Probabilistic damage scenarios from uncertain macroseismic data

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Motivations

We consider the **beta-binomial model for macroseismic attenuation** (Rotondi et al., *BSSA*, 2009; Rotondi et al., *Bull. Earthquake Eng*, 2016), which:

- respects as far as possible the *ordinal nature of the intensity scale*,
- allows for the assumption of *spatial isotropy* or *anisotropy*,
- allows for the Bayesian treatment of uncertainties,
- is a probabilistic tool to produce macroseismic scenarios.

**Critical point:**

The application of the beta-binomial model typically requires rounding-up or -down the observed intensities to the nearest integer values (e.g. intensity VIII-IX must be set equal to VIII or IX).

**Solution:**

We propose an **extension of the beta-binomial model in order to include in the stochastic modelling the uncertainty in the assignment of the intensities.**
Original beta-binomial model of the intensity $l_s$ at site $J$

circular bins are drawn around the epicenter (isotropy)

At a given bin $j$:

- $l_s = l_0 - \Delta l$ follows the **binomial distribution** $Binom(l_s \mid l_0, p_j)$:

  
  \[
  Pr\{l_s = i \mid l_0 = i_0, p_j\} = Binom\{i \mid i_0, p_j\} = \binom{i_0}{i} p_j^i (1 - p_j)^{i_0 - i}, \quad i \in \{0, 1, \ldots, i_0\}
  \]

- random variable $p_j \sim Beta$ distribution

  \[
  Beta(p_j; \alpha_j, \beta_j) = \frac{\Gamma(\alpha_j + \beta_j)}{\Gamma(\alpha_j)\Gamma(\beta_j)} \int_0^{p_j} x^{\alpha_j-1}(1 - x)^{\beta_j-1}dx
  \]

  to account for the variability in ground shaking

The **prior hyperparameters** $\alpha_j, \beta_j$ are assigned on the basis of the macroseismic fields belonging to the same class, but of different $l_0$. 

New beta-binomial model of the intensity $I_s$ at site $J$

circular bins are drawn around the epicenter (isotropy)

At a given bin $j$:

- $I_s = I_0 - \Delta I$ follows the **binomial distribution** $Binom(2I_s \mid 2I_0, \, p_j)$:

$$Pr \{I_s = i \mid I_0 = i_0, \, p_j\} = Binom \{2i \mid 2i_0, \, p_j\} = \binom{2i_0}{2i} p_j^{2i}(1 - p_j)^{2i_0 - 2i} \quad i \in \{0, 0.5, 1, 1.5 \ldots, I_0\}$$

- random variable $p_j \sim** Beta distribution**

$$Beta(p_j; \, \alpha_j, \, \beta_j) = \frac{\Gamma(\alpha_j + \beta_j)}{\Gamma(\alpha_j)\Gamma(\beta_j)} \int_0^{p_j} x^{\alpha_j-1}(1 - x)^{\beta_j-1} dx$$

to account for the variability in ground shaking

The **prior hyperparameters** $\alpha_j, \beta_j$ are assigned on the basis of the macroseismic fields belonging to the **same class, but of different** $I_0$. 

Updating parameters in the Bayesian framework

Given all the earthquakes with epicentral intensity $l_0$ in a class, let $N_j$ be the number of felt intensities $D_j$ in the $j$-th bin.

Original beta-binomial model:

$$\alpha_j = \alpha_{j,0} + \sum_{n=1}^{N_j} i_s^{(n)} \quad \beta_j = \beta_{j,0} + \sum_{n=1}^{N_j} (l_0 - i_s^{(n)})$$

New beta-binomial model:

$$\alpha_j = \alpha_{j,0} + \sum_{n=1}^{N_j} 2i_s^{(n)} \quad \beta_j = \beta_{j,0} + \sum_{n=1}^{N_j} (2l_0 - 2i_s^{(n)})$$

We estimate the parameter $p_j$ through its posterior mean

$$\hat{p}_j = E(p_j | D_j) = \frac{\alpha_j}{\alpha_j + \beta_j} \quad j = 1, \ldots, J$$
Beta-binomial model with smoothed $p = p(d)$

The estimates $\hat{p}_j$ are smoothed by the inverse power function of the distance through the least squares method:

$$p(d) = \left( \frac{c_1}{c_1 + d} \right)^{c_2}$$

Smoothed binomial distribution at any distance $d$

$$Pr(I_s = i | I_0 = i_0, p(d)) = \binom{i_0}{i} p(d)^i [1 - p(d)]^{(i_0 - i)}$$

$$Pr(I_s = i | I_0 = i_0, p(d)) = \binom{2i_0}{2i} p(d)^{2i} [1 - p(d)]^{(2i_0 - 2i)}$$

The mode $i_{\text{smooth}}$ of the smoothed binomial distribution is the predicted value of $I_s$
Learning set

Analysis of 441 macroseismic fields (MFs) of “good quality” from the Italian DBMI15 database:

- occurred since 1500,
- at least epicentral intensity $V$,
- at least 40 felt reports.
Hierarchical agglomerative clustering (R package `cluster`)

The learning set is first analyzed by the Ward’s hierarchical agglomerative clustering method. Four attenuation classes are identified.

### class A
132 MFs

1919/06/29, Io X: observed

### class B
192 MFs

1907/10/23, Io VIII–IX: observed

### class C
82 MFs

1895/04/14, Io VIII–IX: observed

### class D
35 MFs

1914/10/27, Io VII: observed
Smoothed posterior distribution of intensity $I_s$ for **class A**

$$Pr \{ I_s = i \mid I_0 = i_0, \ p_{i_0}(d) \} = Binom \{ 2i \mid 2i_0, \ p_{i_0}(d) \}$$

where

$$p_{i_0}(d) = \left( \frac{c_1}{c_1 + d} \right)^{c_2}$$

<table>
<thead>
<tr>
<th>$I_0$</th>
<th>n. MFs</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
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<tbody>
<tr>
<td>V</td>
<td>22</td>
<td>13846.42</td>
<td>219.98</td>
</tr>
<tr>
<td>V-VI</td>
<td>17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VI</td>
<td>21</td>
<td>113.96</td>
<td>2.16</td>
</tr>
<tr>
<td>VI-VII</td>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VII</td>
<td>17</td>
<td>1989.44</td>
<td>29.01</td>
</tr>
<tr>
<td>VII-VIII</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VIII</td>
<td>8</td>
<td>9674.80</td>
<td>126.72</td>
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<td>VIII-IX</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IX</td>
<td>14</td>
<td>3573.55</td>
<td>37.52</td>
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<tr>
<td>IX-X</td>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>X</td>
<td>10</td>
<td>27752.59</td>
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<tr>
<td>X-XI</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>XI</td>
<td>4</td>
<td>36102.69</td>
<td>332.45</td>
</tr>
</tbody>
</table>

The number of MFs having uncertain epicentral intensity $I_0$ is sometimes very small or even null; it follows that the corresponding estimated coefficients $c_1$ and $c_2$ might be quite unreliable.

In order to obtain more reliable and stable estimates for uncertain epicentral intensity $I_0 = i_0$, its parameter $p_{i_0}(d)$ is chosen as follows:

$$p_{i_0}(d) = \frac{p_{\lceil i_0 \rceil}(d) + p_{\lfloor i_0 \rfloor}(d)}{2}$$
Probability of the intensity attenuation versus distance

Original model

New model

New model

Italy: Io V, class A

Italy: Io V, class A

Italy: Io V−VI, class A

Italy: Io VI, class A

Italy: Io VI−VII, class A

Italy: Io VII, class A

Italy: Io VII−VIII, class A

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Probability of the intensity attenuation versus distance

Original model

New model

New model

Italy: Io VIII, class A

Italy: Io VIII, class A

Italy: Io VIII−IX, class A

Italy: Io IX, class A

Italy: Io IX−X, class A

Italy: Io X, class A

Italy: Io X−XI, class A
Probability of the intensity attenuation versus distance

Original model

New model

References


Zonno G., Rotondi R., and Brambilla C. (2009), Mining macroseismic fields to estimate the probability distribution of the intensity at site, BSSA, 99 (5), 2876-2892
