condition @ 50 Ma



True final condition @ present day

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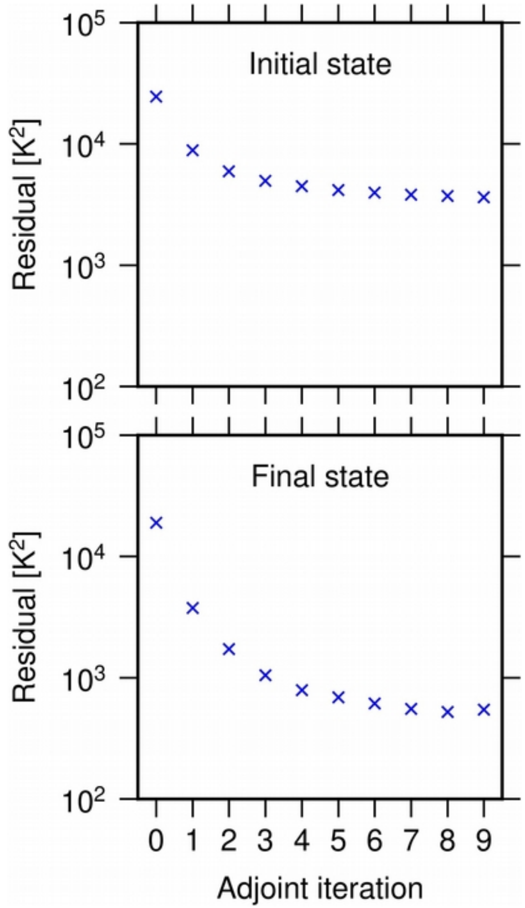
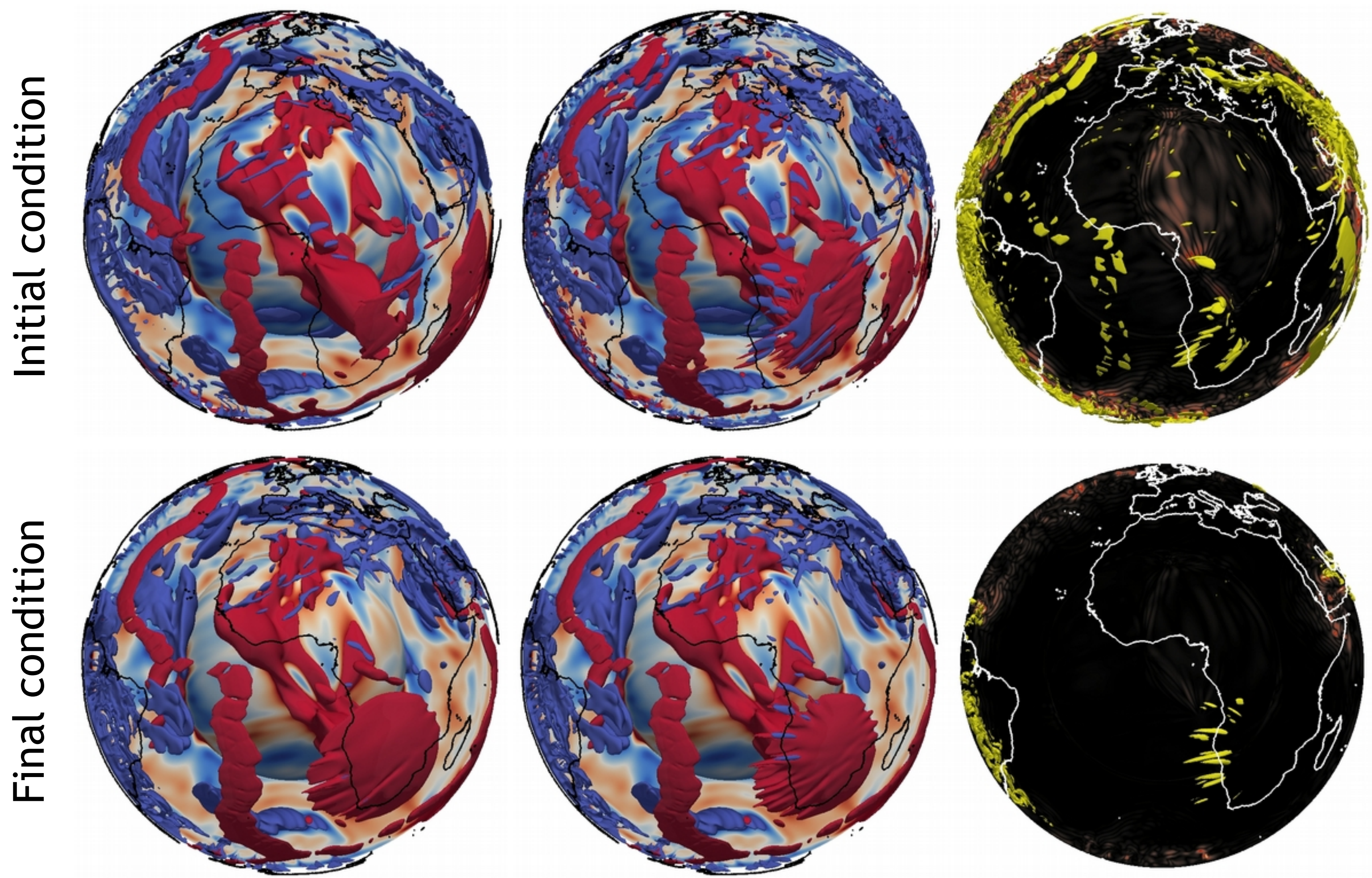
Reference inversion: error free best-case scenario



True | Reconstructed | Difference



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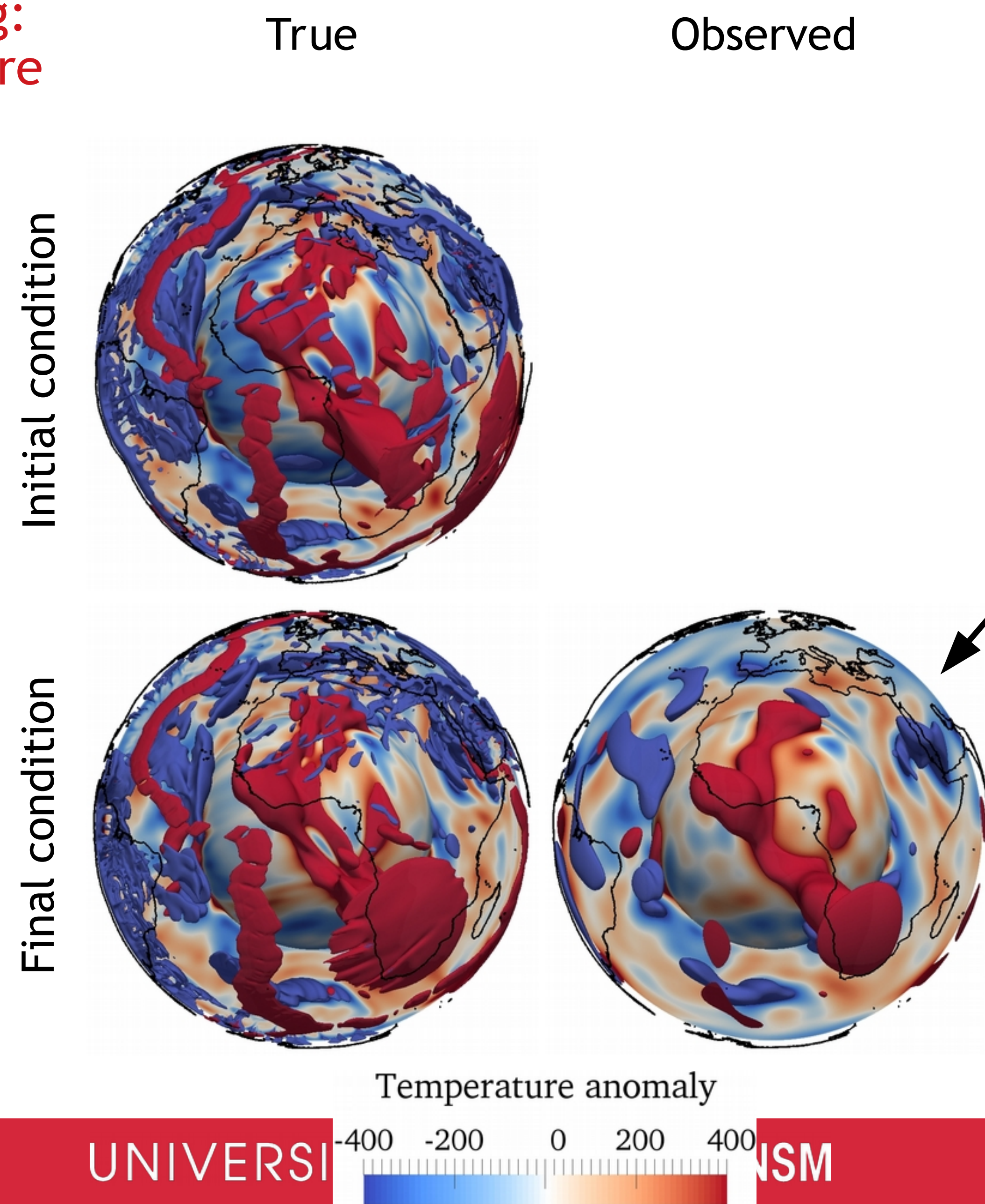
Tomographic filtering: no short-scale structure

Geodynamic model at Earth's convective vigor naturally produces short-scale structures, in particular at subduction zones.

There is a fundamental physical inconsistency between assumed convective vigor, imposed surface motions and estimated final state



This is what we will use as the known present-day state of the planet in our next inversion

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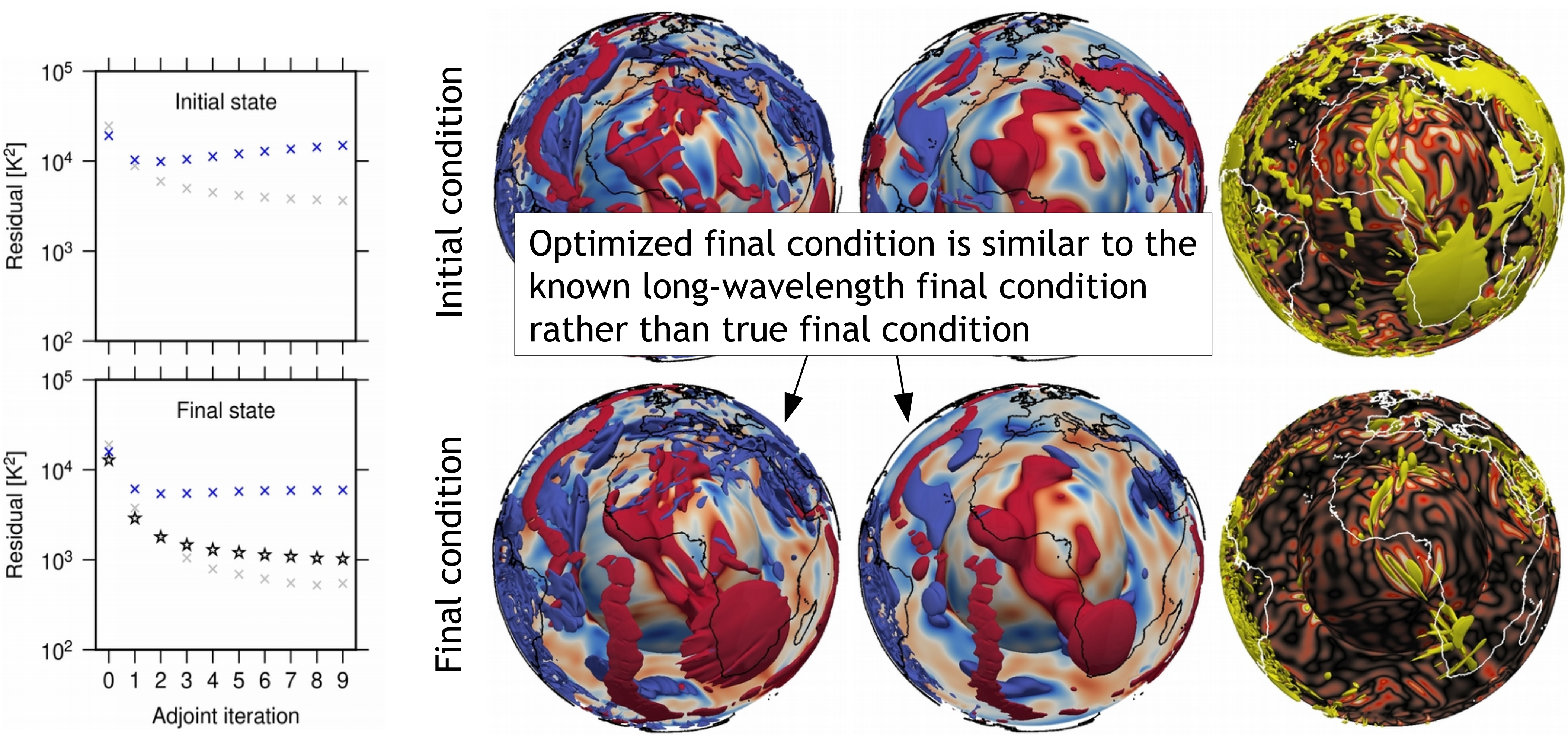
Tomographic filtering: no short-scale structure



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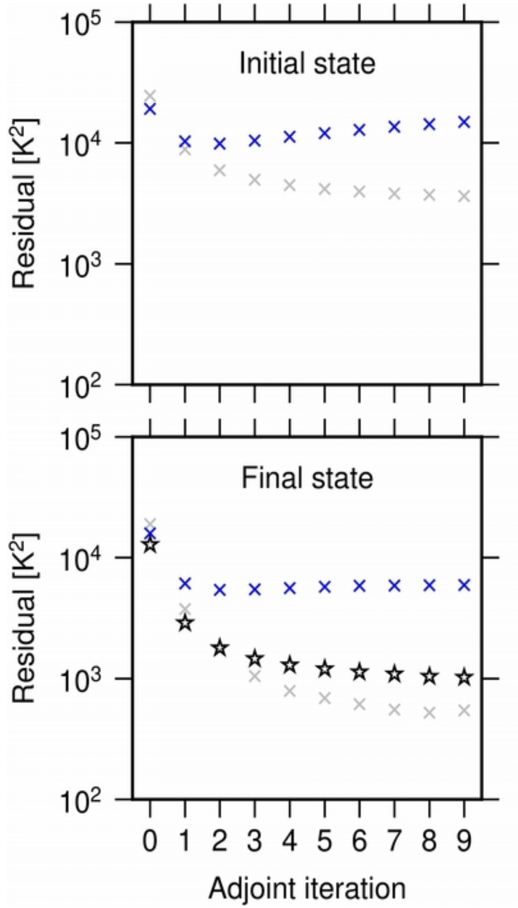
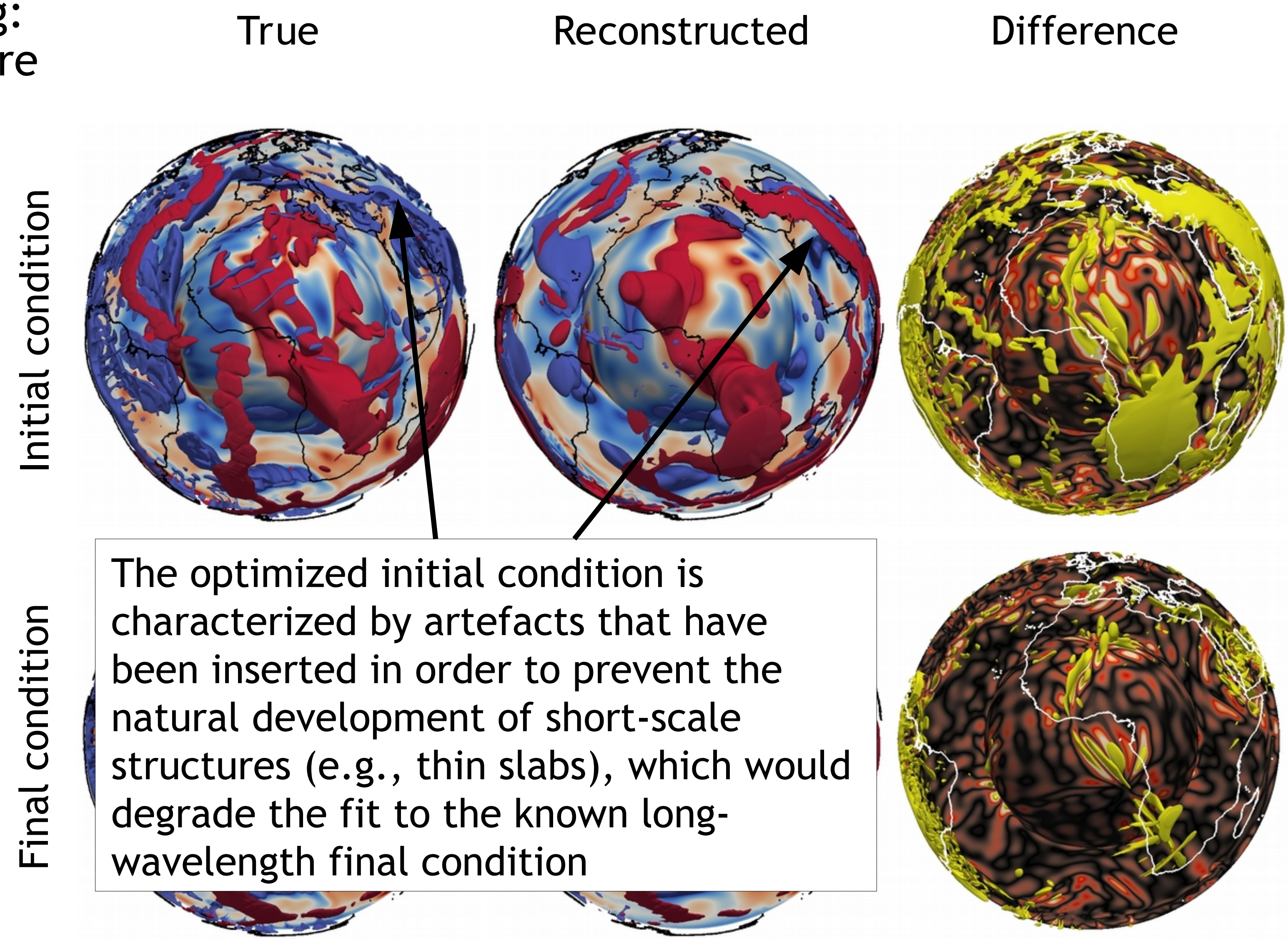
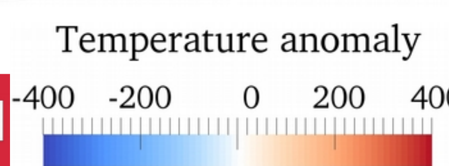
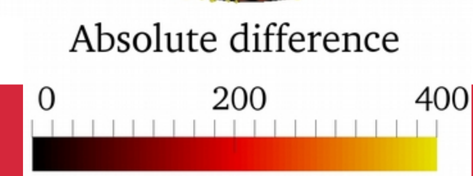




The optimized initial condition is characterized by artefacts that have been inserted in order to prevent the natural development of short-scale structures (e.g., thin slabs), which would degrade the fit to the known long-wavelength final condition





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Synthetic study

- Part of the problem stems from the fact that the commonly used misfit is based on a least-squares formulation.

$$\chi(T) := \frac{1}{2}\int_V \int_I (T(T_0, x, t) - T^E(x))^2\, \delta(t - t_1)\, dt\, dx^3$$

- This means that we are trying to find an initial state that strictly honors the estimated final state, thus in particular its lack of small-scale structure.

- What if we explicitly aim to match only its long-wavelength part?

$$\chi(T) := \frac{1}{2}\int_{R_b}^{R_a} \int_I \sum_{l,m} (T_{lm}(T_0, r, t) - T^E_{lm}(r))^2\, r^2\, \delta(t - t_1)\, dt\, dr$$

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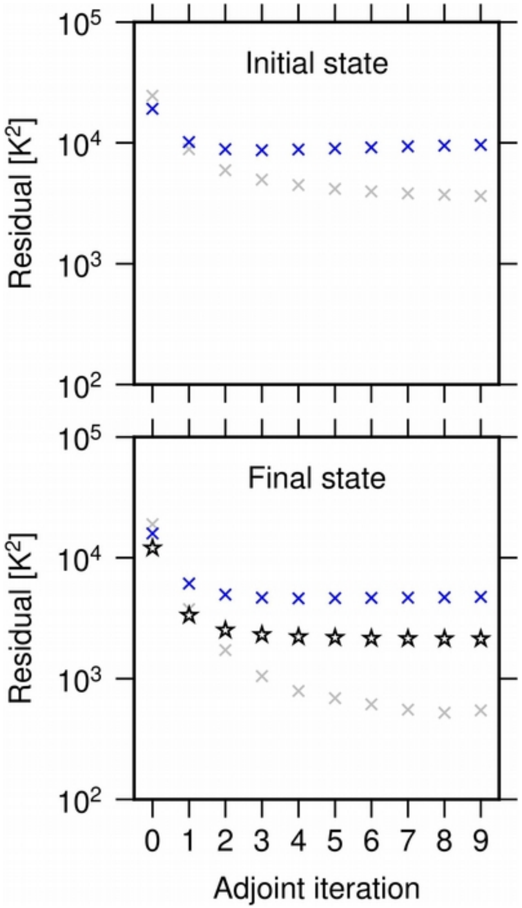
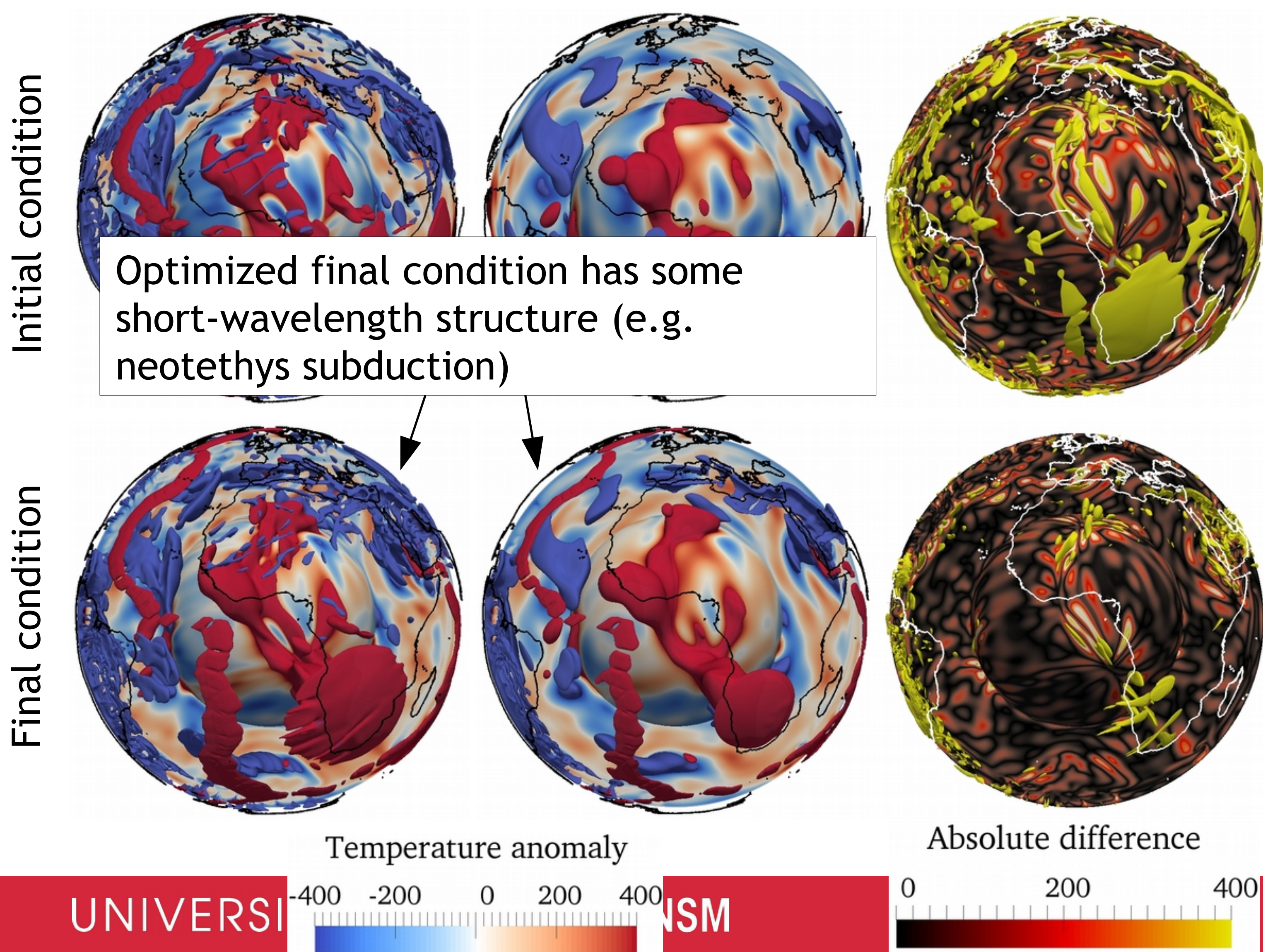
Tomographic filtering: no short-scale structure





Optimized final condition has some short-wavelength structure (e.g. neotethys subduction)

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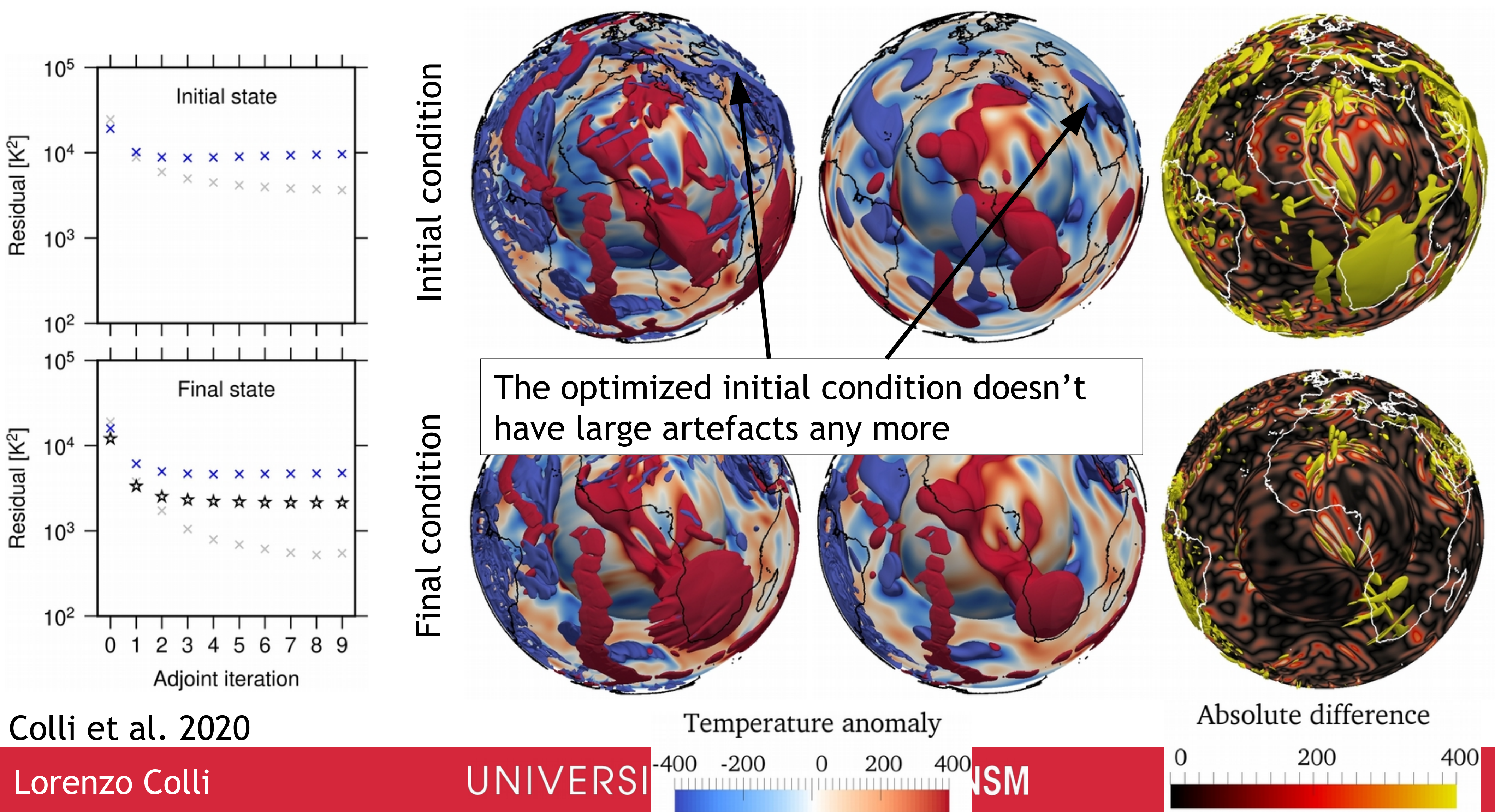
Tomographic filtering: no short-scale structure



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Conclusions

- Inconsistencies between model and datasets are inevitable in real-Earth applications
- Misfit minimization signals an optimized initial condition but not necessarily a good fit to the true initial condition
- Unphysical structures are good diagnostic, but not always present
- Thorough minimization of misfit maximizes artefacts if inconsistencies are present
- Inconsistencies can be mitigated using appropriate formulation for misfit function
- Assimilating one datasets using weight <1 increases importance of other datasets and geodynamic model
- Requires uncertainty/resolution estimate