Geodetic determination of the gravitational potential difference for the optical lattice clock comparison in the Kanto region in Japan

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Acknowledgements: Some geodetic data were provided by the Geospatial Information Authority of Japan and Kanagawa, Saitama and Tokyo Prefectures.

This study was supported by the JST project, “Space-time information platform with a cloud of optical lattice clocks”.

Background

- The gravitational red shift: time runs slower where the gravitational potential is lower.
  \[
  \frac{dt_{\text{high}}}{dt_{\text{low}}} = 1 + \frac{\Delta W}{c^2}, \quad \Delta W = g \Delta H
  \]
- Atomic clocks can detect a relative difference in the clock frequencies.
- Terrestrial clocks can be used as an altimeter.

<table>
<thead>
<tr>
<th>Region (e.g.)</th>
<th>Geology /network scale</th>
<th>Main purpose</th>
<th>Required uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>Stable continent</td>
<td>Unification of height reference systems</td>
<td>10^{-17} or better (cf leveling)</td>
</tr>
<tr>
<td>Japan</td>
<td>Unstable island arc</td>
<td>Crustal deformation monitoring</td>
<td>10^{-18} ≤24h (cf GNSS)</td>
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</table>

- Fiber-linked optical lattice clocks (OLCs) can achieve ~10^{-18} (corresponding to 1-cm height difference) uncertainty within several hours.
• Helmert orthometric height
• Geoid model by the Geospatial Information Authority of Japan (GSI) (Miyahara et al., 2014), SD=1.8 cm
• ~1300 cGNSS stations with average spacing of 20-25 km and the first-order leveling routes over 18,000 km for crustal deformation monitoring
• GNSS-leveling and gravimetric approaches were used for the longer- and shorter-wavelength determination, respectively.

Uncertainty of the geoid model (comparison with GNSS-leveling)
Crustal vertical velocity in Japan


Murakami and Ozawa (2004)
Recent progress regarding OLCs in Japan (selected)

- Chronometric heights obtained by OLCs were compared with geodetic survey results:
  - RIKEN-UTokyo: $5 \times 10^{-18}$, OLCs in laboratory environment (Takano et al., 2016)
  - Observatory of Tokyo Skytree: $1-5 \times 10^{-18}$, portable clock (Takamoto et al., 2020)
  - NTT-RIKEN-UTokyo: Fiber-linked clocks will become available soon.
  - 400-km fiber link toward the NE Japan (Mizusawa) under development

Takamoto et al. (2020)
Purpose of this study

• Our ultimate goal is to utilize OLCs to assist GNSS to monitor vertical deformation.

• In this study, we determine the static potential difference between the NTT and RIKEN clock sites to confirm the uncertainty of the portable clocks over a 100-km-scale fiber network, using geodetic observations.

• We discuss the error budget for the geodetic result.

Red: Expected uncertainty by using OLCs
• Faster positioning of vertical deformation than in GNSS (1 cm in several hours)
• Free from atmospheric noise
• It can separate apparent seasonal variations inherent in space geodetic techniques

Murakami and Ozawa (2004)
Method

Leveling-gravity method

(i) Direct integration of the potential increment

$$\Delta W_{AB} \approx \sum_i \bar{g}_{i,i+1} \Delta H_{i,i+1}$$

where a Bouguer plate with a uniform density (2.67 g/cm³) is assumed.

(ii) Computation based on the definition of Helmert orthometric height

$$W_{A/B} = \bar{g}_{A/B} H_{A/B}$$

$$\bar{g}_{A/B} \approx g_{A/B} + 0.0424 H_{A/B}$$

where a Bouguer plate with a uniform density (2.67 g/cm³) is assumed.

- We calculate $W_B - W_A$ by combining local leveling and gravity surveys near the clock sites (i) and the result of regional leveling surveys regularly measuring the Helmert height (ii).

- We correct for crustal movement on the route (ii) to adjust the epochs to 1/1/2020 with a least-square regression.

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$\bar{g}_{i,i+1}$: average surface gravity between site i and i+1

$\Delta H_{i,i+1}$: observed leveling height between site i and i+1

$\bar{g}_B$: average gravity along the plumb line at site B

$H_{A,B}$: orthometric height

$g_B$: surface gravity at site B

Delva et al., (2019)
Hofmann-Wellenhof & Moritz (1967)
Leveling survey route

Study area

Orthometric Height (m)

Geoid height (m)

Local survey routes

NTT

~5 km

A27

10357

01-02

43-03

Ko (fix.)

Regional survey

(A&B) Regional survey

GSIGEO2011

RIKEN

NTT

01-02

A27

~1 km

RIKEN

Distance along the route [km]

NTT

RIKEN

~1 km

~5 km
Data

• Leveling data
  A. GSI’s crustal deformation monitoring data (1/a) [2013-2019]
  B. Municipal government data for monitoring groundwater movement (0.5-1/a) [2012-2019]
  C. Local (<10 km) survey near the clock sites [2020]
  • A-C are based on 1st order survey (uncertainty $\leq 2.5\sqrt{S}/\text{km}[\text{mm}]$, w temperature correction, no tidal corrections)

• Gravity data
  • Values on routes for A&B were calculated by the GSI, based on JGSN75 (The Japan Gravity Standardization Net 1975) (GSI, 1976). Uncertainty is 0.1 mGal (Kuroishi & Murakami, 1991).
  • Values on route for C were observed with a L-R G-type gravimeter (#705) and an absolute gravimeter FG5#109. Deviations from the linear drift after a tidal correction were ~5 microGals.
Examples of leveling & gravity survey

- Leveling survey inside the buildings: Feb. 4 and 18, 2020 (Showa holdings Co. Ltd.)
- Gravity measurement inside the buildings: Feb. 18 and Mar. 24, 2020

The mask is probably for preventing the bubble from being warmed by the breath.
Preliminary result

<table>
<thead>
<tr>
<th>Sites</th>
<th>01-02</th>
<th><strong>RIKEN</strong></th>
<th>A27</th>
<th><strong>NTT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>*Helmert height [m]</td>
<td>36.1236</td>
<td>35.9523(^a)</td>
<td>25.9868</td>
<td>99.2568(^b)</td>
</tr>
<tr>
<td>Potential [m(^2)/s(^2)]</td>
<td>353.936</td>
<td>352.257(^c)</td>
<td>254.616</td>
<td>972.499(^d)</td>
</tr>
</tbody>
</table>

*Height at the Tokyo Origin (Ko) is fixed at 22.9994 m
**Height at the highest point on the clock chamber
(exact location of the atom clouds: t.b.d.)

• \(dH\) (b-a) and \(dW\) (d-c) = \(63.3045 \pm 0.0114\) m, \(620.242 \pm 0.112\) m\(^2\)/s\(^2\)

• The biases associated with the origin of height and the potential value on the domestic geoid model vanish when taking the difference between the two sites.
The error budget (height)

- Allowable measurement error = $\pm 2.5\sqrt{S/\text{km}} \text{ [mm]} \cong \pm 25 \text{ mm}$

- Postseismic deformation of the 2011 Tohoku eq and secular plate motion
  - Leveling data over 4-6 yr time spans show average vertical velocity on the route $|V| < 2 \pm 1 \text{ mm/yr}$ (figure)

- Routes A&B: Fitting $y = a(t - 2020) + b$ against the repeated survey data from 2013-2018. The resultant correction for epochs $= -1.6 \pm 1.8 \text{ mm}@A27$ (NTT) and $0.3 \pm 1.3 \text{ mm}@01-02$ (RIKEN).

- Route C: Average closure of round-trip surveys/1 km x distance (2.3 mm)

- Tidal potential changes during each observation
  - OLC data are typically averaged over $>1$ day.
  - Kuroishi (2010) estimated the effects of the solid-Earth and ocean tides on four representative routes across Japan. The total error is 11 mm at the maximum for 100-km distance, comparable to the estimate of Vanicek (1980): 0.1 mm/km for the solid-Earth tides.

These lead to the maximum uncertainty of $\pm 11.4 \text{ mm}$ in $dH$ and $9.8 \text{ m/s}^2 \times \pm 11.4 \text{ mm} = \pm 0.112 \text{ m}^2/\text{s}^2$. 
The velocity obtained in our study probably reflects plate motion (faster subsidence toward South)
The error budget (gravity)

• Uncertainty from surface gravity (±0.1 mGal on routes A&B and ±0.005 mGal on route C)
  • The largest height difference between BMs adjacent to each other is 30 m.
  • The corresponding maximum height difference = 0.1 mGal/980 Gal x 30 m = 0.0031 mm
  • # of BMs ≈ 70. The maximum unc. = 0.0031 mm x 70 = 0.22 mm or 0.002 m²/s², which is negligible.

• Uncertainty due to the simple Bouguer correction (applied to sites A27 and 01-02)
  • \((\gamma + 2 \times 2\pi G \rho) H/2 = -0.0424\) mGal/m
  • When \(\rho = 1\) g cm⁻³, the factor = -0.1124 mGal/m. \((-0.1124+0.0424)\) mGal/m x 26/36 m = -1.8/-2.5 mGal.
  • The resultant max. unc. for the potential difference could be \(2.5 \times 36 \times 10^{-5} = 0.0009\) m²/s², which is negligible.

• The effect of the permanent tide should be theoretically restored in the analyses of gravity data, but it is also negligible (<0.1 mGal (Ekman, 1989)).

→ The uncertainty of the potential is dominated by the uncertainty of the height determination.
Summary and future work

• The 100-km-scale optical fiber network connecting RIKEN and NTT with portable OLCs with $10^{-18}$-order uncertainties will become available soon in Japan.

• We estimated the potential difference between the two clock sites in advance, based on the leveling-gravity method.

• The maximum uncertainty for the potential difference originating from the height and gravity measurements was estimated as $\pm 1.1$ cm in the unit of height. This uncertainty is dominated by the tidal effects on the inclination of the potential surface during measurements, which was only roughly estimated in this study.

• We will estimate the tidal effects through the observation route more realistically.

• Temporal changes in the potential due to groundwater variations $\rightarrow$ GRACE-FO

• Effects of non-tidal variations in the sea-level on the inclination of the surface potential $\rightarrow$ Numerical simulation

• We will carry out an independent confirmation by the GNSS-geoid method.