



Geodetic determination of the gravitational potential difference for the optical lattice clock comparison in the Kanto region in Japan

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https://www.jst.go.jp/mirai/jp/uploads/saitaku2018/JPMJMI18A1_katori.pdf

Background

- The gravitational red shift: time runs slower where the gravitational potential is lower.

$$\frac{dt_{high}}{dt_{low}} = 1 + \Delta W / c^2, \Delta W = g\Delta H$$

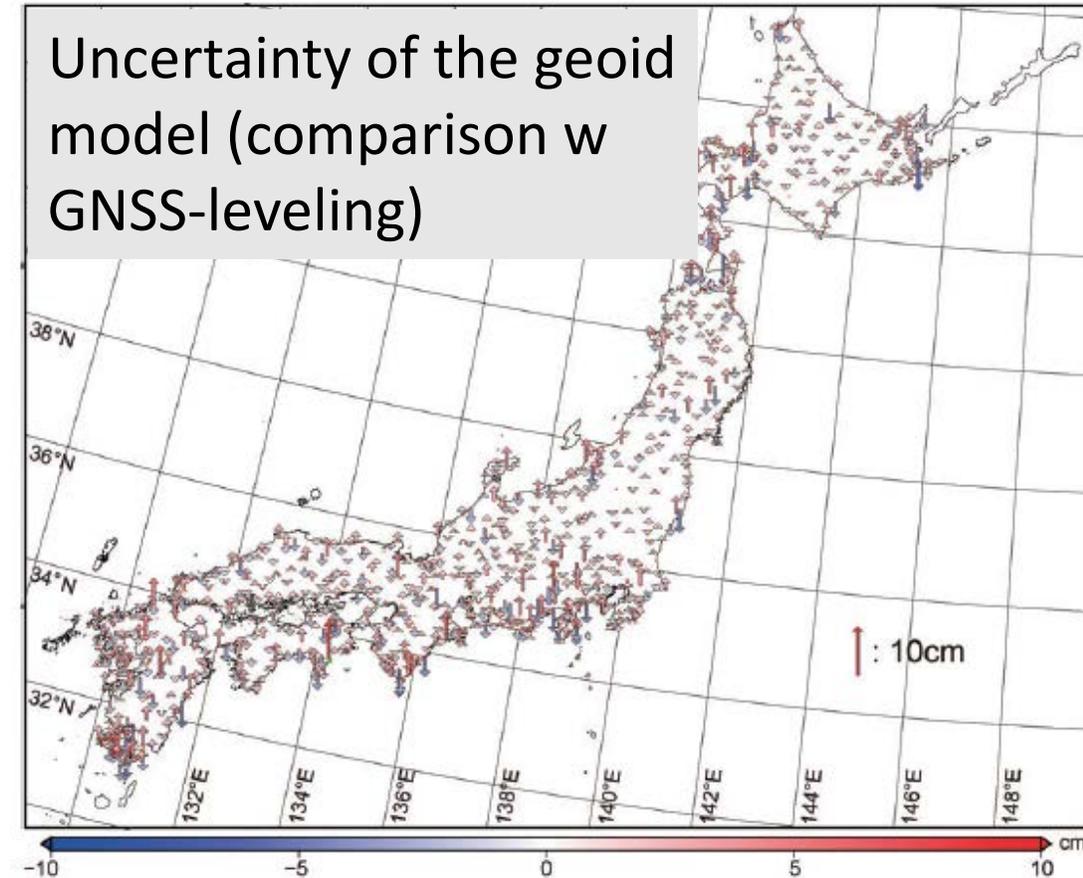
- Atomic clocks can detect a relative difference in the clock frequencies.
- Terrestrial clocks can be used as an altimeter.

Region (e.g.)	Geology /network scale	Main purpose	Required uncertainty
Europe	Stable continent	Unification of height reference systems	10^{-17} or better (cf leveling)
Japan	Unstable island arc	Crustal deformation monitoring	$10^{-18} \leq 24\text{h}$ (cf GNSS)

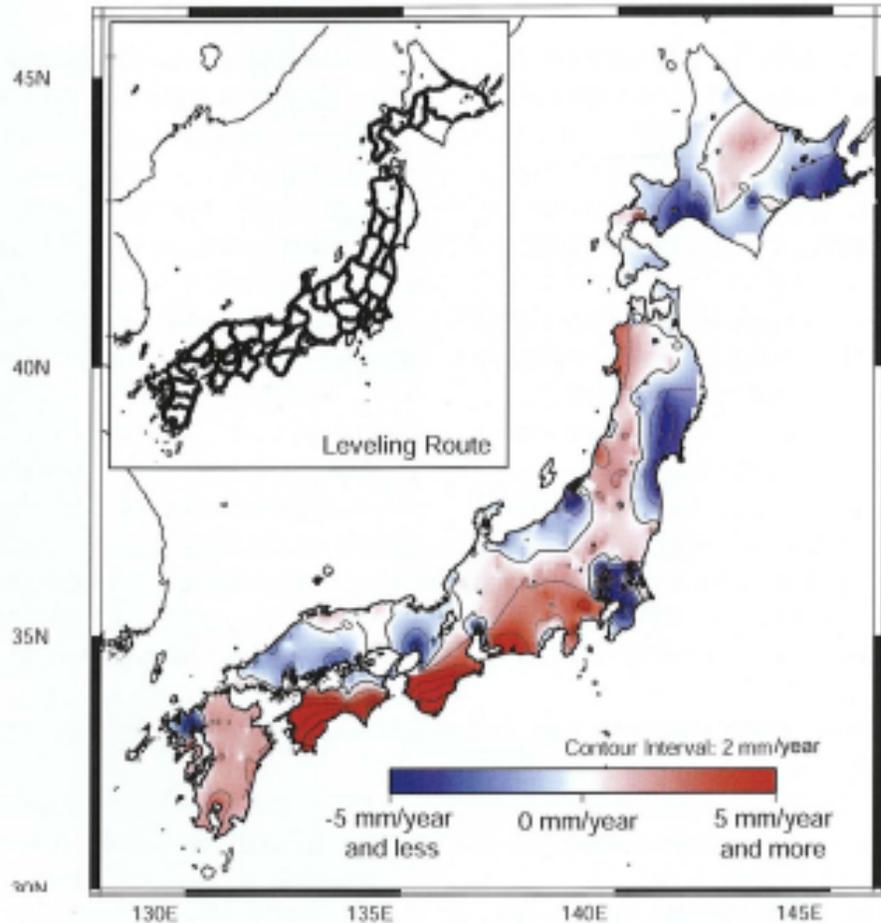
- Fiber-linked optical lattice clocks (OLCs) can achieve $\sim 10^{-18}$ (corresponding to 1-cm height difference) uncertainty within several hours.

Height reference system in Japan

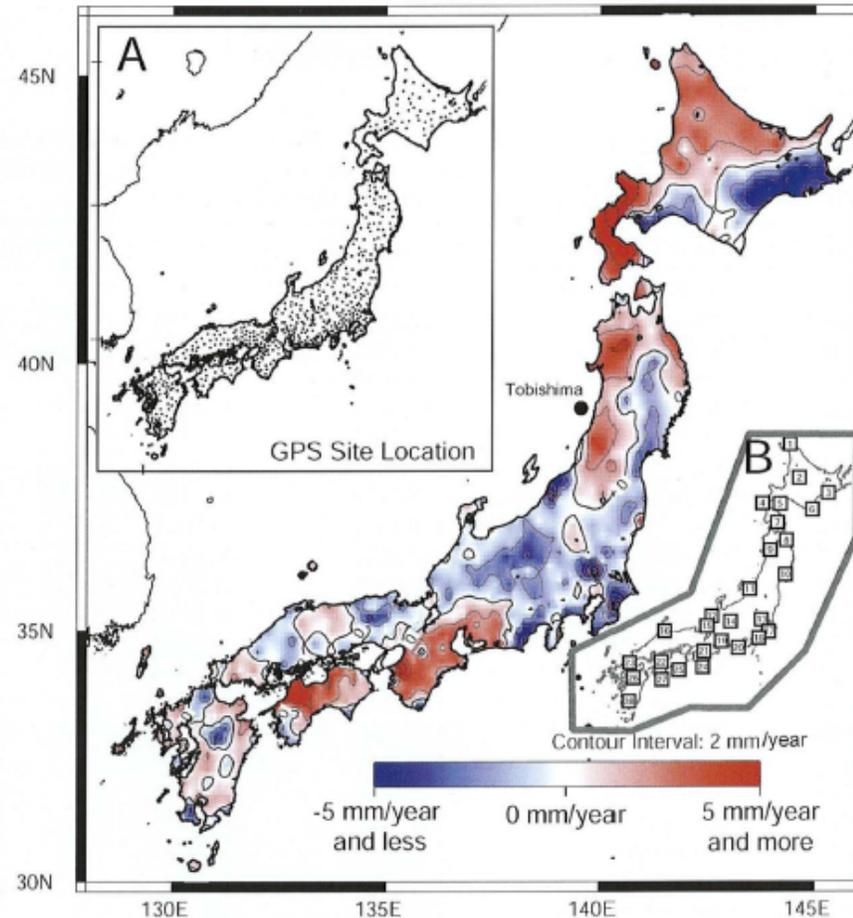
- **Helmert orthometric height**
- Geoid model by the Geospatial Information Authority of Japan (GSI) (Miyahara et al., 2014), $SD=1.8$ cm
- ~1300 cGNSS stations with average spacing of 20-25 km and the first-order leveling routes over 18,000 km for crustal deformation monitoring
- GNSS-leveling and gravimetric approaches were used for the longer- and shorter-wavelength determination, respectively.



Crustal vertical velocity in Japan



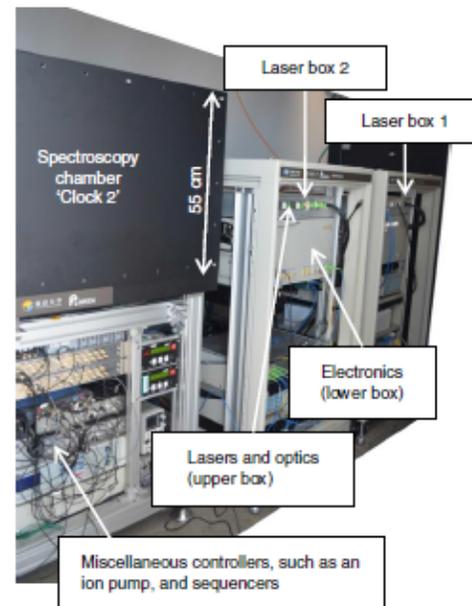
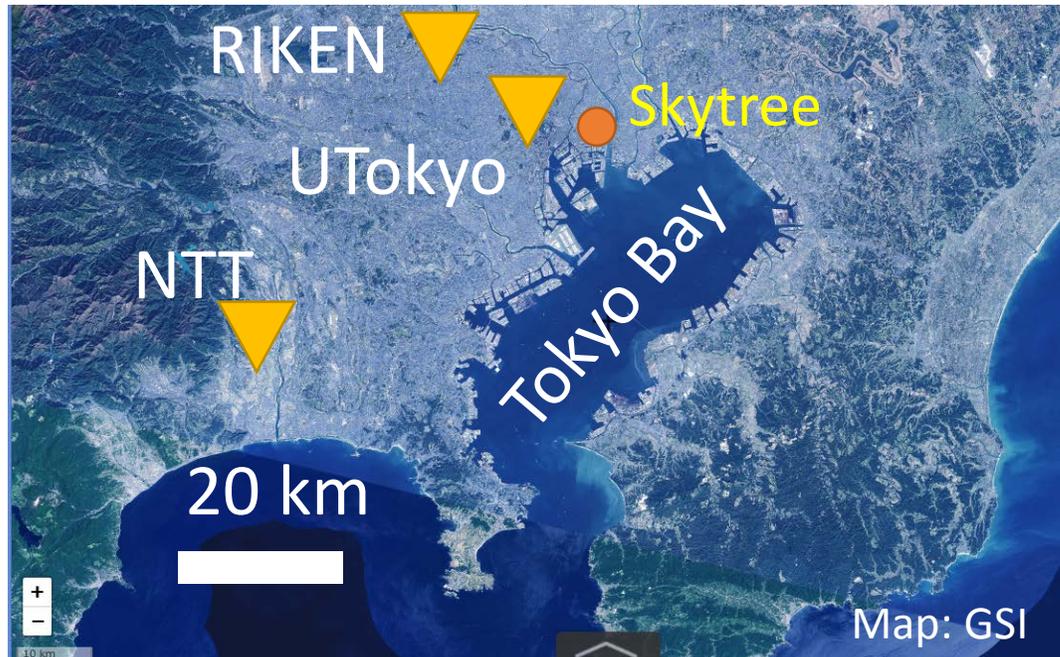
Left: leveling (1947–1961, 1988–1999)



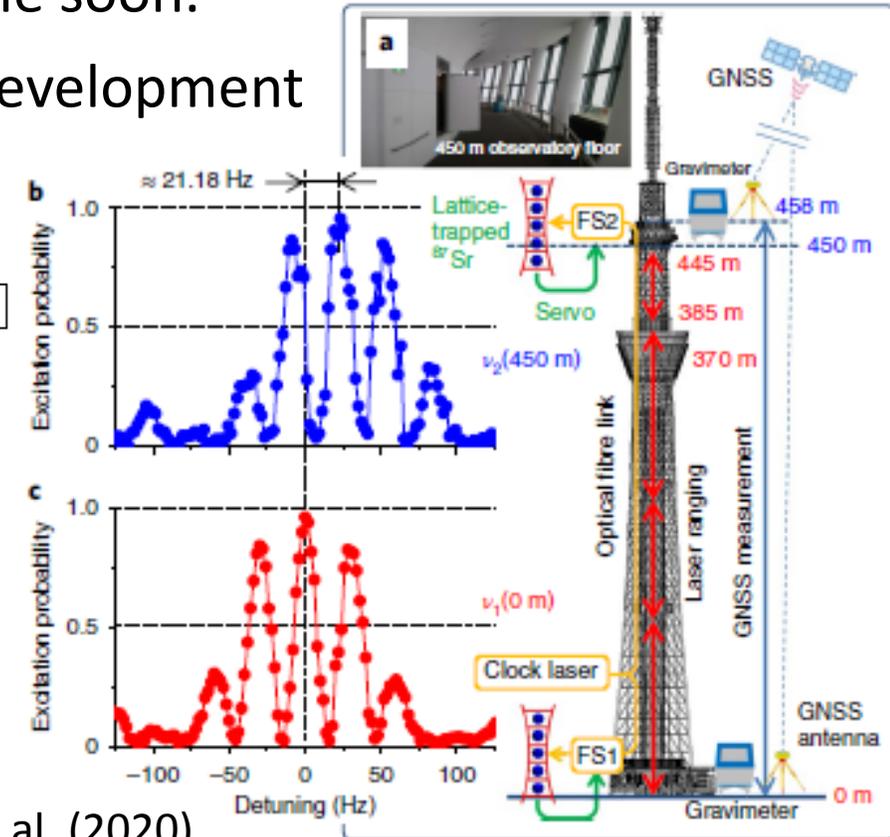
Right: GPS (1996–2003)

Recent progress regarding OLCs in Japan (selected)

- Chronometric heights obtained by OLCs were compared with geodetic survey results:
 - RIKEN-UTokyo: 5×10^{-18} , OLCs in laboratory environment (Takano et al., 2016)
 - Observatory of Tokyo Skytree: $1-5 \times 10^{-18}$, portable clock (Takamoto et al., 2020)
- NTT-RIKEN-UTokyo: Fiber-linked clocks will become available soon.
- 400-km fiber link toward the NE Japan (Mizusawa) under development



Takamoto et al. (2020)

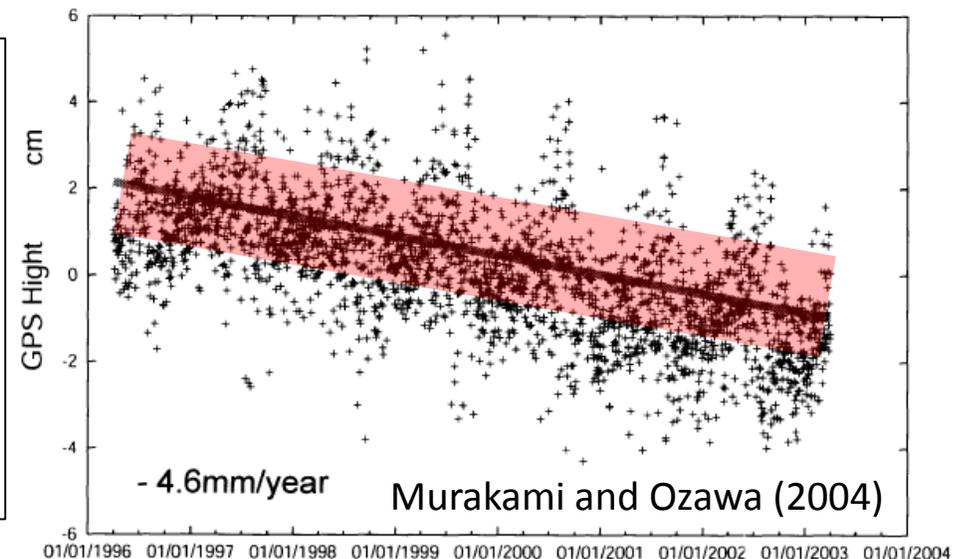


Purpose of this study

- Our ultimate goal is to utilize OLCs to assist GNSS to monitor vertical deformation.
- In this study, we determine the static potential difference between the NTT and RIKEN clock sites to confirm the uncertainty of the portable clocks over a 100-km-scale fiber network, using geodetic observations.
- We discuss the error budget for the geodetic result.

Red: Expected uncertainty by using OLCs

- Faster positioning of vertical deformation than in GNSS (1 cm in several hours)
- Free from atmospheric noise
- It can separate apparent seasonal variations inherent in space geodetic techniques



Method

Leveling-gravity method

(i) Direct integration of the potential increment

$$\Delta W_{AB} \cong \sum_i \bar{g}_{i,i+1} \Delta H_{i,i+1} \quad A, B: \text{clock sites}$$

(ii) Computation based on the definition of Helmert orthometric height

$$W_{A/B} = \bar{g}_{A/B} H_{A/B} \quad \bar{g}_{A/B} \cong g_{A/B} + 0.0424 H_{A/B}$$

where a Bouguer plate with a uniform density (2.67 g/cm³) is assumed.

- We calculate $W_B - W_A$ by combining **local leveling and gravity surveys near the clock sites (i)** and the result of **regional leveling surveys regularly measuring the Helmert height (ii)**.
- We correct for crustal movement on the route (ii) to adjust the epochs to 1/1/2020 with a least-square regression.

$\bar{g}_{i,i+1}$: average surface gravity between site i and i+1

$\Delta H_{i,i+1}$: observed leveling height between site i and i+1

\bar{g}_B : average gravity along the plumb line at site B

$H_{A,B}$: orthometric height

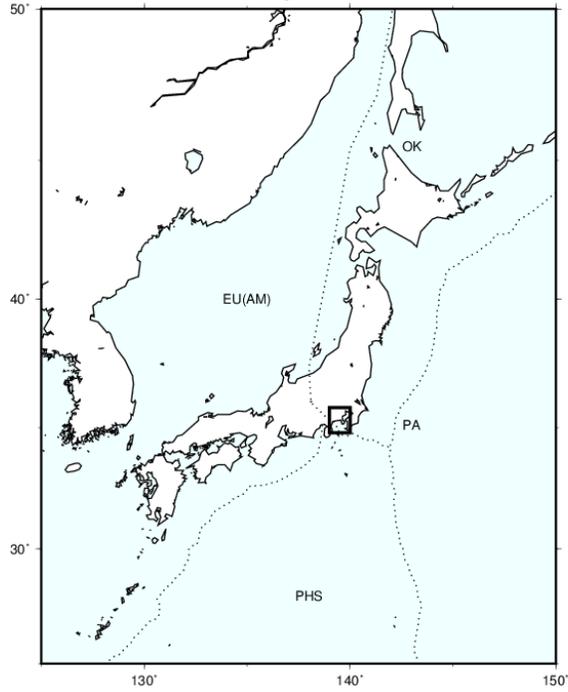
g_B : surface gravity at site B

Delva et al., (2019)

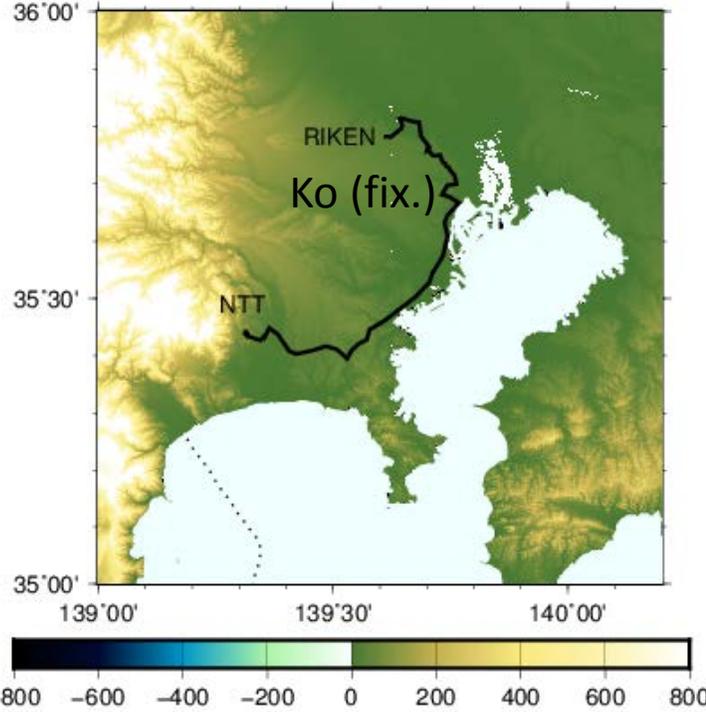
Hofmann-Wellenhof & Moritz (1967)

Leveling survey route

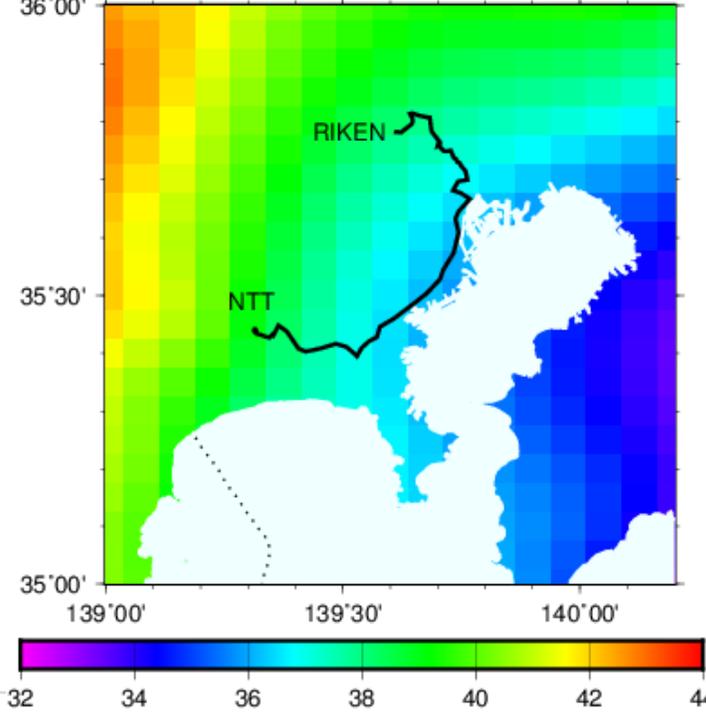
Study area



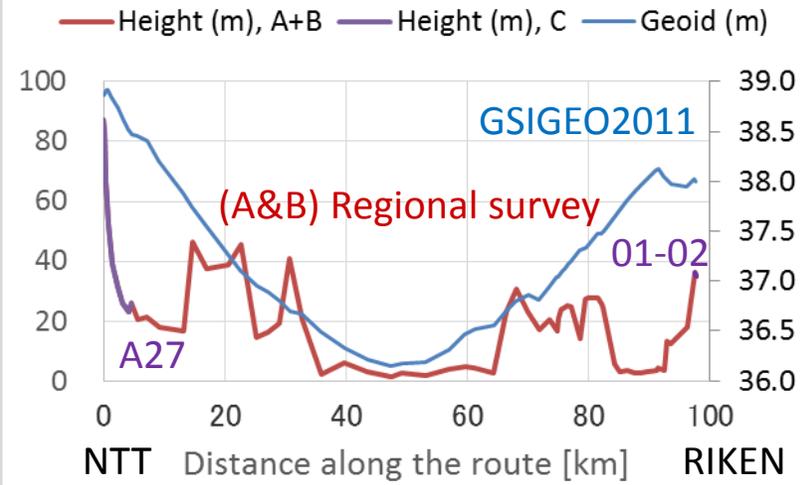
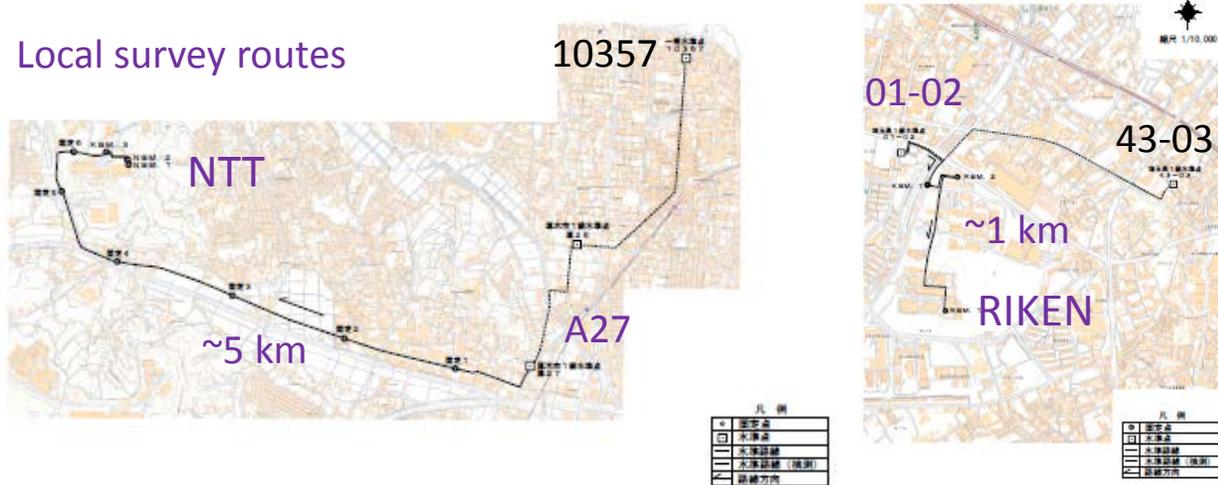
Orthometric Height (m)



Geoid height (m)



Local survey routes



Data

- Leveling data

- A. GSI's crustal deformation monitoring data (1/a) [2013-2019]

- B. Municipal government data for monitoring groundwater movement (0.5-1/a) [2012-2019]

- C. Local (<10 km) survey near the clock sites [2020]

- A-C are based on 1st order survey (uncertainty $\leq 2.5\sqrt{S/\text{km}}$ [mm], w temperature correction, no tidal corrections)

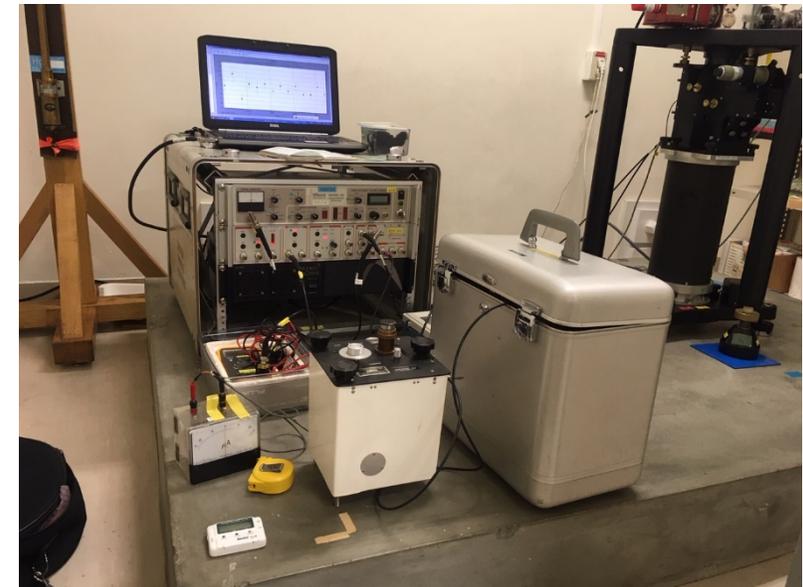
- Gravity data

- Values on routes for A&B were calculated by the GSI, based on JGSN75 (The Japan Gravity Standardization Net 1975) (GSI, 1976). Uncertainty is 0.1 mGal (Kuroishi & Murakami, 1991).

- Values on route for C were observed with a L-R G-type gravimeter (#705) and an absolute gravimeter FG5#109. Deviations from the linear drift after a tidal correction were ~5 microGals.

Examples of leveling & gravity survey

- Leveling survey inside the buildings: Feb. 4 and 18, 2020 (Showa holdings Co. Ltd.)
- Gravity measurement inside the buildings: Feb. 18 and Mar. 24, 2020



The mask is probably for preventing the bubble from being warmed by the breath.

Preliminary result

Sites	01-02	**RIKEN	A27	**NTT
*Helmert height [m]	36.1236	35.9523 ^a	25.9868	99.2568 ^b
Potential [m ² /s ²]	353.936	352.257 ^c	254.616	972.499 ^d

*Height at the Tokyo Origin (Ko) is fixed at 22.9994 m

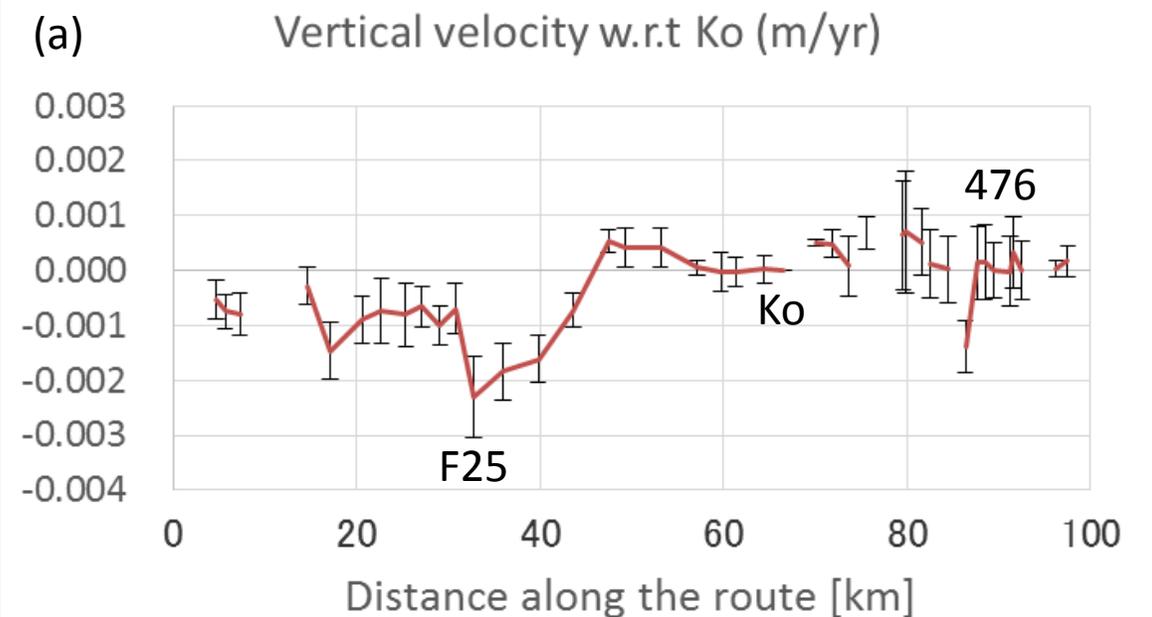
** Height at the highest point on the clock chamber
(exact location of the atom clouds: t.b.d.)

- dH (b-a) and dW (d-c) = 63.3045 ± 0.0114 m, 620.242 ± 0.112 m²/s²
- The biases associated with the origin of height and the potential value on the domestic geoid model vanish when taking the difference between the two sites.

The error budget (height)

- Allowable measurement error = $\pm 2.5\sqrt{S/\text{km}}$ [mm] $\cong \pm 25$ mm
- Postseismic deformation of the 2011 Tohoku eq and secular plate motion
 - Leveling data over 4-6 yr time spans show average vertical velocity on the route $|V| < 2 \pm 1$ mm/yr (figure)
- Routes A&B: Fitting $y = a(t - 2020) + b$ against the repeated survey data from 2013-2018. The resultant correction for epochs = -1.6 ± 1.8 mm@A27 (NTT) and 0.3 ± 1.3 mm@01-02 (RIKEN).
- Route C: Average closure of round-trip surveys/1 km x distance (**2.3 mm**)

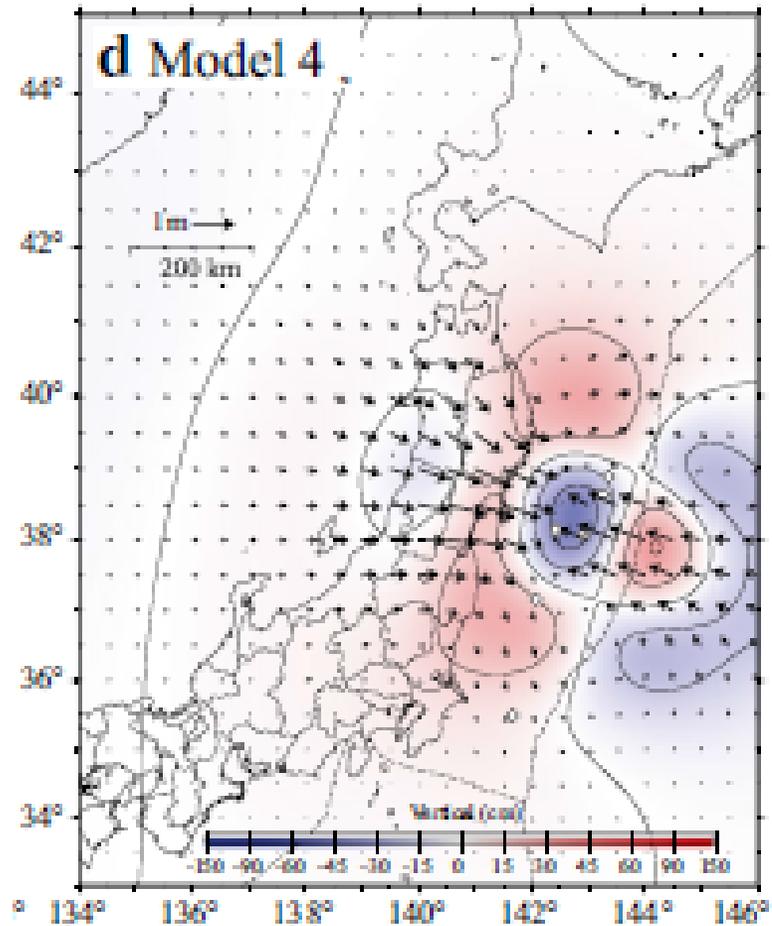
- Tidal potential changes during each observation
 - OLC data are typically averaged over >1 day.
 - Kuroishi (2010) estimated the effects of the solid-Earth and ocean tides on four representative routes across Japan. The total error is **11 mm at the maximum** for 100-km distance, comparable to the estimate of Vanicek (1980): 0.1 mm/km for the solid-Earth tides.



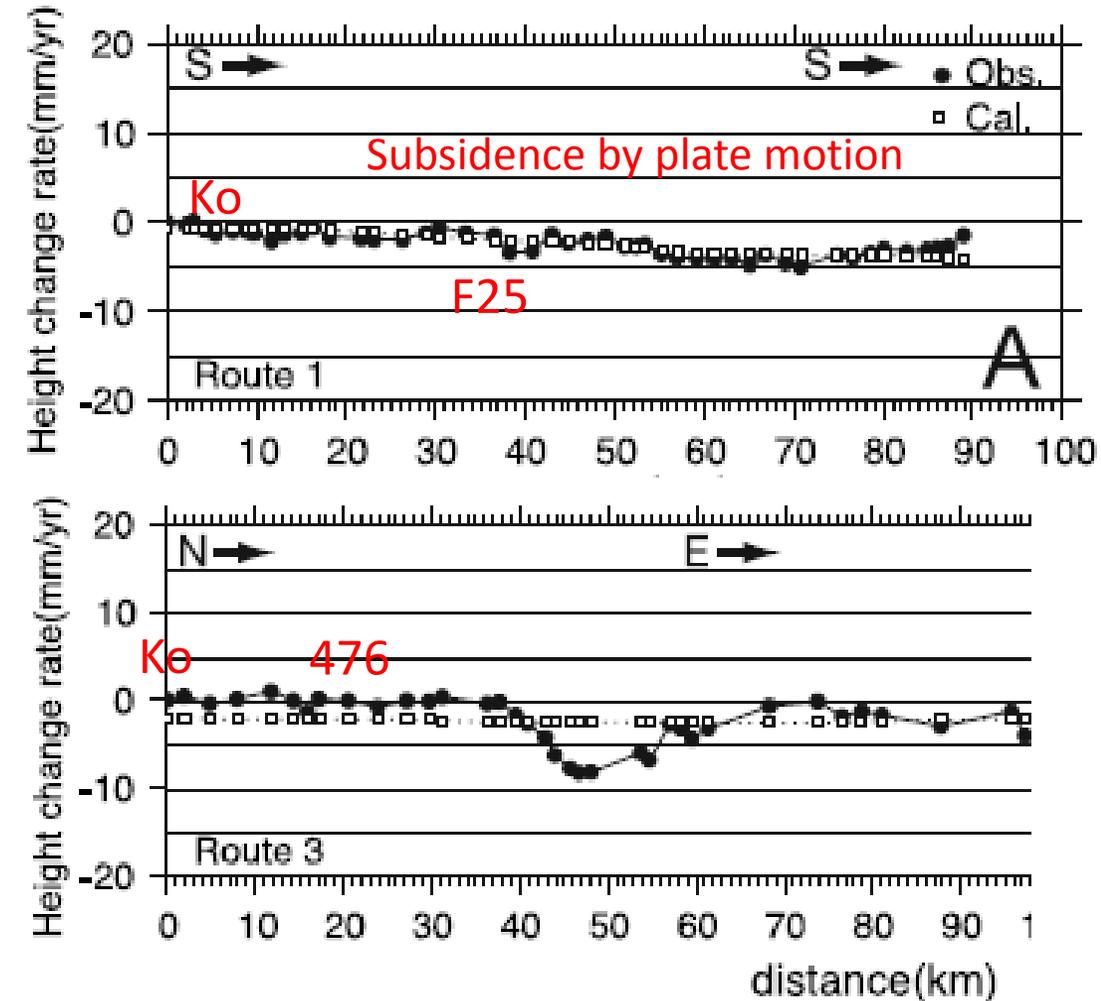
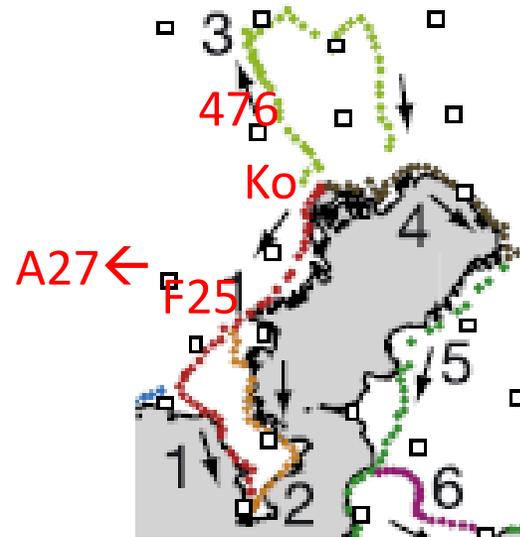
These lead to the maximum uncertainty of **± 11.4 mm** in dH and $9.8 \text{ m/s}^2 \times \pm 11.4 \text{ mm} = \pm 0.112 \text{ m}^2/\text{s}^2$.

Spatial pattern of the vertical velocity

Postseismic relaxation (2011-2016)
(Suito, 2017)



Secular plate motion before the 2011 event
(Nishimura et al., 2007)



- The velocity obtained in our study probably reflects plate motion (faster subsidence toward South)

The error budget (gravity)

- Uncertainty from surface gravity (± 0.1 mGal on routes A&B and ± 0.005 mGal on route C)
 - The largest height difference between BMs adjacent to each other is 30 m.
 - The corresponding maximum height difference $\cong 0.1$ mGal/980 Gal x 30 m = 0.0031 mm
 - # of BMs $\cong 70$. The **maximum unc.** = 0.0031 mm x 70 = 0.22 mm or **0.002 m²/s²**, which is negligible.
- Uncertainty due to the simple Bouguer correction (applied to sites A27 and 01-02)
 - $(\gamma + 2 \times 2\pi G\rho)H/2 = -0.0424$ mGal/m
 - When $\rho = 1$ g cm⁻³, the factor = -0.1124 mGal/m. $(-0.1124 + 0.0424)$ mGal/m x 26/36 m = -1.8/-2.5 mGal.
 - The resultant max. unc. for the potential difference could be $2.5 \times 36 \times 10^{-5} = \mathbf{0.0009$ m²/s², which is negligible.
- The effect of the permanent tide should be theoretically restored in the analyses of gravity data, but it is also negligible (<0.1 mGal (Ekman, 1989)).

→ The uncertainty of the potential is dominated by the uncertainty of the height determination.

Summary and future work

- The 100-km-scale optical fiber network connecting RIKEN and NTT with portable OLCs with 10^{-18} –order uncertainties will become available soon in Japan.
- We estimated the potential difference between the two clock sites in advance, based on the leveling-gravity method.
- The maximum uncertainty for the potential difference originating from the height and gravity measurements was estimated as ± 1.1 cm in the unit of height. This uncertainty is dominated by the tidal effects on the inclination of the potential surface during measurements, which was only roughly estimated in this study.
- We will estimate the tidal effects through the observation route more realistically.
- Temporal changes in the potential due to groundwater variations → GRACE-FO
- Effects of non-tidal variations in the sea-level on the inclination of the surface potential → Numerical simulation
- We will carry out an independent confirmation by the GNSS-geoid method.