# The maximum likelihood climate change under influence of extreme events


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## Introduction

Abrupt climatic transitions happened several times in the past [1], but their mechanisms are poorly understood. There is an alternative view that the abrupt climatic changes could be triggered by extreme events, and an $\alpha$-stable Lévy process is thought to be an appropriate model to generate such extreme events [2]. The transition path is a geometric characterization of the dynamic behavior of the system under the noise. It is is the crucial step to explore such abrupt shift events between metastable states. In contrast with the classic Gaussian noise, a comprehensive approach of the most probable transition path for systems under $\alpha$-stable Lévy noise is still lacking. We develop here a probabilistic framework, based on the conditional transition probability, to investigate the maximum likelihood climate change for an energy balance system under the influence of greenhouse effect and extreme events when global warming is $1.5^oC$.

## Stochastic Energy Balance Model

The short-timescale fluctuating processes, such as extreme events, modeled as a scalar symmetric $\alpha$-stable Lévy stochastic process $L_t^\alpha$ can be thought of as driving long-term climate variations [3]. The global mean surface temperature $T(t)$ evolution throughout the entire system [4] for global warming 1.5°C,

$$\frac{\mathrm{dT}}{\mathrm{dt}} = \frac{1}{C}\Big(\frac{1}{4}(1-\alpha(T))S_0 - \gamma\theta T^4\Big) + \frac{\tilde{\epsilon}}{C}\dot{L}_t^\alpha. \qquad (1)$$

The planetary albedo $\alpha(T)$ on temperature is expressed as [4]

$$\alpha(T) = 0.5 - 0.2\tanh\Big(\frac{T-265}{10}\Big).$$

## Method

The conditional probability density function $\mathcal{P}_A(x,t)$ for the $\mathbb{R}^1$-valued solution $X(t)$ defined by SDE (1) under the condition $X(0)=x_0$ and $X(T_f)=x_f$ exists and can be expressed as

$$\mathcal{P}_A(x,t) = p(X(t)=x|X(0)=x_0; X(T_f)=x_f) = \frac{Q(x_f,T_f|x,t)Q(x,t|x_0,0)}{Q(x_f,T_f|x_0,0)}.$$

The **maximum likelihood transition path** is formed by

$$x_m(t) = \arg\max_x \mathcal{P}_A(x,t). \qquad (2)$$

At a given moment $t$, the maximizer $x_m(t)$ for the conditional probability density $\mathcal{P}_A(x,t)$ indicates the maximum likelihood location of this stochastic trajectories. The transitional densities $Q(u;t|\omega,s)$ for SDEs is expressed in terms of the nonlocal Fokker-Planck equation [5]

$$\frac{\partial}{\partial t}Q(x;t|\omega,s) = -\frac{\partial}{\partial x}(f(x)Q(x;t|\omega,s)) + \epsilon^\alpha\int_{\mathbb{R}^1\setminus\{0\}}[Q(x+y;t|\omega,s) - Q(x;t|\omega,s) - \frac{\partial}{\partial x}I_{|y|<1}yQ(x;t|\nu,s)]\nu_\alpha(dy). \qquad (3)$$

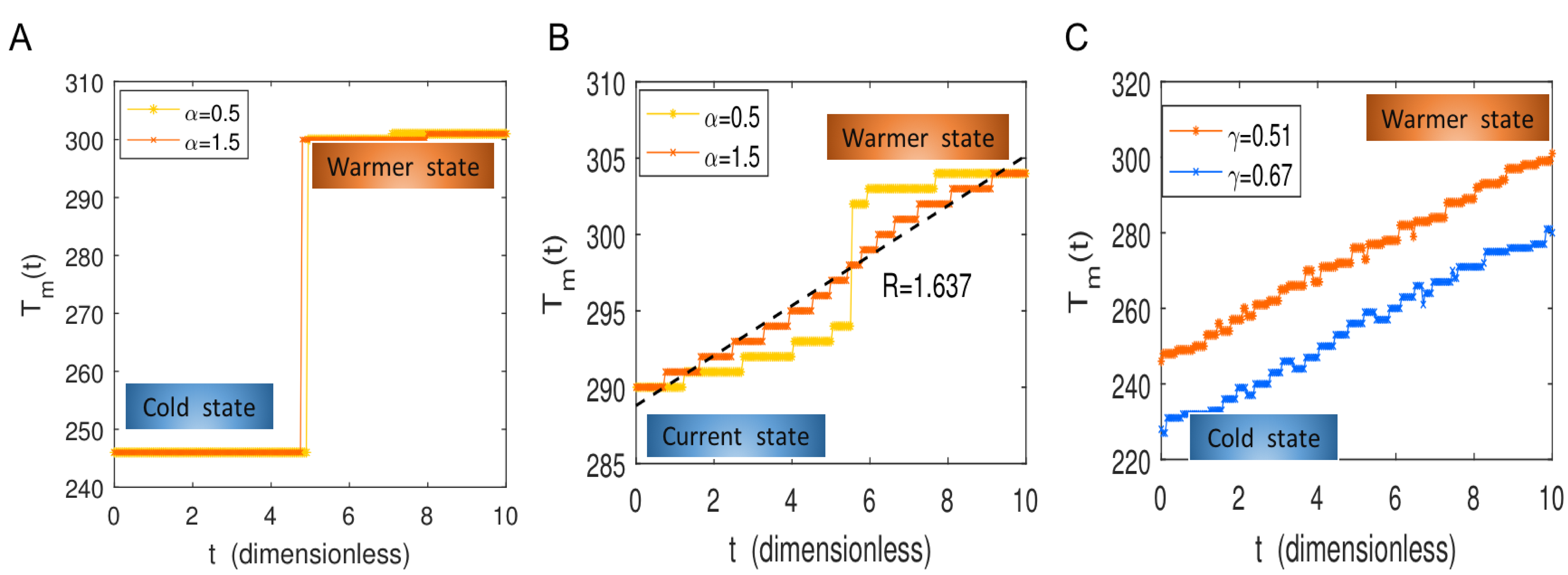
It fulfills the condition

$$\lim_{t\to s} Q(x,t|\omega,s) = \delta(x-\omega).$$

We apply the "punched-hole" trapezoidal numerical algorithm of Gao *et al.* [6] to solve the solution $Q(u;t|\omega,s)$ of nonlocal Fokker-Planck equation (3).

## Numerical Results

We investigate the maximum likelihood transition path (Eq.(2)) of climate change starting in a cold glacial metastable state and landing in a warmer interstadials metastable state, when the climate system (Eq.(1)) is under influence of **$\alpha$-stable Lévy noise** (Fig. (A)) and **Gaussian noise** (Fig. (C)) with the same noise intensity, respectively. Additionally, we look at effect of $\alpha$-stable Lévy noise on the transition from a current state to a metastable state under **an enhanced greenhouse effect** (Fig. (B)).




## Conclusions

Numerical simulations have revealed the dependence of the climate change on the the Lévy noise intensity, the jump frequency and the jump size. We find that a period of the relatively stable climate has been interrupted by sharp transitions to the warmer state attributing to larger jumps with lower frequency. Additionally, the climate change for warming 1.5°C under an enhanced greenhouse effect generates a step-like growth process. These results provide important insights into

## References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