Probabilistic damage scenarios from uncertain macroseismic data

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Motivations

We consider the **beta-binomial model for macroseismic attenuation** (*Rotondi et al., BSSA, 2009; Rotondi et al., Bull. Earthquake Eng, 2016*), which:

- respects as far as possible the *ordinal nature of the intensity scale*,
- allows for the assumption of *spatial isotropy* or *anisotropy*,
- allows for the Bayesian treatment of uncertainties,
- is a probabilistic tool to produce macroseismic scenarios.

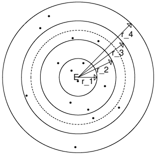
Critical point:

The application of the beta-binomial model typically requires rounding-up or -down the observed intensities to the nearest integer values (e.g. intensity VIII-IX must be set equal to VIII or IX).

Solution:

We propose **an extension of the beta-binomial model in order to include in the stochastic modelling the uncertainty in the assignment of the intensities**.

Original beta-binomial model of the intensity I_s at site



J circular bins are drawn around the epicenter (isotropy)

At a given bin j:

- $\mathbf{I}_s = I_0 - \Delta I$ follows the **binomial distribution** $Binom(\mathbf{I}_s \mid I_0,\ p_j)$:

$$
\begin{aligned}
Pr\{\mathbf{I}_s = i \mid I_0 = i_0,\ p_j\} &= Binom\{i \mid i_0,\ p_j\} = \\
&= \binom{i_0}{i} p_j^i (1 - p_j)^{i_0 - i} \qquad i \in \{0, 1, \dots, I_0\}
\end{aligned}
$$

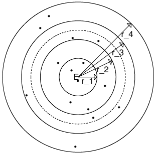
- random variable $p_j \sim$ **Beta distribution**

$$
Beta(p_j;\ \alpha_j,\ \beta_j) = \frac{\Gamma(\alpha_j + \beta_j)}{\Gamma(\alpha_j)\Gamma(\beta_j)} \int_0^{p_j} x^{\alpha_j - 1}(1 - x)^{\beta_j - 1} dx
$$

to account for the variability in ground shaking

The prior hyperparameters α_j, β_j are assigned on the basis of the macroseismic fields belonging to the *same class, but of different* I_0.

New beta-binomial model of the intensity I_s at site



J circular bins are drawn around the epicenter (isotropy)

At a given bin j:

- $\mathbf{I}_s = I_0 - \Delta I$ follows the **binomial distribution** $Binom(\mathbf{2}I_s \mid 2I_0,\ p_j)$:

$$
\begin{aligned}
Pr\{\mathbf{I}_s = i \mid I_0 = i_0,\ p_j\} &= Binom\{2i \mid 2i_0,\ p_j\} = \\
&= \binom{2i_0}{2i} p_j^{2i}(1-p_j)^{2i_0-2i} \qquad i \in \{0, 0.5, 1, 1.5 \ldots, I_0\}
\end{aligned}
$$

- random variable $p_j \ \sim$ **Beta distribution**

$$Beta(p_j;\ \alpha_j,\ \beta_j) = \frac{\Gamma(\alpha_j+\beta_j)}{\Gamma(\alpha_j)\Gamma(\beta_j)} \int_0^{p_j} x^{\alpha_j-1}(1-x)^{\beta_j-1}dx$$

  to account for the variability in ground shaking

The prior hyperparameters α_j, β_j are assigned on the basis of the macroseismic fields belonging to the *same class, but of different* I_0.

Updating parameters in the Bayesian framework

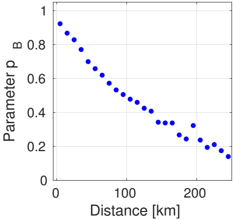
Given all the earthquakes with epicentral intensity I_0 in a class, (Fortran software)

let N_j be the number of felt intensities $\mathcal{D}_j$ in the j-th bin

Original beta-binomial model: $\alpha_j = \alpha_{j,0} + \sum_{n=1}^{N_j} i_s^{(n)}$ $\beta_j = \beta_{j,0} + \sum_{n=1}^{N_j} (I_0 - i_s^{(n)})$

New beta-binomial model: $\alpha_j = \alpha_{j,0} + \sum_{n=1}^{N_j} 2i_s^{(n)}$ $\beta_j = \beta_{j,0} + \sum_{n=1}^{N_j} (2I_0 - 2i_s^{(n)})$

I_0 VIII, class B



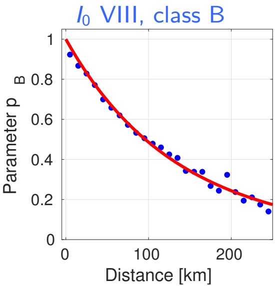
We estimate the parameter p_j through its posterior mean

$$\hat{p}_j = E\left(p_j \mid \mathcal{D}_j\right) = \frac{\alpha_j}{\alpha_j + \beta_j} \qquad j = 1, \ldots, J$$

Beta-binomial model with smoothed $p = p(d)$

The estimates $\hat{p}_j$ are smoothed by the **inverse power function** of the distance through the least squares method:

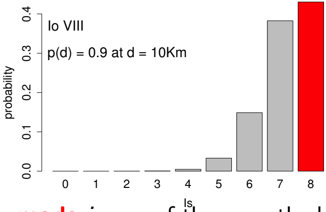
$$p(d) = \left(\frac{c_1}{c_1 + d} \right)^{c_2}$$

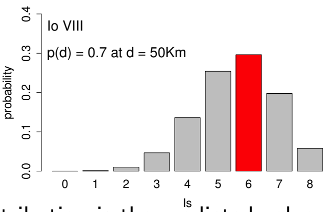


Smoothed binomial distribution at any distance d

$$Pr(\mathbf{I}_s = i | I_0 = i_0, p(d)) = \binom{i_0}{i} p(d)^i \, [1 - p(d)]^{(i_0 - i)}$$

$$Pr(\mathbf{I}_s = i | I_0 = i_0, p(d)) = \binom{2i_0}{2i} p(d)^{2i} \, [1 - p(d)]^{(2i_0 - 2i)}$$



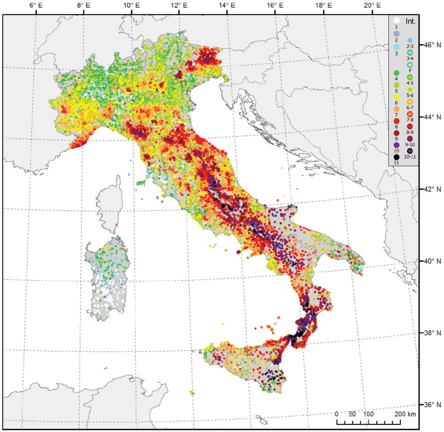


The **mode** i_{smooth} of the smoothed binomial distribution is the predicted value of I_s

Learning set

Analysis of **441 macroseismic fields (MFs) of "good quality"** from **the Italian DBMI15 database**:

- occurred since 1500,
- at least epicentral intensity V,
- at least 40 felt reports.



Database DBMI15
Distribution of maximum macroseismic intensity

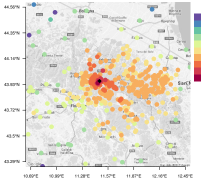
Hierarchical agglomerative clustering (R package *cluster*)

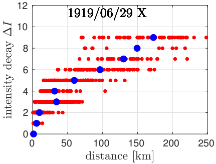
The learning set is first analyzed by the Ward's hierarchical agglomerative clustering method.

Four attenuation classes are identified.

class A
132 MFs

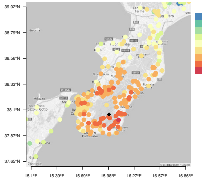
1919/06/29, Io X: observed

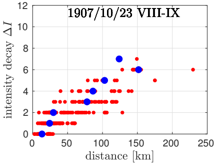




class B
192 MFs

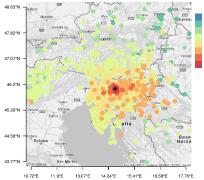
1907/10/23, Io VIII–IX: observed

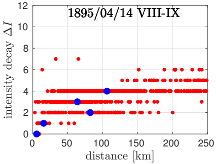




class C
82 MFs

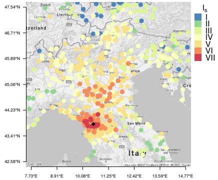
1895/04/14, Io VIII–IX: observed

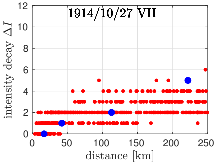




class D
35 MFs

1914/10/27, Io VII: observed





Smoothed posterior distribution of intensity I_s for class A

$$Pr\{\mathbf{I}_s = i \mid I_0 = i_0,\ p_{i_0}(d)\} = Binom\{2i \mid 2i_0,\ p_{i_0}(d)\} \quad \text{where} \quad p_{i_0}(d) = \left(\frac{c_1}{c_1 + d}\right)^{c_2}$$

I_0	$n.MFs$	c_1	c_2
V	22	13846.42	219.98
V-VI	17	-	-
VI	21	113.96	2.16
VI-VII	3	-	-
VII	17	1989.44	29.01
VII-VIII	10	-	-
VIII	8	9674.80	126.72
VIII-IX	0	-	-
IX	14	3573.55	37.52
IX-X	4	-	-
X	10	27752.59	254.40
X-XI	2	-	-
XI	4	36102.69	332.45

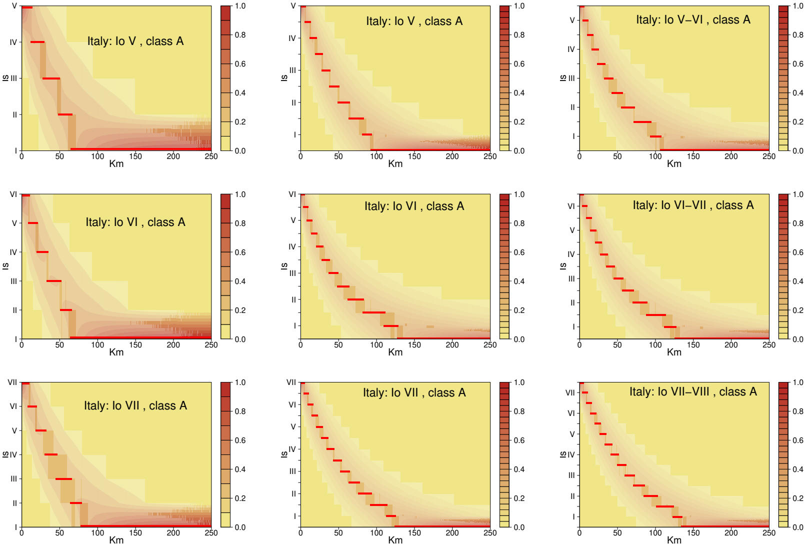
The number of MFs having uncertain epicentral intensity I_0 is sometimes very small or even null; it follows that the corresponding estimated coefficients c_1 and c_2 might be quite unrealiable.

In order to obtain more reliable and stable estimates for uncertain epicentral intensity $I_0 = i_0$, its parameter $p_{i_0}(d)$ is chosen as follows:

$$p_{i_0}(d) = \frac{p_{\lceil i_0 \rceil}(d) + p_{\lfloor i_0 \rfloor}(d)}{2}$$

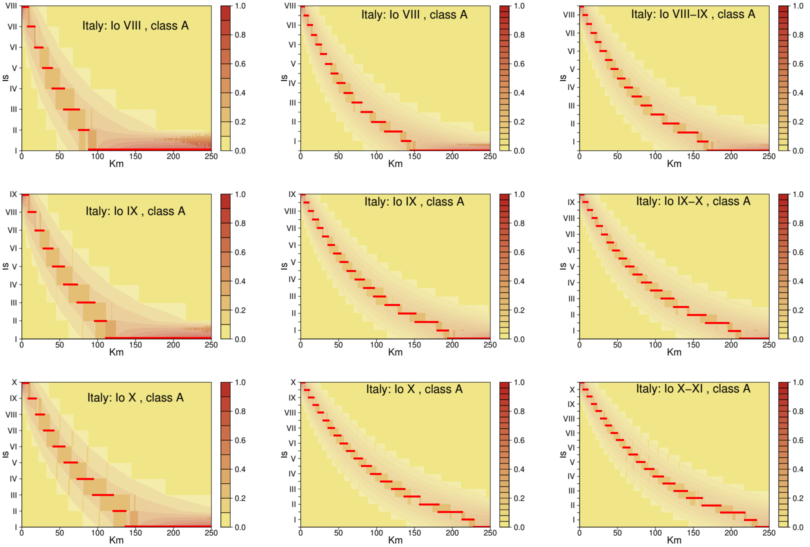
Probability of the intensity attenuation versus distance

Original model | New model | New model



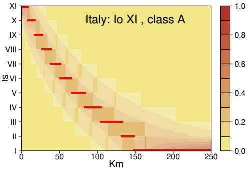
Probability of the intensity attenuation versus distance

Original model | New model | New model

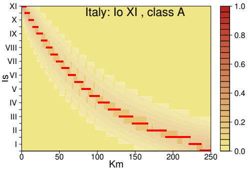


Probability of the intensity attenuation versus distance

Original model



New model



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