Study on the Variance Component Estimation in relative weighting of the Inter-Satellite Links and GNSS observations for orbit determination

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1. Introduction

Inter-Satellite Links (ISL) provide precise range measurements between satellites in the specific GNSS constellation which is one of the key requirements for improving accuracy and reliability of the orbit determination. Our previous investigation based on various ISL connectivity schemes (observation scenarios) indicates that by using ISL measurements in addition to GNSS observations, it is possible to improve orbit estimation mainly by reducing RMS errors in cross-track and along-track components.

This research aims at evaluation of the Variance Component Estimation (VCE) to derive a combined orbit solution from the ISL and GNSS measurements. We would like to focus on comparison of weighting methods based on presupposed measurement accuracies (described here as an empirical weighting) and three approaches to the VCE method. VCE is used to determine proper weighting factors for different types of measurements, e.g. of diverse nature or based on distinct techniques and thus of various accuracy. In this simulation-based study we assess orbit solutions using both types of weighting and evaluate properties of the simulated ISLs measurements including the connectivity schemes and observation accuracy.

2. Methodology

Performed analysis are based on simulations done with software currently developed at Space Research Centre Polish Academy of Sciences. Its key modules are:
- precise orbit propagator with specific support for navigation satellites,
- simulator of the GNSS and the ISL observations,
- orbit estimator.

For the purpose of this study, Galileo constellation is simulated as a Walker 56°:24/3/1. Nominal conditions of the performed simulations are presented in the Table 1.

Simulations are performed with three values of empirical coefficients and three approaches to VCE: Helmert, Förstner (I) and Förstner (II). To assess the impact of each concept, four sets of relative accuracies (Tests I—IV) of the ISLs and GNSS measurements were tested (Table 2). In the Table 3 are presented weighting factors obtained with VCE for each test. In the figures 2—5 are shown examples of orbit determination for Tests I—IV. To obtain full error assessment each simulation was repeated 10 times (imitating Monte-Carlo method), then mean RMS values are shown together with their standard deviations as error bars.

Orbit simulation	
Galileo Walker 56°:24/3/1 Galileo FOC box-wing model Gravitational and non-gravitational perturbations	
GNSS observations	**ISL observations**
44 ground stations 30 seconds sampling Observation noise Zenith Wet Delays	Three connectivity schemes 30 seconds sampling Observation noise
Orbit estimation - Weighted Least Squares Algorithm with epoch-wise clock estimation	
Satellite positions ECOM2 - 9 parameters Epoch-wise satellite clocks Epoch-wise station clocks Zenith Wet Delay - piecewise linear model	

REFERENCE

Table 1. Methodology summary

For purpose of this study we assumed that:
- the ISL and the GNSS hardware are connected to the same on-board atomic clock,
- the ISL observations are synchronized with the GNSS measurements,
- possible hardware delays are not taken directly into account during the simulation,
- no clock jumps, pulses and other accidental errors were simulated.

Two connection types are considered - one-way and dual one-way (simultaneous) (Figure 1.). In the first, only one connection is established between scheduled pair of the satellites in the current epoch and the clock estimation is required. In the second type, two connections are established in the same time (we neglect possible inaccuracies in synchronization). In the post-processing the impact of the clock errors on the link is reduced.

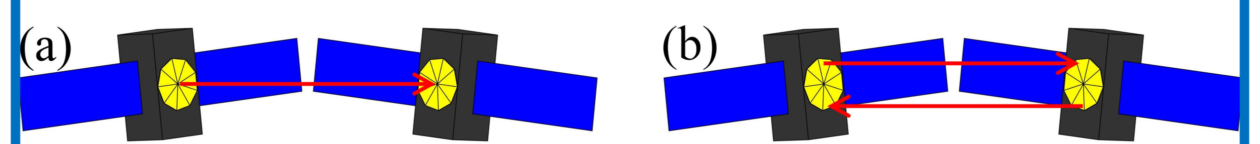

Figure 1. Connection type: (a) one-way and (b) dual one-way

3. Theory and Results

Scenario	Test I	Test II	Test II	Test IV
GNSS	σ = 1 cm	σ = 1 cm	σ = 15 cm	σ = 5 cm
ISL	σ = 0.5 cm	σ = 1 cm	σ = 5 cm	σ = 15 cm
α	α = 0.5	α = 0.75	α = 1	

Table 2. Parameters provided for simulations

	mean $1/S^2$ [GNSS | ISL]					
	Helmert		Förstner (I)		Förstner (II)	
Test I						
Sequential dual one-way	2.8962	0.8466	2.8940	0.8463	2.8908	0.8479
Sequential one-way	2.8728	0.3746	2.8774	0.3756	2.8699	0.3756
Intra-plane closed	2.8448	0.5544	2.8420	0.5546	2.8473	0.5534
Intra-plane open	2.8535	0.5572	2.8541	0.5593	2.8541	0.5563
Test II						
Sequential dual one-way	2.8977	1.4956	2.8958	1.5023	2.8926	1.5022
Sequential one-way	2.8747	0.8189	2.8801	0.8275	2.8723	0.8225
Intra-plane closed	2.8511	0.9730	2.8487	0.9789	2.8539	0.9726
Intra-plane open	2.8590	0.9774	2.8600	0.9814	2.8593	0.9771
Test III						
Sequential dual one-way	0.0330	28.5117	0.0177	1.9481	0.0183	1.9471
Sequential one-way	0.0331	1.8633	0.0175	0.5650	0.0183	0.5660
Intra-plane closed	0.0348	0.8946	0.0174	0.5775	0.0182	0.5767
Intra-plane open	0.0352	0.9092	0.0174	0.5745	0.0182	0.5744
Test IV						
Sequential dual one-way	0.1680	2.0505	0.1643	2.0182	0.1649	2.0181
Sequential one-way	0.1594	1.3371	0.1550	1.3086	0.1557	1.3042
Intra-plane closed	0.1515	1.2536	0.1469	1.2426	0.1477	1.2355
Intra-plane open	0.1532	1.2604	0.1488	1.2452	0.1496	1.2448

Table 3. Weighting factors $1/S^2$ computed with VCE

Empirical weighting

$$P_{ISL} = \frac{\alpha}{\sigma_{ISL}^2}$$
$$\alpha = (0{,}1\rangle$$
$$P_{GNSS} = \frac{\cos^2(Z)}{\sigma_{GNSS}^2}$$

Z – zenith angle

$\sigma_{ISL}, \sigma_{GNSS}$ – a priori values

Variance Component Estimation

Helmert

$$\boldsymbol{N}_{GNSS} = \boldsymbol{A}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{A}_{GNSS}$$
$$\boldsymbol{N}_{ISL} = \boldsymbol{A}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{A}_{ISL}$$
$$iteration\ k = 0 \rightarrow S_{GNSS}^{(k)} = \sigma_{GNSS}, S_{ISL}^{(k)} = \sigma_{ISL}$$
$$\boldsymbol{N}^{(k)} = \frac{1}{S_{GNSS}^{2\,(k)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k)}} \boldsymbol{N}_{ISL}$$
$$\begin{pmatrix} \boldsymbol{v}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{v}_{GNSS} \\ \boldsymbol{v}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{v}_{ISL} \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} S_{GNSS}^{2(k+1)} \\ S_{ISL}^{2(k+1)} \end{pmatrix}$$
$$s_{ii} = n_i - 2trace(\boldsymbol{N}^{-1}\boldsymbol{N}_i) + trace(\boldsymbol{N}^{-1}\boldsymbol{N}_i\boldsymbol{N}^{-1}\boldsymbol{N}_i)$$
$$s_{ij} = s_{ij} = trace\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\boldsymbol{N}^{-1}\boldsymbol{N}_j\right),$$
$$(i \neq j, 1 - GNSS, 2 - ISL)$$
$$S_{GNSS}^{(k+1)} - S_{GNSS}^{(k)} > \varepsilon$$
$$S_{ISL}^{(k+1)} - S_{ISL}^{(k)} > \varepsilon$$
$$k = k + 1$$
$$\boldsymbol{N}^{(k+1)} = \frac{1}{S_{GNSS}^{2\,(k+1)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k+1)}} \boldsymbol{N}_{ISL}$$

Förstner (I)

$$\boldsymbol{N}_{GNSS} = \boldsymbol{A}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{A}_{GNSS}$$
$$\boldsymbol{N}_{ISL} = \boldsymbol{A}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{A}_{ISL}$$
$$iteration\ k = 0 \rightarrow S_{GNSS}^{(k)} = \sigma_{GNSS}, S_{ISL}^{(k)} = \sigma_{ISL}$$
$$\boldsymbol{N}^{(k)} = \frac{1}{S_{GNSS}^{2\,(k)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k)}} \boldsymbol{N}_{ISL}$$
$$r_{GNSS}^{(k)} = n_{GNSS} - trace(\boldsymbol{N}_{GNSS}\boldsymbol{N}^{-1})$$
$$r_{ISL}^{(k)} = n_{ISL} - trace(\boldsymbol{N}_{ISL}\boldsymbol{N}^{-1})$$
$$S_{GNSS}^{2\,(k+1)} = \frac{\boldsymbol{v}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{v}_{GNSS}}{r_{GNSS}^{(k)}}$$
$$S_{ISL}^{2\,(k+1)} = \frac{\boldsymbol{v}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{v}_{ISL}}{r_{ISL}^{(k)}}$$
$$S_{GNSS}^{(k+1)} - S_{GNSS}^{(k)} > \varepsilon$$
$$S_{ISL}^{(k+1)} - S_{ISL}^{(k)} > \varepsilon$$
$$k = k + 1$$
$$\boldsymbol{N}^{(k+1)} = \frac{1}{S_{GNSS}^{2\,(k+1)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k+1)}} \boldsymbol{N}_{ISL}$$

Förstner (II)

$$\boldsymbol{N}_{GNSS} = \boldsymbol{A}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{A}_{GNSS}$$
$$\boldsymbol{N}_{ISL} = \boldsymbol{A}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{A}_{ISL}$$
$$iteration\ k = 0 \rightarrow S_{GNSS}^{(k)} = \sigma_{GNSS}, S_{ISL}^{(k)} = \sigma_{ISL}$$
$$\boldsymbol{N}^{(k)} = \frac{1}{S_{GNSS}^{2\,(k)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k)}} \boldsymbol{N}_{ISL}$$
$$r_{GNSS}^{(k)} = n_{GNSS} - \frac{1}{S_{GNSS}^{2\,(k)}} trace(\boldsymbol{N}_{GNSS}\boldsymbol{N}^{-1})$$
$$r_{ISL}^{(k)} = n_{ISL} - \frac{1}{S_{ISL}^{2\,(k)}} trace(\boldsymbol{N}_{ISL}\boldsymbol{N}^{-1})$$
$$S_{GNSS}^{2\,(k+1)} = \frac{\boldsymbol{v}_{GNSS}^T \boldsymbol{P}_{GNSS} \boldsymbol{v}_{GNSS}}{r_{GNSS}^{(k)}}$$
$$S_{ISL}^{2\,(k+1)} = \frac{\boldsymbol{v}_{ISL}^T \boldsymbol{P}_{ISL} \boldsymbol{v}_{ISL}}{r_{ISL}^{(k)}}$$
$$S_{GNSS}^{(k+1)} - S_{GNSS}^{(k)} > \varepsilon$$
$$S_{ISL}^{(k+1)} - S_{ISL}^{(k)} > \varepsilon$$
$$k = k + 1$$
$$\boldsymbol{N}^{(k+1)} = \frac{1}{S_{GNSS}^{2\,(k+1)}} \boldsymbol{N}_{GNSS} + \frac{1}{S_{ISL}^{2\,(k+1)}} \boldsymbol{N}_{ISL}$$

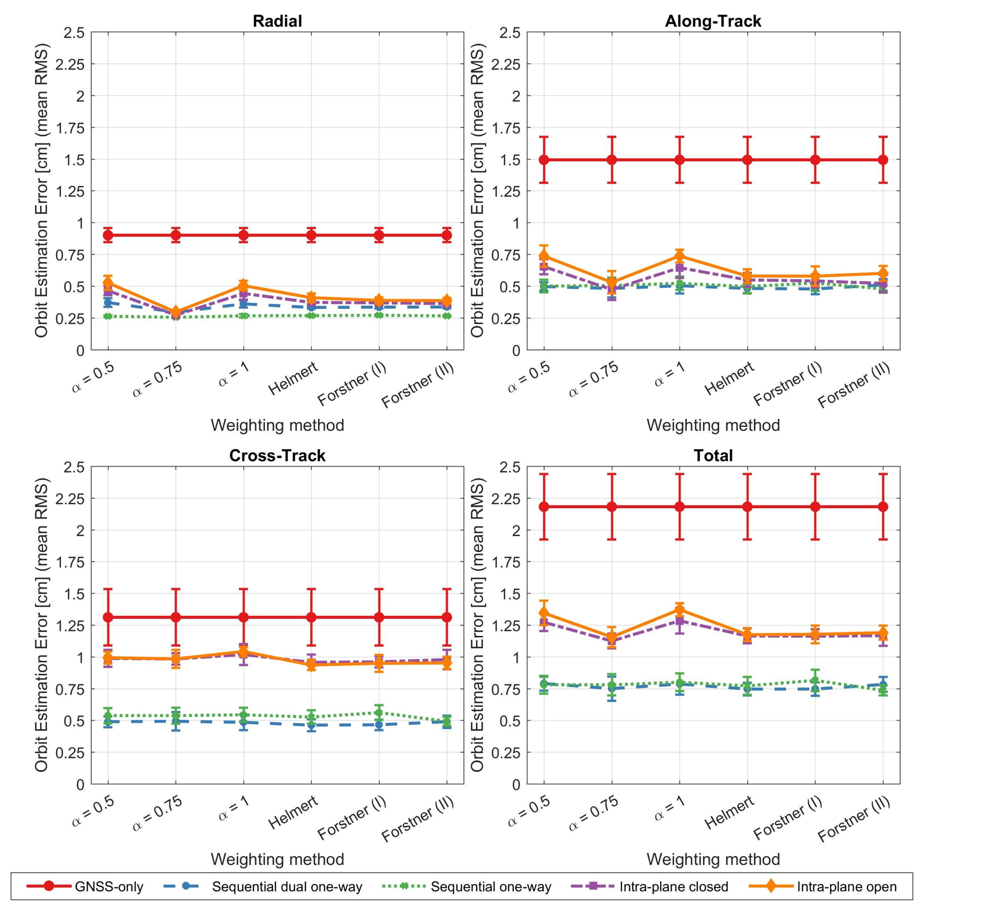

Figure 2. Example 1—results of **Test I** for orbit determination

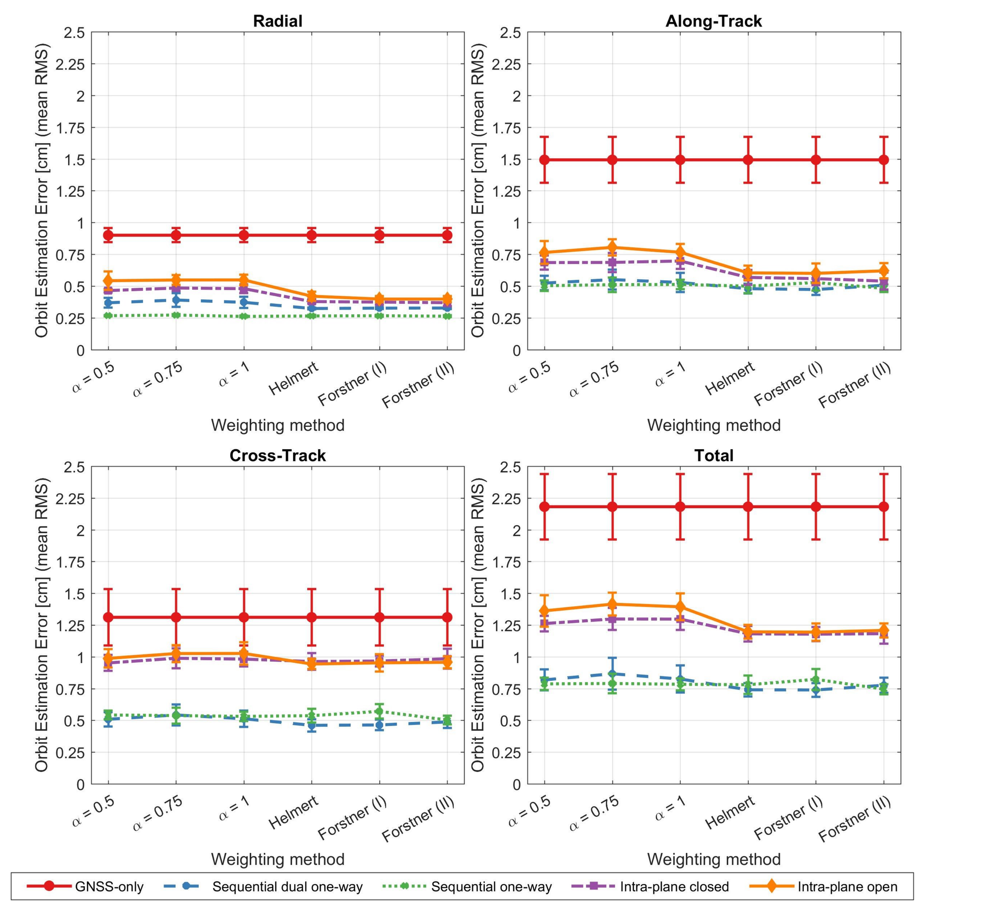

Figure 3. Example 2—results of **Test II** for orbit determination

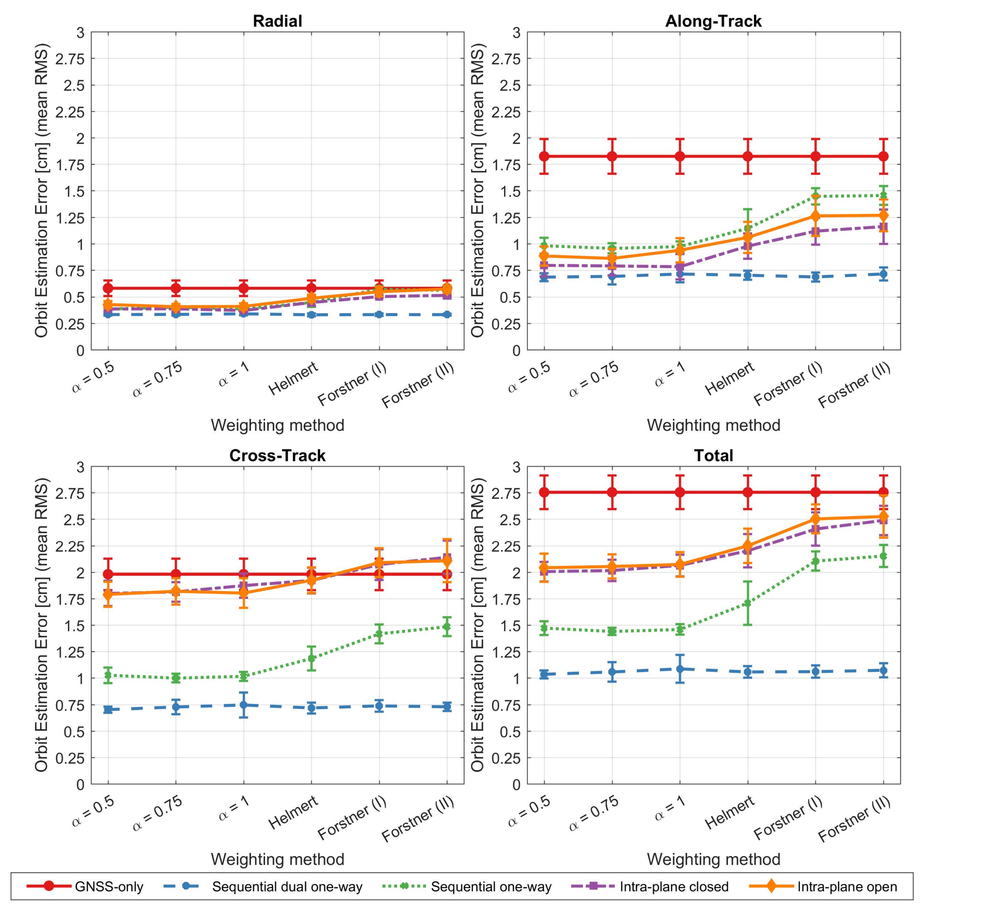

Figure 4. Example 3—results of **Test III** for orbit determination

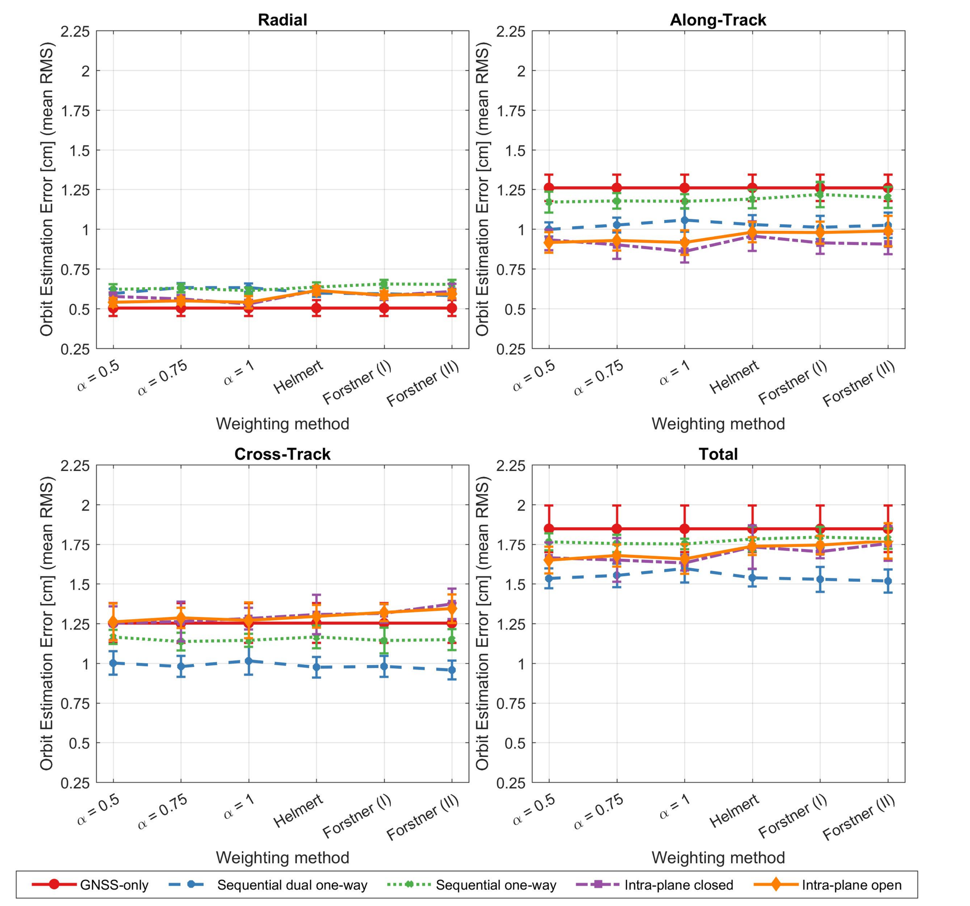

Figure 5. Example 4—results of **Test IV** for orbit determination

CONNECTIVITY SCHEMES

Sequential | Intra-plane closed (Ring) | Intra-plane open (Open ring)

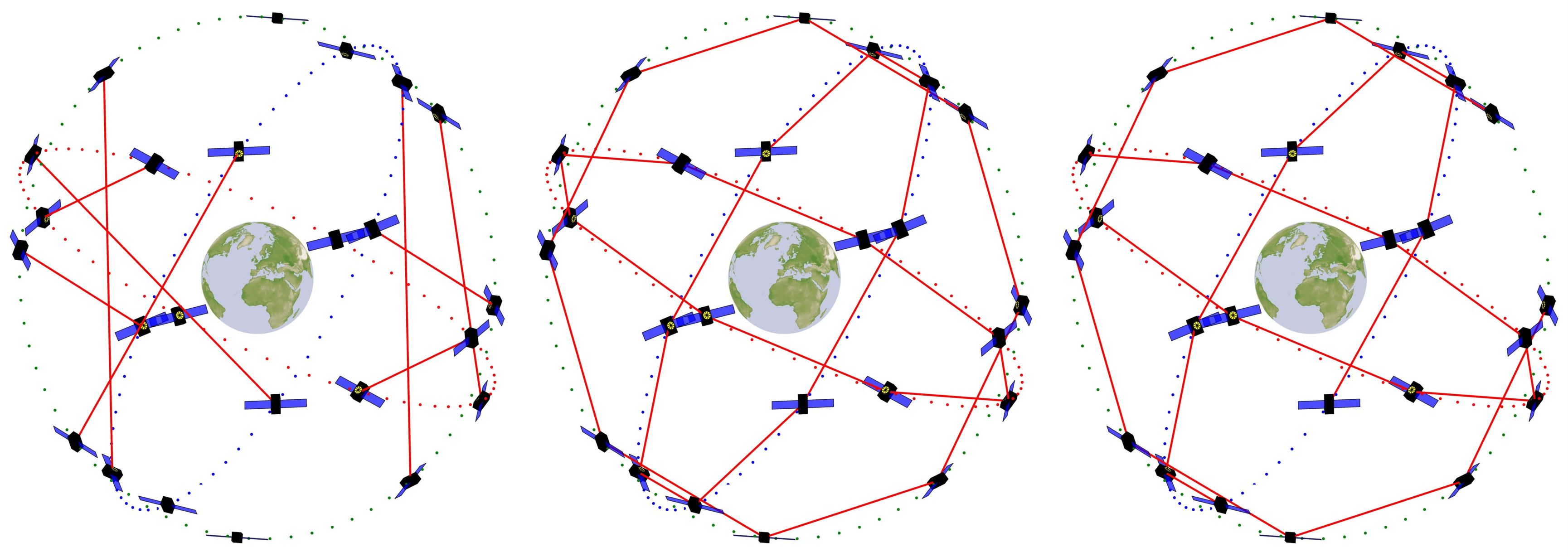

Figure 6. Graphic representation of the simulated ISL measurements

4. Conclusions

The simulation results show that the Variance Component Estimation can be used for better consideration of ISL and GNSS measurement errors. In general:

1. the differences between results obtained with various VCE approaches are mostly negligible (except Test III),
2. values of empirical coefficients for Test II, Test III and Test IV do not significantly change the results of the orbit determination,
3. VCE helps to minimize orbit 3D RMS errors in Test I and Test II, when the most realistic simulation scenarios are realized, in contrary to the results obtained with empirical weighting,
4. VCE enlarge 3D RMS orbit determination errors in Test III except sequential dual one-way scheme,
5. due to limited space, we resigned from showing results of clock estimation, but for each test they can be considered as not impacted by the choice of weighting method.

It should be noticed that the simulations were carried out according to simplified scenarios. The weighting coefficients show the differences between the connectivity schemes or relative accuracy of measurements. However, there remains the issue of estimation of additional parameters such as biases of ISL measurements or distance to the centre of mass, which we hope to include in subsequent analysis.

Remarks

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