Examining the extremal dependence structure of precipitation in Norway

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April 28, 2020

Motivation

- We wish to model extreme precipitation using spatial extreme value theory.
- "Space-poor" hourly precipitation data is available from weather stations all over Norway: https://frost.met.no.
- A gridded data product of resolution $1 \times 1\text{km}^2$ contains post-processed predictions of hourly precipitation data over Norway from 2013-2020: https://thredds.met.no/thredds/catalog/metpparchivev1/catalog.html.
- We examine extremal properties of the gridded data product to test whether it can be used for spatial modelling of extreme precipitation.

Research questions

- What is the extremal dependence structure of hourly precipitation in the southern parts of Norway?
- Can the gridded data product be used as a surrogate for unobserved precipitation data?

Extremal dependence

- For the random variables X and Y with distribution functions F_X and F_Y, denote

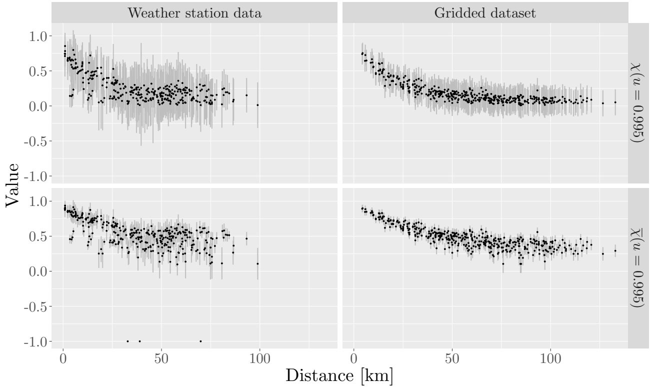
$$\chi = \lim_{u \to 1} \chi(u) = \lim_{u \to 1} P\left(F_X(X) > u | F_Y(Y) > u\right).$$

- If $\chi > 0$ we say that X and Y are asymptotically dependent, while they are asymptotically independent for $\chi = 0$.
- Extremes values of asymptotically dependent variables tend to occur simultaneously. This can have great consequences, as e.g. extreme precipitation over an entire catchment area can have much more devastating effects than extreme precipitation at only one small part of a catchment.

Method

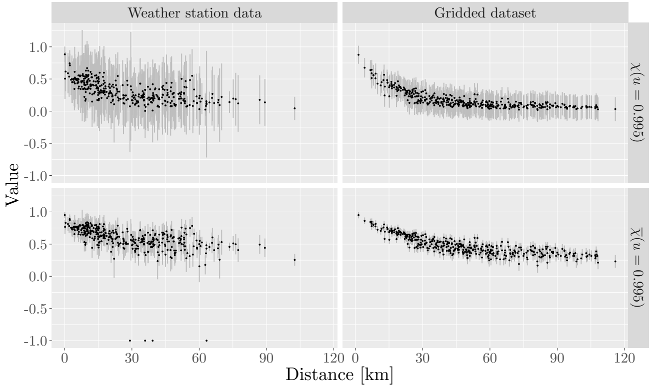
- We examine bivariate properties of extremal dependence by estimating $\chi(u = 0.995, h)$ and $\overline{\chi}(u = 0.995, h)$ for different distances h (e.g. Coles et al., 1999).
- The dependence measures are estimated in an area around Bergen and Stavanger, as these areas contain many weather stations that observe hourly precipitation.
- Bivariate precipitation data is modelled with copulas exhibiting different extremal dependence properties.

Stavanger



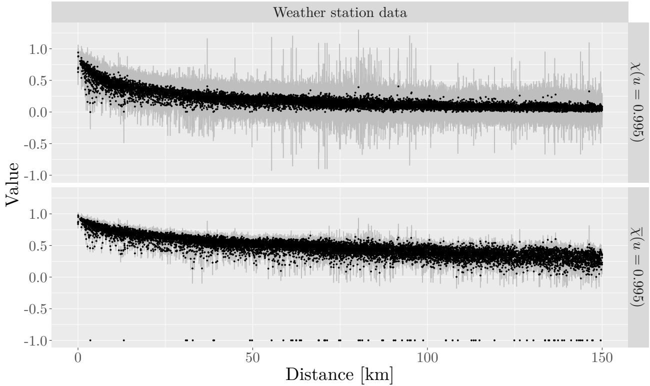
χ goes quickly towards zero as the distance increases. The rate of decline is similar for both data sets. $\overline{\chi}$ converges to a value that is significantly different from one as the distance increases.

Bergen



One again, χ goes quickly towards zero as the distance increases, and the rate of decline is similar for both data sets. Values of $\overline{\chi}$ also seems to be coherent between both data sets.

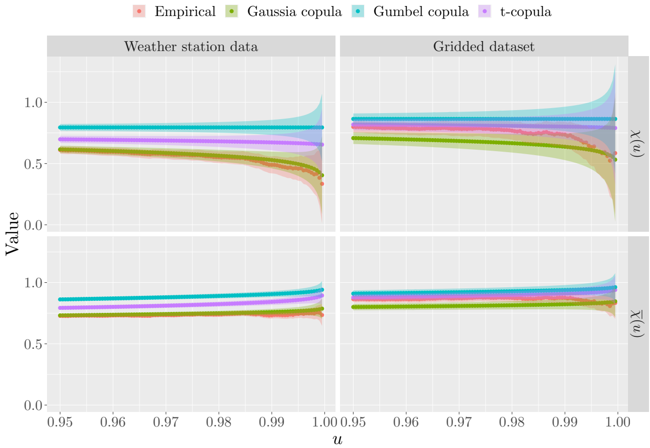
All of southern Norway



The same relationship between χ, $\overline{\chi}$ and distance can be found for all weather stations in the southern parts of Norway.

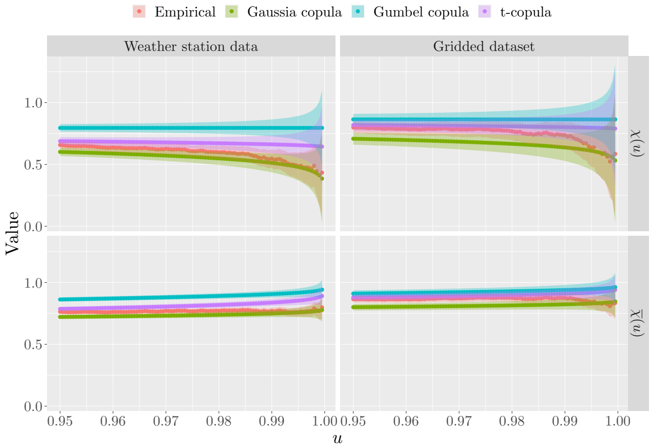
- The data exhibits properties of extremal independence for large distances, but what about for small distances?
- Following Dawkins and Stephenson (2018) we fit copulas with different extremal dependence properties to bivariate precipitation data, and calculate χ and $\overline{\chi}$ with the estimated copulas.

$h = 10$km



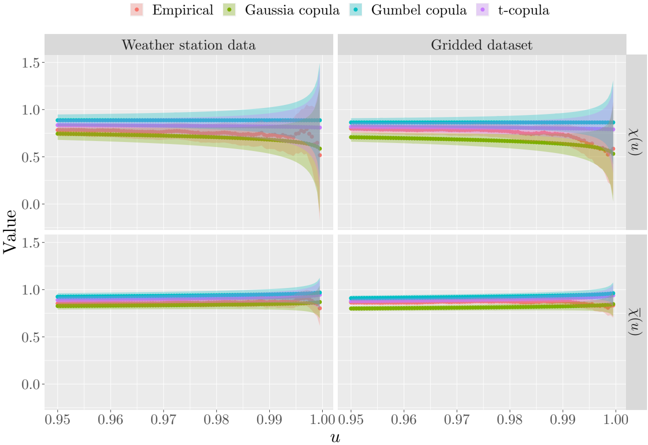
The Gaussian copula seems to have a good fit to data. Values of χ and $\overline{\chi}$ are similar to the empirical values. There is a considerable difference between the weather station data and the gridded data product, even though data from the weather stations are included in the creation of the data product.

$h = 5\text{km}$



Results are similar to those of $h = 10\text{km}$.

$h = 2$km



There is no significant difference between the values of χ and $\overline{\chi}$ for the different copulas.

Conclusion

- The gridded data product can be used as a surrogate for unobserved precipitation data. However, there are considerable differences between bivariate extremal dependence characteristics at specific locations, so care should be taken.
- Extreme precipitation in Norway is asymptotically independent for large distances. For short distances where the data is expected to be asymptotically dependent, the Gaussian copula, which is asymptotically independent, still provides a good model fit.

Bibliography

Coles, S., Heffernan, J., & Tawn, J. (1999). Dependence measures for extreme value analyses. *Extremes*, *2*(4), 339–365. https://doi.org/10.1023/A:1009963131610

Dawkins, L. C., & Stephenson, D. (2018). Quantification of extremal dependence in spatial natural hazard footprints: Independence of windstorm gust speeds and its impact on aggregate losses. *European Geosciences Union (EGU)/Copernicus Publications*. https://doi.org/10.5194/nhess-18-2933-2018