

# On the connection between heat waves and large deviations of temperature

Vera Melinda Galfi <sup>1</sup>

Valerio Lucarini <sup>1,2</sup>

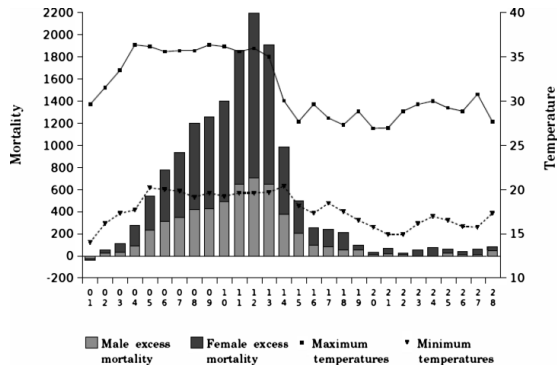
Jeroen Wouters <sup>2</sup>

<sup>1</sup> University of Hamburg

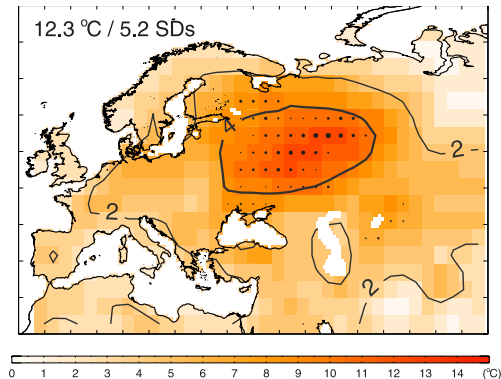
<sup>2</sup> University of Reading

May 4, 2020 - EGU General Assembly 2020

Heat waves are high impact events with **long duration** and **large spatial extension**.



Number of excess deaths in France during the 2003 European heat wave (Poumadere et al., 2005)



15-day max. temperature during the 2010 Russian heat wave (Barriopedro et al., 2011)

We use large deviation theory to study persistent events, like heat waves.

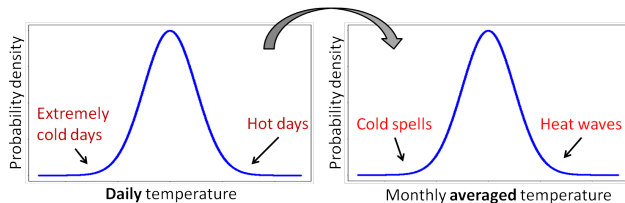
By using **extreme indices**, subjective decision are needed to select extremes.

**Extreme value theory (EVT)** is based on a **limit law for probabilities of very rare events**.

Problem: in case of persistent extreme events subsequent extremes can be correlated.

**Large deviation theory (LDT)** is based on a **limit law for probabilities of averages**.

**Extremes of averages can be related to persistent events.**



We analyze persistent events of **temperature** in two numerical models (atm. and climate).

By using LDT we can study persistent events  
in chaotic systems  
if we construct adequate observables.

# Large Deviation Theory:

limit law for averages over large averaging blocks.

Random variables  $X_1, X_2, \dots$

→ averages over blocks of length  $n$ :  $A_n = \frac{1}{n} \sum_{i=1}^n x_i$

Large Deviation Principle (LDP):

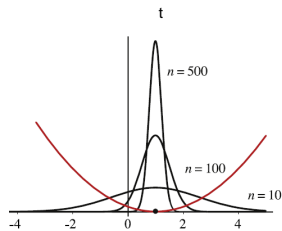
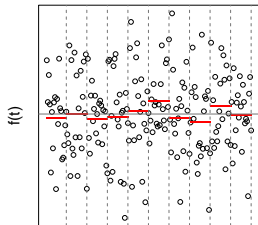
for  $n \rightarrow \infty$

the probability of  $A_n$ :

$$p(A_n = a) \approx e^{-nI(a)}$$

Rate function: speed of decay

Cramer, 1938; Ellis, 1985



Rate function (red line)  
of Gaussian random variables.  
Touchette, 2009

According to the large deviation analysis of temperature in the used numerical models we find that:

1. A universal function describes both temporal and spatial (and spatio-temporal) large deviations of temperature (PUMA).
2. Heat waves (cold spells) can be studied as large deviations of temperature averaged over adequate spatial or temporal scales (PUMA & MPI-ESM).

# We analyse large deviations of near-surface temperature in the atmospheric model PUMA

The **P**ortable **U**niversity **M**odel of the **A**tmosphere (PUMA) is a general circulation model developed at the University of Hamburg.

- primitive equations on the sphere
- hydrostatic approximation
- simple parametrisations for friction, diffusion and diabatic heating
- can be downloaded for free:

<https://www.mi.uni-hamburg.de/en/arbeitsgruppen/theoretische-meteorologie/modelle/plasim.html>

We perform simulations for 10000 years without orography or annual cycle, symmetric N-S forcing, T42 resolution, 10 vertical levels.

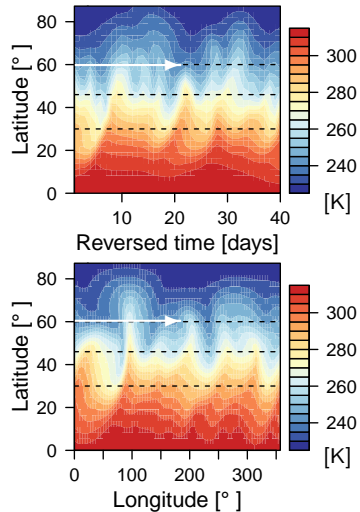
We obtain temporal and spatial rate functions by averaging the near-surface temperature in time and in space, respectively.

temporal averages  $A_{n_t}$  for increasing  
 $n_t = 1\tau_t, 5\tau_t, 10\tau_t, \dots$

temporal rate function estimates  $I_{n_t} = \frac{\ln p(A_{n_t})}{n_t/\tau_t}$

spatial averages  $A_{n_x}$  for increasing  
 $n_x = 1\tau_x, 5\tau_x, 10\tau_x, \dots$

spatial rate function estimates  $I_{n_x} = \frac{\ln p(A_{n_x})}{n_x/\tau_x}$

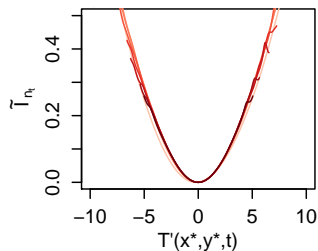


Galfi et al., 2018



The temporal and spatial rate functions converge in PUMA.

temporal

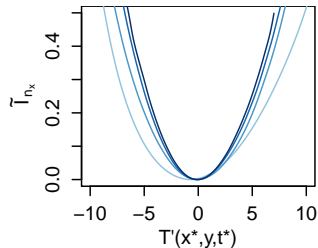


$$\begin{aligned} & n_t = 5\tau_t \\ & n_t = 10\tau_t \\ & \vdots \\ & n_t = 40\tau_t \end{aligned}$$

Optimal averaging length  $n^* \approx 20\tau$ .

Predictive power for averages over  $n > n^*$ .

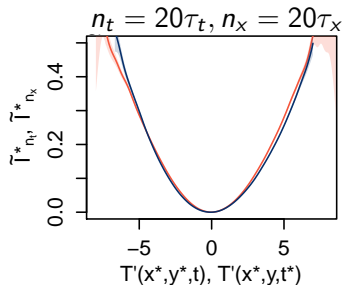
spatial



$$\begin{aligned} & n_x = 5\tau_x \\ & n_x = 10\tau_x \\ & \vdots \\ & n_x = 20\tau_x \end{aligned}$$

How about the connection between temporal and spatial large deviations?

The universal function describes both temporal and spatial large deviations.



Rate functions re-normalized based on the integrated auto-correlation:

$$I_{n_t} = \frac{\ln p(A_{n_t})}{n_t/\tau_t}, I_{n_x} = \frac{\ln p(A_{n_x})}{n_x/\tau_x}$$

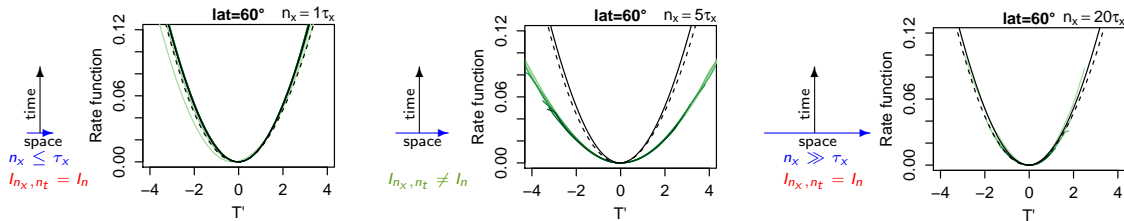
Temporal and spatial (re-normalized) rate functions are equal.  
 $\Rightarrow$  Universal function  $I_n$ .

Galfi et al., 2018

Problem: Valid for large  $n$ , persistent events are averaged out.

Idea: average in 2 dimensions, 1. in space and then 2. in time.

We capture heat waves (cold spells) if we average over intermediate spatial scales and then in time.



Temperature averaged over intermediate spatial scales:

Probabilities of large deviations are higher than in the universal case (black curves), due to the non-trivial spatial correlations.

We can study heat waves (cold spells) as large deviations of temperature averaged spatially over large, but not too large, synoptic scales.

How do things look like in an Earth system model?

# We analyse large deviations of surface temperature in the Earth system model MPI-ESM

We use the pre-industrial control runs (1000 years) of MPI-ESM-LR developed at the Max-Planck-Institute for Meteorology in Hamburg (part of the CMIP5 multi-model ensemble).

Model output downloaded from: <https://esgf-data.dkrz.de/search/cmip5-dkrz/>

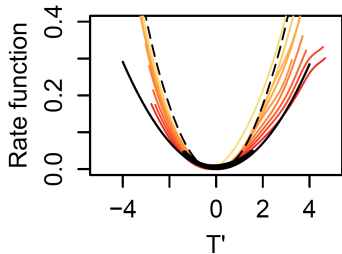
Unlike in case of the PUMA simulations, we have here a seasonal cycle, orography, and the whole Earth system.

We eliminate the seasonal cycle, and consider only summer temperature values.

Heat waves can be analysed as large deviations of local temperature averaged over adequate temporal scales.

We define heat waves as long lasting local events.

We average the surface temperature at one grid point in Europe (lon 22.5° E, lat 46° N) in time over consecutive daily values (until seasonal average) and estimate the rate function.



2 $\tau$	8 $\tau$	14 $\tau$
4 $\tau$	10 $\tau$	16 $\tau$
6 $\tau$	12 $\tau$	18 $\tau$

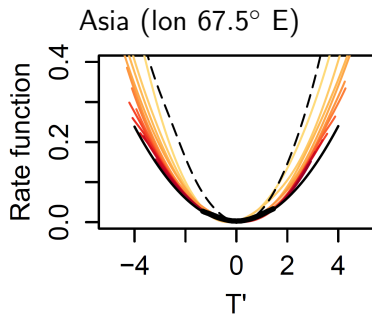
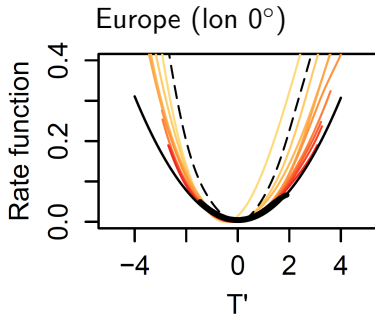
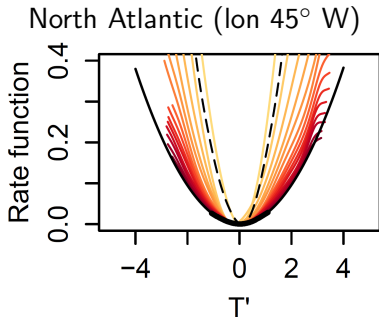
The rate function estimates do not converge, as an effect of serial correlation: compare the colored lines with the black dashed line (serial correlation eliminated).

Nonetheless, we obtain a rate function if we average additionally the seasonal averages of different years (solid black line).

Heat waves can be analysed as large deviations of local temperature averaged over adequate temporal scales.

Considering the local temperature in other locations (lat  $46^\circ$  N) we find that:

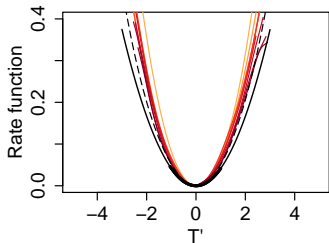
- Rate functions of seasonal averages are good estimates for the probability of persistent events on seasonal scale.
- Convergence of rate functions seems to be slower for summer temperature over the oceans.



Heat waves can be analysed as large deviations of spatially averaged temperature over adequate spatial scales.

We define heat waves as events with large spatial extension.

We take the European averaged temperature, and average it in time over consecutive daily values (until seasonal average) and estimate the rate function.



The rate function estimates converge very fast. Serial correlation seems to have almost no effect. Compare the colored lines with the black dashed line (serial correlation eliminated).

Through spatial averaging we have created an observable, which can be analysed directly as large deviation.



1. In the absence of orography, we find a **universal function** describing both **temporal and spatial** large deviations.
2. Heat waves are events with a **long duration**.  
Thus, they can be analysed as large deviations of temperature **averaged in time** over adequate **temporal scales**.
3. Heat waves are events with a **large spatial extension**.  
Thus, they can be analysed as large deviations of temperature **averaged in space** over adequate **spatial scales**.

By using LDT we can study persistent events  
in chaotic systems  
if we construct adequate observables.

The large deviation analysis in PUMA was published in:

Galfi, V.M., V. Lucarini, and J. Wouters (2018)

A Large Deviation Theory-based Analysis of Heat Waves and Cold Spells in a Simplified Model of the General Circulation of the Atmosphere, *J.Stat.Mech.*, 033404.



# References

- Barriopedro, D., E. Fischer, J. Luterbacher, R. Trigo, and R. Garcia-Herrera (2011) The hot summer of 2010: Redrawing the temperature record map of Europe *Science* 332, 220-224.
- Cramér, H. (1938) Sur un nouveau théorème limite dans la théorie des probabilités, *Colloque consacré à la théorie des probabilités* 3, 2-23.
- Ellis, R.S. (1985) Entropy, Large Deviations, and Statistical Mechanics, *Springer*, New York.
- Poumadere, M., C. Mays, S. Le Mer, and R. Blong (2005) The 2003 heat wave in France: dangerous climate change here and now *Risk. Anal.* 25, 1483-94.
- Touchette, H. (2009) The large deviations approach to statistical mechanics *Physics Reports* 478, 1-69.